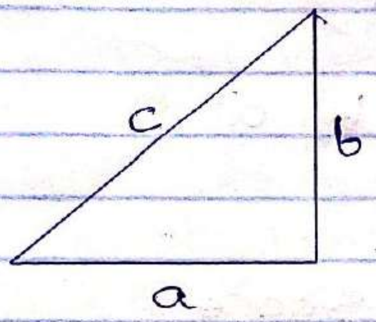
TRIGONOMETRY

It is a branch of mathematics which deals with the measurement of sides of triangles and how they are related to each other

Trigonometric ratios of acute angles

Consider a right angled triangle shown below



Using SOH CAH TOA

$\text{Sine }\theta = \frac{\text{opposite}}{\text{hypotenuse}} \Rightarrow \sin\theta = \frac{b}{c}$

$\text{Cosine }\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \Rightarrow \cos\theta = \frac{a}{c}$

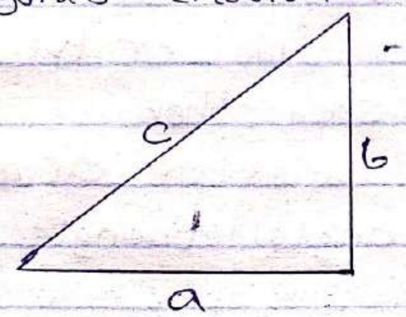
$\text{Tan }\theta = \frac{\text{opposite}}{\text{adjacent}} \Rightarrow \tan\theta = \frac{b}{a}$

$\text{Sec }\theta = \frac{1}{\cos\theta} \Rightarrow \sec\theta = \frac{c}{a}$

$\text{Cosec }\theta = \frac{1}{\sin\theta} \Rightarrow \text{cosec}\,\theta = \frac{c}{b}$

$\text{Cot }\theta = \frac{1}{\tan\theta} \Rightarrow \cot\theta = \frac{a}{b}$

Using Pythagoras' theorem to the triangle shown below



i) $a^2+b^2=c^2$

divide through by a^2

$$\frac{a^2}{a^2}+\frac{b^2}{a^2}=\frac{c^2}{a^2}$$

$$1+\tan^2\theta = \sec^2\theta$$

ii) $a^2+b^2=c^2$

divide through by b^2

$$\frac{a^2}{b^2}+\frac{b^2}{b^2}=\frac{c^2}{b^2}$$

$$\cot^2\theta+1=\cos^2\theta$$

iii) $a^2+b^2=c^2$

divide through by c^2

$$\frac{a^2}{c^2}+\frac{b^2}{c^2}=\frac{c^2}{c^2}$$

$$\cos^2\theta+\sin^2\theta=1$$

Negative Identities

i) $\cos(-x)=\cos x$

Cos of a positive angle equal to Cos of negative angle

ii) $\sin(-x)=-\sin x$

iii) $\tan(-x)=-\tan x$

Cofunction Identities

① ~~Sin (90 − x)~~

Compound angle Formular

① $\sin(A+B) = \sin A\cos B + \cos A\sin B$

② $\sin(A-B) = \sin A\cos B - \cos A\sin B$

③ $\cos(A+B) = \cos A\cos B - \sin A\sin B$

4 $\cos(A-B) = \cos A\cos B + \sin A\sin B$

5 $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A + \tan B}$

6 $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A + \tan B}$

Example

① Solve: $\sin(90-x)$

Solution

From Compound angles

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\sin(90-x) \therefore = \sin 90\cos(-x) - \cos 90\sin(-x)$

$= \underline{\cos(-x)}$ or $\underline{\cos x}$

because Cos of a negative angle is equal to cos of a positive angle

ii) Solve $\cos(90-x)$

From Compound angles

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\cos(90-x) = \cos 90\cos(-x) + \sin 90\sin(-x)$

$= \sin x$

iii) Solve $\tan(90-x)$

$\tan = \dfrac{\sin}{\cos}$

therefore $\tan(90-x) = \dfrac{\sin(90-x)}{\cos(90-x)} = \dfrac{\cos x}{\sin x} \Rightarrow \underline{\underline{\cot x}}$

Worked Problems on trigonometric Identities

Prove the following Identities

① $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$

$(\cos^2\theta)^2 - (\sin^2\theta)^2$

From trigonometry

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

replace $\cos^2\theta$ with $1-\sin^2\theta$

$(1-\sin^2\theta)^2 - (\sin^2\theta)^2$

expand

$(1-\sin^2\theta)(1-\sin^2\theta)$

$1 - \sin^2\theta - \sin^2\theta + (\sin^2\theta)^2$

equit it back

$1 - 2\sin^2\theta + (\sin^2\theta)^2 - (\sin^2\theta)^2$

$= 1 - 2\sin^2\theta$

but

$2\sin^2\theta = 1 - \cos 2\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$

therefore $\cos 2\theta$ from the table

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ hence proved

B) $\cot\theta + \tan\theta = \dfrac{1}{\sin\theta\cos\theta}$

$\cot\theta = \dfrac{1}{\tan}$ $\quad \tan = \dfrac{\sin\theta}{\cos\theta}$

$\dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta}$ $\quad$ $\dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$

lcm

$\cos^2\theta + \sin^2\theta = 1$

$\therefore \dfrac{1}{\sin\theta\cos\theta}$ hence proved

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Trigonometric ratios of angles within different quadrants

All Student take chemistry

Trigonometric ratio of angles within second quadrant in terms of trigonometric ratios of accute angle

If x is in the second quadrant then

i) $\sin x = \sin(180 - x)$

ii) $\cos x = -\cos(180 - x)$

iii) $\tan x = -\tan(180 - x)$

If x is in the third quadrant then

i) $\sin x = -\sin(x - 180)$

ii) $\cos x = -\cos(x - 180)$

iii) $\tan x = \tan(x - 180)$

If x is in the Forth quadrant then

i) $\sin x = -\sin(360 - x)$

ii) $\cos x = \cos(360 - x)$

iv) $\tan x = -\tan(360 - x)$

Example 1

① Express the following trigonometric ratios interms of trigonometric ratio of accute angles

i) $\sin 120^\circ$

Solution

It is in the second quadrant

$\sin x = \sin(180 - x)$

$\sin x = \sin(180 - 120)$

$= \underline{\sin 60^\circ}$

ii) $-\cos(180 - 140)$

$= -\cos(180 - x)$

$= \underline{-\cos 40^\circ}$

iii) tan 535

Soln

$\tan(535-360)$

$\tan 175$

$\tan(175-180)$

$-\tan 5$

iv) $\sin 330^0$

Soln

$-\sin(360-x)$

$-\sin(360-330)$

$-\sin 30^0$

v) $\sin(-669^0)$

$-\sin(669^0-360)$

$-\sin 309$

$-\sin(360-309)$

$-\sin 51$

Example 2

Express the following into a single trigonometric ratio

i) $\sin 34 \cos 15^0 + \cos 34 \sin 15^0$

Soln

From compound angle

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(34+15)$

$\sin 49$

ii) $\cos 14 \cos 69^0 + \sin 14^0 \sin 69^0$

Soln

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\cos 14-69$

$\cos -55$ or $\cos 55$

iii) $\cos 47^\circ \cos 20^\circ - \sin 47 \cos 20^\circ$

Soln

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(47+20)$

$\underline{\underline{\cos 67}}$

iv) $\dfrac{\tan 20^\circ + \tan 30^\circ}{1 - \tan 20^\circ \tan 30^\circ}$

Soln

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan \cancel{A} \tan B}$

$\tan(20+30)$

$\underline{\underline{\tan 50}}$

Example

Given that angle A and B are accute and $\sin A = \frac{24}{25}$ and $\cos B = \frac{8}{17}$ without using a calculator determine

i) $\sin(A-B)$

ii) $\sec(A+B)$

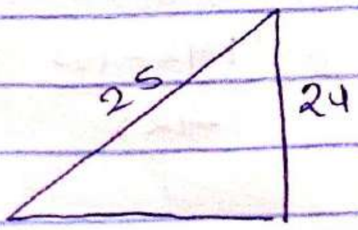
iii) $\cot(A+B)$

Soln

i) $\sin(A-B)$

SOH CAH TOA

$\sin A = \frac{opp}{Hy} \quad \frac{24}{25}$



$\sqrt{25^2 - 24^2}$

$\sqrt{49} = 7$

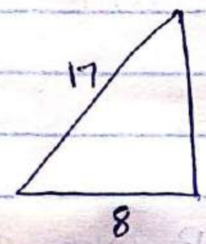
$$\cos_A = \frac{Adj}{Hyp} \quad \frac{7}{25}$$

$$\sin_A = \frac{24}{25}$$

$$\cos B = \frac{adj}{Hyp}$$

$$\cos B = \frac{8}{17}$$

Using pythagoras theorem



$$\sqrt{17^2 - 8^2} = \sqrt{225}$$

$$= 15$$

$$\sin = \frac{opp}{Hyp} \quad \frac{15}{17}$$

$$\therefore \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\sin\left(\frac{24}{25}\right) \times \cos\left(\frac{8}{17}\right) - \cos\left(\frac{7}{25}\right) \times \sin\left(\frac{15}{17}\right)$$

$$\frac{24}{25} \times \frac{8}{17} - \frac{7}{25} \times \frac{15}{17}$$

$$\frac{192}{425} - \frac{105}{425} \quad \frac{192-105}{425} = \frac{87}{425}$$

ii) Sec (A+B)

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\frac{1}{}$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\frac{1}{\cos\left(\frac{7}{25}\right) \times \cos\left(\frac{8}{17}\right) - \sin\left(\frac{24}{25}\right) \cdot \times \sin\left(\frac{15}{17}\right)}$$

$$\frac{1}{\left(\frac{7}{25} \times \frac{8}{17}\right) - \left(\frac{24}{25} \times \frac{15}{17}\right)}$$

$$\frac{1}{\frac{56}{425} - \frac{72}{85}} \quad \frac{1}{\frac{-304}{425}}$$

$$1 \div \frac{-304}{425}$$

$$1 \times \frac{-425}{304} = \frac{-425}{304}$$

iii) Cot (A+B)

$$\cot\theta = \frac{1}{\tan\theta}$$

$$= \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\frac{1}{= \frac{\sin\theta}{\cos\theta}} \Rightarrow \frac{\cos\theta}{\sin\theta}$$

$$\cot \in \frac{\cos(A+B)}{\sin(A+B)}$$

$\cos(A+B) = \frac{-304}{425}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin\left(\frac{24}{25}\right) \times \cos\left(\frac{8}{17}\right) + \cos\left(\frac{7}{25}\right) \times \sin\left(\frac{15}{17}\right)$

$\left(\frac{24}{25} \times \frac{8}{17}\right) + \left(\frac{7}{25} + \frac{15}{17}\right)$

$\frac{192}{425} + \frac{21}{85} \quad \frac{297}{425}$

$\therefore \frac{-304}{425} \times \frac{425}{297} = \frac{-304}{297}$

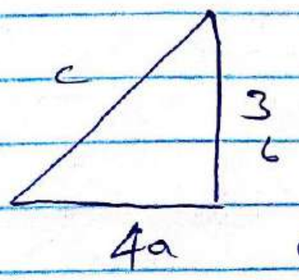
Example

Given that $\tan\theta = \frac{3}{4}$ determine 5 other trigonometric ratios of θ

We know that $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Using SOH CAH TOA

$\text{Tan} = \frac{\text{OPP}}{\text{ADJ}} \quad \frac{3}{4}$



$a^2 + b^2 = c^2$

$4^2 + 3^2 = \sqrt{25}$

$= \underline{\underline{5}}$

i) $\text{Sin} = \frac{\text{OPP}}{\text{HYP}} \quad \underline{\underline{\frac{3}{5}}}$

ii) $Cos = \frac{Adj}{HPY}$ $\frac{4}{5}$

iii) $Sec = \frac{1}{Cos}$ $\frac{1}{\frac{4}{5}} \Rightarrow \frac{5}{4}$

iv) $Csc = \frac{1}{Sin} = \frac{1}{\frac{3}{5}} \Rightarrow \frac{5}{3}$

v) Cot (θ)

$Cot = \frac{1}{tan}$ $\frac{1}{\frac{3}{4}} \Rightarrow \frac{4}{3}$

Example

Solve the Following Equation

① $Cos2\theta + Cos\theta + 1 = 0$ For $0^0 < \theta \le 360^0$

From the table

$2cos^2\theta = 1 + cos2\theta$

$cos2\theta = 1 - 2cos^2\theta$

replace $cos2\theta$ with $1 - 2cos^2\theta$

$1 - 2cos^2\theta + cos\theta + 1 = 0$

$2cos^2\theta + cos\theta = 0$

$cos\theta[2cos\theta + 1] = 0$

either $cos\theta = 0$ or $2cos\theta + 1 = 0$

$\theta = cos^{-1}(0)$ $\frac{2cos\theta}{2} = \frac{-1}{2}$

$\theta = 90$

$\theta = cos^{-1}(-\frac{1}{2})$

$\theta = 120$

NB Important Formulae to get angles over range

For $cos\theta$: $\Rightarrow \theta = 360n \pm \theta$

$sin\theta$: $\Rightarrow \theta = 180n + (-1)^n \times \theta$

$tan\theta$: $\Rightarrow \theta = 180n + \theta$

For $\cos\theta = 0$

For $\theta = 90^\circ \rightarrow$ Principal Value

$\theta = 360n \pm 90$

$n=0 \quad \theta = 90, -90$

$n=1 \quad \theta = 450; 270$

$n=2 \quad \theta = 810, 630$

$\theta = \{90, 270\}$

For $\theta = 120 \rightarrow$ Principal Value

$\theta = 360n \pm 120$

$n=0 \quad \theta = 120, -120$

$n=1 \quad \theta = 480, 240$

$n=2 \quad \theta = 840, 600$

$\theta = \{120, 240\}$

$\theta = \{90^\circ, 120^\circ, 240^\circ, 270^\circ\}$

(2) $\sin 2\theta + \cos\theta = 0$ For $0^\circ \leq \theta \leq 360$

From the table

$\sin 2\theta = 2\sin\theta\cos\theta$

replace $\sin 2\theta$

$2\sin\theta\cos\theta + \cos\theta = 0$

$\cos\theta[2\sin\theta + 1] = 0$

$\cos\theta = 0$ or $2\sin\theta + 1 = 0$

$\frac{2\sin\theta}{2} = \frac{-1}{2}$

$\theta = \cos^{-1} = \theta$

$\theta = 90 \rightarrow$ Principal Value $\quad \sin\theta = -\frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{-1}{2}\right)$

$= -30 \rightarrow$ Principal Value

For $\cos 90$

$\theta = 360n \pm 90$

$n=0 \quad \theta = 90, -90$

$n=1 \quad \theta = 450, 270$

$\theta = \{90, 270\}$

For $\sin -30$

$\theta = 180n + (-1)^n \times \theta$

$n=0 \quad \theta = -30$

$n=1 \quad \theta = 210$

$n=2 \quad \theta = 330$

$\theta = [210, 330]$

$\therefore \theta = \{90^\circ, 210^\circ, 270^\circ, 330\}$

3 $\sec^2\theta + 5\tan\theta = 7$ For $0^\circ \leq \theta \leq 360$

From the table

$1 + \tan^2\theta = \sec^2\theta$

replace $\sec^2\theta$

$1 + \tan^2\theta + 5\tan\theta = 7$

$\tan^2\theta + 5\tan\theta = 6$

$\tan^2\theta + 5\tan\theta - 6 = 0$

Let $\tan\theta$ be t

$t^2 + 5t - 6 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\cancel{\theta}a = t^2, \ b = 5t \quad c = 6$

$\frac{-5 \pm \sqrt{25 - 4(1 \times -6)}}{2 \times 1}$

$\frac{-5 \pm 7}{2} \qquad \frac{-5+7}{2} \quad \text{or} \quad \frac{-\cancel{5}-7}{2}$

$1 \quad \text{or} \quad -6$

$t = 1 \quad \text{or} \quad -6$

$\tan\theta = 1 \qquad \text{or} \qquad \tan\theta = -6$

$\theta = \tan^{-1}(1) \qquad \theta = \tan^{-1}(-6)$

$\theta \doteq 45^\circ \rightarrow P.V \qquad -80.54^\circ \rightarrow P.V$

~~For $\tan\theta = 1$~~

~~$\theta = \tan^{-1}(1)$~~

$\theta = 180n + \theta$

$\theta = 180n + 45$

When

$n = 0 \quad \theta = 45$

$n = 1 \quad \theta = 225$

$\theta \{45, 225\}$

$\theta = 180n + (-80.54)$

When

$n = 0 \quad \theta = -80.54$

$n = 1 \quad \theta = 99.46$

$n = 2 \quad \theta = 279.46$

$\theta \{99.46^\circ, 279.46\}$

$\theta \ [45^\circ, 99.46, 225^\circ, 279.46\}$

(4) $\cot\theta + 7\csc^2\theta = 15$ For $0^\circ \leq \theta \leq 360$

From the table

$\csc^2\theta = 1 + \cot^2\theta$

replace

$\cot\theta + 7[1 + \cot^2\theta] = 15$

$\cot\theta + 7 + 7\cot^2\theta = 15$

$7\cot^2\theta + \cot\theta = 8$

$7\cot^2\theta + \cot\theta - 8 = 0$.

let $t = \cot\theta$

$7t^2 + t - 8 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = 1$ or $t = -\frac{8}{7}$

$\cot\theta = 1$

$\frac{1}{\tan\theta} = 1$

$1 = \tan\theta$

$\theta = \tan^{-1}(1)$

$\theta = 45^\circ$ P.V

Taking $t = -\frac{8}{7}$

$\cot\theta = -\frac{8}{7}$

$\frac{1}{\tan\theta} = -\frac{8}{7}$

$7 = -8\tan\theta$

$\tan\theta = -\frac{7}{8}$ $\quad \theta = \tan^{-1}\left(\frac{-7}{8}\right)$

$\theta = -41.2^\circ$

For tan $45°$ | For tan $-41.2°$
$\theta = 180n + \theta$ | $\theta = 180n + (-41.2)$
$n=0$ $\theta = 45$ | $n = 0$ $\theta = -41.2$
$n=1$ $\theta = 225$ | $n=1$ $\theta = 138.8$
$n=2$ $\theta = 405$ | $n=2$ $\theta = 318.8$

θ [$45°, 138.8°, 225°, 318.8°$]

Solution of tringonometric equation of the form
$A\cos\theta + B\sin\theta = C$

Example1

i) Express $4\cos\theta + 3\sin\theta$ in the form $R\sin(\theta+\alpha)$ where $R > 0$ $0° \leq \alpha \leq 90°$

ii) Hence solve the equation

$4\cos\theta + 3\sin\theta = 4.8$ For $0° \leq \theta \leq 360°$

Solun

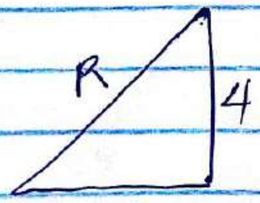
Expand $R\sin(\theta+\alpha)$

$4\cos\theta + 3\sin\theta = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

SOHCAHTOA

$\frac{4}{R} = \frac{R\sin\alpha}{R}$ $\sin\alpha = \frac{4}{R}$

$\frac{3}{R} = \frac{R\cos\alpha}{R}$ $\cos\alpha = \frac{3}{R}$



$R = \sqrt{3^2 + 4^2}$

$R = 5$

$\alpha \tan^{-1}\left(\frac{4}{3}\right) = 53.13$

$4\cos\theta + 3\sin\theta = 5\sin(\theta + 53.13^\circ)$

ii) $4\cos\theta + 3\sin\theta = 4.8$ For $0^\circ \le \theta \le 360^\circ$

$5\sin(\theta + 53.13^\circ) = 4.8$

$\frac{5\sin}{5} = \frac{4.8}{5}$ $\sin = 0.96$

$\theta + 53.13 = \sin^{-1}(0.96)$

$\theta + 53.13 = 73.74$

$\theta = 73.74 - 53.13$

$\theta = 20.61^\circ$

For $\sin 73.74$ $\theta = 180n + (-1)^n \times \alpha$

$n = 0$ $= 20.61$

$n = 1$ $= 106.26$

$n = 2$ $= 433.74$

$\theta = [20.61^\circ, 53.13^\circ]$

Example

1. Express $5\cos\theta - 12\sin\theta$ in the Form $R\cos(\theta + \alpha)$ Where $R > 0$ and $0^\circ \le \alpha \le 90^\circ$.

ii) Hence solve the equation

$5\cos\theta - 12\sin\theta = 6$ For $0^\circ \le \theta \le 360^\circ$

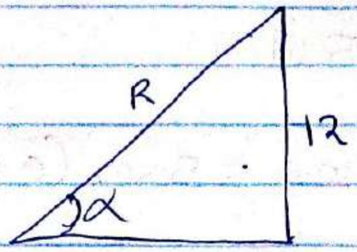
Soln

Expand $R\cos(\theta + \alpha)$

$5\cos\theta - 12\sin\theta = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

$\frac{5}{R} = \frac{R\cos\alpha}{R}$ $\cos\alpha = \frac{5}{R}$

$\frac{12}{R} = \frac{R\sin\alpha}{R}$ $\sin\alpha = \frac{12}{R}$



$$R = \sqrt{5^2+12^2}$$
$$= 13$$
$$\alpha = \tan^{-1}\left(\frac{12}{5}\right) = \underline{\underline{67.38^0}}$$

$$5\cos\theta - 12\sin\theta = 13\cos(\theta + 67.38^0)$$

ii) $5\cos\theta - 12\sin\theta = 6$

$$13\cos(\theta + 67.38^0) = 6$$
$$\cos(\theta + 67.38^0) = \frac{6}{13}$$
$$\theta + 67.38^0 = \cos^{-1}\left(\frac{6}{13}\right)$$
$$\theta + 67.38^0 = 62.51^0$$

$$\theta = 62.51^0, 297.49^0$$
$$\theta = (62.51^0 - 67.38^0), (297.49^0 - 67.38^0)$$
$$\theta (-4.87^0), 230.11^0$$
$$\theta = \underline{230.11^0}$$

USE OF T-Formulae in solving equation of the form

$A\cos\theta + B\sin\theta = C$

If $t = \frac{1}{2}\theta$ then $\sin\theta = \frac{2t}{1+t^2}$ $\cos\theta = \frac{1-t^2}{1+t^2}$ $\tan\theta = \frac{2t}{1-t^2}$

$$\frac{d\theta}{dt} = \frac{2}{1+t^2}$$

Example 1 :

Use t - Formulae to solve equation

$2\cos\theta + 3\sin\theta = 2$ For $0° \leq 360°$

let $t = \tan\frac{\theta}{2}$

$$2\left(\frac{1-t^2}{1+t^2}\right) + 3\left(\frac{2t}{1+t^2}\right) = 2$$

$$\frac{2-2t^2}{1+t^2} + \frac{6t}{1+t^2} = 2$$

$$1+t^2 \times \frac{2-2t^2+6t}{1+t^2} = 2 \times 1+t^2$$

$$2-2t^2+6t = 2+2t^2$$

$$-4t^2+6t = 0$$

$$2t[-2t+3] = 0$$

$2t = 0$ or $-2t+3 = 0$

$t = \frac{0}{2}$ $\quad t = \frac{-3}{-2}$

but $t = \tan\frac{\theta}{2}$ $\quad t = \frac{3}{2}$

or $\tan^{-1} = 1.5$

$\tan^{-1}(0) = 0$ $\quad = 56.31$

For tan

$180n + \theta$ $\quad n = 0 = 56.31$

$n = 0 \quad 0$ $\quad n = 1 = 236.31$

$n = 1 \quad 180$ $\quad n = 2 = 416.31$

$n = 2 \quad 360$

$2t = \tan\theta$

$\theta = [0, 360, 720, 112.62, 472.26]$

$\theta\{0, 112.62, 360\}$

Example 2

Use the t- formulae to solve the equation

$3\cos\theta + 4\sin\theta = 4.9$ For $0^\circ \leq \theta \leq 360^\circ$

Soln

let $t = \tan\frac{\theta}{2}$

$2t = \tan\theta$

$\sin\theta = \frac{2t}{1+t^2}$

$\cos\theta = \frac{1-t^2}{1+t^2}$

$3\left(\frac{1-t^2}{1+t^2}\right) + 4\left(\frac{2t}{1+t^2}\right) = 4.9$

$\frac{3-3t^2}{1+t^2} + \frac{8t}{1+t^2} = 4.9$

$1+t^2 \times \frac{3-3t^2+8t}{1+t^2} = 4.9 \times 1+t^2$

$3 - 3t^2 + 8t = 4.9 + 4.9t^2$

$-1.9 - 7.9t^2 + 8t = 0$

$-7.9t^2 + 8t - 1.9 = 0$

divide by -

$7.9t^2 - 8t + 1.9 = 0$

Using

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$t = 0.6323$ or 0.3834

$\tan\frac{\theta}{2} = 0.6323$ or $\tan\frac{\theta}{2} = 0.3834$

$\frac{\theta}{2} = \tan^{-1} 0.6323$ or $\frac{\theta}{2}$ $\tan^{-1} 0.3834$

32.3° 20.98

$\tan\theta = 180n + \theta$

$32.3^\circ, 212.3$ or $20.98, 200.98$

but $2t = \tan\theta$

$(64.6, 41.96)$

FACTOR - FORMULAE

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ ----(i)

$\sin(A-B) = \sin A\cos B - \cos A\sin B$ ----(ii)

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ ----(iii)

$\cos(A-B) = \cos A\cos B + \sin A\sin B$ ----(iv)

Add (i) to (ii)

$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$ ----(v)

Subtract (ii) from (i)

$\sin(A+B) - \sin(A-B) = 2\cos A\sin B$ --(vi)

Add (iii) to (iv)

$\cos(A+B) + \cos(A-B) = 2\cos A\cos B$ ---(vii)

Subtract (iii) from (iv)

$\cos(A+B) - \cos(A+B) = 2\sin A\sin B$ (viii)

let $P = A+B$

$Q = A-B$

$P+Q = 2A$

$\therefore A = \frac{P+Q}{2}$

and $P-Q = 2B$

$B = \frac{P-Q}{2}$

Substitute for A and B in equation (v), (vi), (vii) and (viii)

v) $\sin P + \sin Q = 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$

vi) $\sin P - \sin Q = 2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}$

vii) $\cos P + \cos Q = 2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$

vii $\cos q - \cos p = 2\sin\frac{p+q}{2}\sin\frac{p-q}{2}$

Example 1

Solve the equation

$\cos 2\theta + \cos 4\theta = 0$ For $0^{\circ} \le \theta \le 180^{\circ}$

Soln

$2\cos\frac{4\theta+2\theta}{2}\cos\frac{4\theta-2\theta}{2} = 0$

$2\cos\frac{6\theta}{2}\cos\frac{2\theta}{2} = 0$

$2\cos 3\theta \cos\theta = 0$

$2\cos 3\theta = 0$ or $\cos\theta = 0$

$\cos 3\theta = \frac{0}{2}$ $\quad \cos^{-1} = 0$

$= 90$

$3\theta = \cos^{-1} 0$

$3\theta = 90$

$\cos\theta : \theta = 360n \pm \theta$

$n=0$	90, -90
$n=1$	450, 270
$n=2$	810 630
$n=3$	1170 990

3θ [90, 270, 450, 630 810, 990 1170]

θ [30, 90 150]

Example 2

Solve the equation

$\sin\theta + \sin 3\theta + \sin 5\theta = 0$

Soln

$\sin p + \sin q = 2\sin\frac{p+q}{2}\cos\frac{p-q}{2}$

$2\sin\frac{5\theta+\theta}{2}\cos\frac{5\theta-\theta}{2}$

$2\sin 3\theta \cos 2\theta + \sin 3\theta = 0$ For $0^\circ \le \theta \le 360$

$\sin 3\theta [2\cos 2\theta + 1] = 0$

$\sin 3\theta = 0$ or $2\cos 2\theta + 1 = 0$

$3\theta = \sin(0)$ or $2\cos 2\theta = -1$

$2\cos 2\theta = -1$

$3\theta = 0$ $\cos 2\theta = \left(\frac{-1}{2}\right)$

$2\theta = 120$

$\theta = 180n + (-1)^n \times \theta$ $\theta = 360n \pm \theta$

$n = 1$ 180 $n = 0$ 120

$n = 2$ 360 $n = 1$ 480

$n = 3$ 540

$n = 4$ 720

3θ [180, 360, 540, 720] 2θ [120, 480]

θ [60, 120, 180, 240, 300, 360]

Example 10

Solve the equation

$\sin 2x + \sin 3x = 0$ For $0^\circ \le x \le 360^\circ$

$2\sin \frac{3x + 2x}{2} \cos \frac{3x - 2x}{2} = 0$

$2\sin \frac{5x}{2} \cos \frac{x}{2} = 0$

$2\sin 2.5x = 0$ or $\cos \frac{x}{2} = 0$

$\sin 2.5x = \frac{0}{2}$ or $\cos \frac{x}{2} = 0$

$2.5x = \sin^{-1}(0)$ or $\frac{x}{2} = \cos^{-1}(0)$

$2.5x = 0$ $\frac{x}{2} = 90$

Sin		Cos
$180n+(-1)^n \times \theta$		$360n \pm \theta$
$n=0$	0	90, -90
$n=1$	180	450, 270
$n=2$	360	810, 630
$n=3$	540	1170, 990
$n=4$	720	
$n=5$	900	

$x\,[0°; 72°, 144°, 180°, 216°, 288°, 360°]$

Example

Solve the equation

$\cos 6x + \cos 4x + \cos 2x = 0$ For $0° \leq x \leq 180°$

Soln

$2\cos\frac{P+q}{2}\cos\frac{P-q}{2}$

$2\cos\frac{6x+2x}{2}\cos\frac{6x-2x}{2}$

$2\cos 4x \cos 2x + \cos 4x = 0$

$\cos 4x\,[2\cos 2x + 1] = 0$

$\cos 4x = 0$ or $\frac{2\cos 2x}{2} = \frac{-1}{2}$

$4x = \cos^{-1}(0)$ or $2x = \cos^{-1}(0.5)$

$360n \pm 0$

$4x = 90$ | $2x = 120$

$n=0$	90, -90		$n=0$	120, -120
$n=2$	450, 270		$n=1$	480 240
$n=3$	810, 630		$n=2$	840 600
$n=4$	1170 990			
$n=5$	1440 1350			

$x = [22.5°, 60°, 67.5°, 112.5°, 157.5°, 120°]$

Example 3

If A, B and C are angles of a triangle show that
$\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

$A + B + C = 180$

$B + C = 180 - A$

$\frac{B+C}{2} = 90 - \frac{A}{2}$

$\cos\left(\frac{B+C}{2}\right) = \cos\left(90 - \frac{A}{2}\right)$

$= \sin\frac{A}{2}$

$\sin\left(\frac{B+C}{2}\right) = \sin\left(90 - \frac{A}{2}\right)$

$= \cos\frac{A}{2}$

$\sin A + \sin B + \sin C = \sin A + 2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)$

replace

$= \sin A + 2\cos\frac{A}{2}\cos\left(\frac{B-C}{2}\right)$

$= 2\sin\frac{A}{2}\cos\frac{A}{2} + 2\cos\frac{A}{2}\cos\left(\frac{B-C}{2}\right)$

$2\cos\frac{A}{2}\left\{\sin\frac{A}{2} + \cos\left(\frac{B-C}{2}\right)\right\}$

$2\cos\frac{A}{2}\left\{2\cos\frac{\frac{B}{2}+\frac{C}{2}+\frac{B}{2}-\frac{C}{2}}{2}\right.$

$\left.\cos\left(\frac{\frac{B}{2}+\frac{C}{2}-\frac{B}{2}+\frac{C}{2}}{2}\right)\right\}$

$4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ hence shown

Example

Given that A, B, and C are angles of a triangle show that

$\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

$A + B + C = 180$

$B + C = 180 - A$

$\sin B + C = \sin 180 - A$

$\sin A$

$\cos(B + C) = \cos(180 - A)$

$= \cos A$

$\sin 2A + \sin 2B + \sin 2C = \sin 2A + 2 \sin \frac{1}{2}(2B + 2C) \cos \frac{1}{2}(2B - 2C)$

$= \sin 2A + 2 \sin(B + C) \cos(B - C)$

$2 \sin A \cos A + 2 \sin A \cos(B - C)$

$2 \sin A [\cos A + \cos(B - C).]$

$2 \sin A [-\cos(B + C) + \cos(B - C)]$

$2 \sin A [2 \sin \frac{1}{2}(B - C + B + C) \sin \frac{1}{2}(B + C - B + C)]$

$2 \sin A [2 \sin \frac{1}{2}(2B) \sin \frac{1}{2} 2C]$

$2 \sin A [2 \sin B \sin C]$

$\underline{\underline{4 \sin A \sin B \sin C}}$