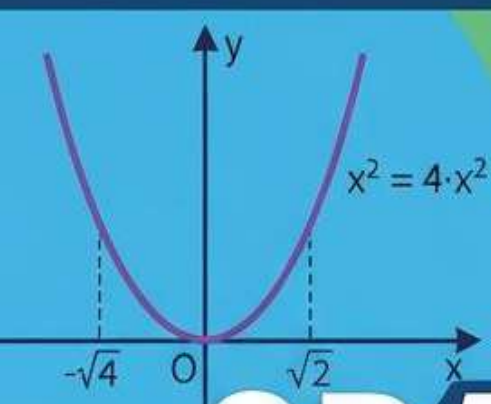
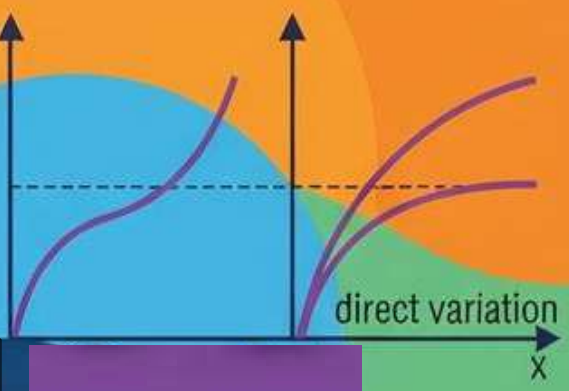
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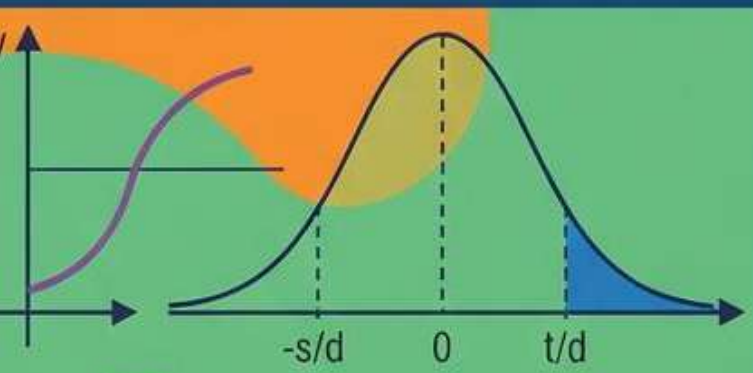


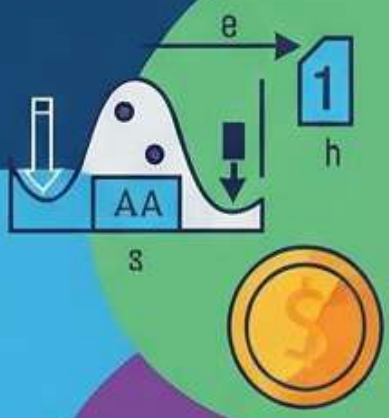
GRADE 10 ESSENTIAL MATHEMATICS NOTES





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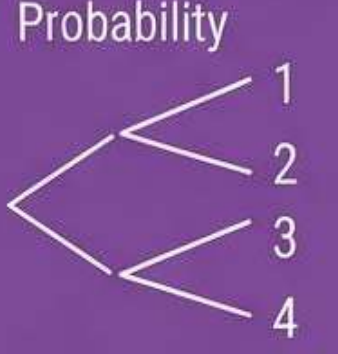












AUTHORS: COMPETENCE EDUCATION GROUP

STRAND 1.0: NUMBERS AND ALGEBRA

SUB-STRAND 1.1: QUADRATIC EQUATIONS

1.0 Introduction to quadratic equations

A quadratic equation is an algebraic equation in which the highest power of the unknown variable is two.

Quadratic equations are useful in solving mathematical problems involving areas, dimensions, heights, distances, profit, motion and other real-life situations.

In Kenya, quadratic equations can be applied in:

1. Determining the dimensions of rectangular farms and school gardens.
2. Calculating the maximum height reached by a ball thrown into the air.
3. Designing curved structures such as arches and bridges.
4. Determining the dimensions of buildings and construction materials.
5. Solving problems involving areas and land measurement.
6. Analysing business situations involving revenue and profit.

Definition

A quadratic equation is an equation that can be expressed in the general form:

$$ax^2 + bx + c = 0$$

Where:

- a is the coefficient of x^2.
- b is the coefficient of x.
- c is the constant term.
- x is the unknown variable.
- a, b and c are real numbers.
- $a \neq 0$.

The condition $a \neq 0$ is important because if $a = 0$, the equation ceases to be quadratic.

1.1.1 Examples of quadratic equations

The following are examples of quadratic equations:

1. $x^2 + 5x + 6 = 0$
2. $2x^2 - 7x + 3 = 0$
3. $3x^2 + 4x - 8 = 0$
4. $5x^2 - 20 = 0$
5. $x^2 - 16 = 0$

Identifying the coefficients

Consider the equation:

$$3x^2 + 7x - 10 = 0$$

Comparing it with $ax^2 + bx + c = 0$:

- $a = 3$
- $b = 7$
- $c = -10$

Important: Always include the sign of each coefficient when identifying a, b and c.

1.1.2 Equations that are not quadratic

The following equations are not quadratic:

Equation	Reason
$3x + 5 = 0$	Highest power is 1
$x^3 + 2x = 0$	Highest power is 3
$4x^2 + 3x = 7$	Can be quadratic after rearranging
$5x^4 - 2 = 0$	Highest power is 4

Notice that $4x^2 + 3x = 7$ is quadratic because it can be rearranged into:

$$4x^2 + 3x - 7 = 0$$

2.0 Formation of quadratic equations

2.1 Meaning of forming quadratic equations

Formation of quadratic equations involves translating real-life situations, statements or mathematical relationships into algebraic equations of the form:

$$ax^2 + bx + c = 0$$

This requires identifying the unknown quantity, expressing the given information algebraically and simplifying the resulting expression.

2.2 Steps followed when forming quadratic equations

The following systematic procedure can be used:

1. Read the problem carefully.
2. Identify the unknown quantity.
3. Represent the unknown quantity using a variable, usually x.
4. Express other quantities in terms of x.
5. Translate the given relationship into a mathematical equation.
6. Expand brackets where necessary.
7. Collect like terms.
8. Rearrange the equation into the standard quadratic form.
9. Check whether the equation correctly represents the given situation.

2.3 Formation of quadratic equations from real-life situations

Example 1: Rectangular school garden

A school in Nakuru County plans to establish a rectangular vegetable garden. Its length is 5 metres longer than its width. The area of the garden is 84 square metres. Form a quadratic equation to represent the situation.

Solution

Step 1: Let the width be x metres.

Step 2: Length is 5 metres longer than width.

$$\text{Length} = x + 5$$

Step 3: Recall that:

$$\text{Area} = \text{Length} \times \text{Width}$$

Step 4: Substitute the given values.

$$84 = x(x + 5)$$

Step 5: Expand the brackets.

$$84 = x^2 + 5x$$

Step 6: Rearrange into standard form.

$$\boxed{x^2 + 5x - 84 = 0}$$

Therefore, the quadratic equation is:

$$\boxed{x^2 + 5x - 84 = 0}$$

Example 2: Two consecutive integers

The product of two consecutive positive integers is 72. Form a quadratic equation representing the situation.

Solution

Let the first integer be x.

The next consecutive integer is $x + 1$.

Their product is 72.

$$x(x + 1) = 72$$

Expanding:

$$x^2 + x = 72$$

Rearranging:

$$\boxed{x^2 + x - 72 = 0}$$

Example 3: A square piece of land

A square piece of land has an area of 121 square metres. Form a quadratic equation to represent its dimensions.

Solution

Let the length of one side be x metres.

Area of a square:

$$A = x \times x$$

Substituting:

$$x^2 = 121$$

Rearranging:

$$\boxed{x^2 - 121 = 0}$$

3.0 Solving quadratic equations

Solving a quadratic equation involves finding the values of the unknown variable that make the equation true.

These values are called **roots**, **solutions** or **zeros** of the quadratic equation.

There are several methods of solving quadratic equations:

1. Factorisation.
2. Completing the square.
3. The quadratic formula.
4. Graphical method.

3.1 Solving quadratic equations by factorisation

Factorisation involves expressing a quadratic expression as a product of two linear factors.

For example:

$$x^2 + 5x + 6 = 0$$

This can be factorised as:

$$(x + 2)(x + 3) = 0$$

Since the product of two factors is zero, at least one factor must be zero.

Therefore:

$$x + 2 = 0 \quad \text{or} \quad x + 3 = 0$$

Hence:

$$\boxed{x = -2 \text{ or } x = -3}$$

Steps followed when solving by factorisation

1. Arrange the equation in standard form.
2. Identify the coefficients a, b and c.
3. Find two numbers whose product is ac and whose sum is b, where splitting the middle term is applicable.
4. Split the middle term.
5. Group the terms.
6. Factorise each group.
7. Equate each factor to zero.
8. Solve for the unknown variable.
9. Verify the answers by substitution.

Worked Example 4: Solve by factorisation

Solve:

$$x^2 + 7x + 12 = 0$$

Solution

Step 1: Identify the coefficients.

$$a = 1, \quad b = 7, \quad c = 12$$

Step 2: Find two numbers whose product is 12 and sum is 7.

The numbers are 3 and 4 because:

$$3 \times 4 = 12$$

$$3 + 4 = 7$$

Step 3: Split the middle term.

$$x^2 + 3x + 4x + 12 = 0$$

Step 4: Group the terms.

$$x(x + 3) + 4(x + 3) = 0$$

Step 5: Factorise.

$$(x + 3)(x + 4) = 0$$

Step 6: Equate each factor to zero.

$$x + 3 = 0 \quad \text{or} \quad x + 4 = 0$$

Therefore:

$$\boxed{x = -3 \text{ or } x = -4}$$

Worked Example 5: Solve a quadratic equation where the coefficient of x^2 is not 1.

Solve:

$$2x^2 + 7x + 3 = 0$$

Solution

Multiply the coefficient of x^2 by the constant term:

$$2 \times 3 = 6$$

Find two numbers whose product is 6 and sum is 7.

The numbers are 6 and 1.

Split the middle term:

$$2x^2 + 6x + x + 3 = 0$$

Group:

$$2x(x + 3) + 1(x + 3) = 0$$

Factorise:

$$(2x + 1)(x + 3) = 0$$

Therefore:

$$2x + 1 = 0 \quad \text{or} \quad x + 3 = 0$$

$$2x = -1 \quad \text{or} \quad x = -3$$

$$\boxed{x = -\frac{1}{2} \text{ or } x = -3}$$

3.2 Solving quadratic equations by completing the square

Completing the square is a method of solving quadratic equations by transforming the quadratic expression into a perfect square.

A perfect square is an expression that can be written as the square of a binomial.

For example:

$$x^2 + 6x + 9 = (x + 3)^2$$

Steps followed when completing the square

1. Arrange the equation in standard form.
2. Ensure the coefficient of x^2 is 1.
3. Move the constant term to the right-hand side.
4. Take half the coefficient of x.
5. Square the result.
6. Add this value to both sides.
7. Express the left-hand side as a perfect square.
8. Find the square roots of both sides.
9. Solve for the unknown variable.

Worked Example 6

Solve the equation by completing the square:

$$x^2 + 6x + 5 = 0$$

Solution

Step 1: Move the constant to the right.

$$x^2 + 6x = -5$$

Step 2: Take half the coefficient of x and square it.

$$\left(\frac{6}{2}\right)^2 = 9$$

Step 3: Add 9 to both sides.

$$x^2 + 6x + 9 = -5 + 9$$

Step 4: Express the left-hand side as a perfect square.

$$(x + 3)^2 = 4$$

Step 5: Find the square roots of both sides.

$$x + 3 = \pm 2$$

Step 6: Solve.

$$x + 3 = 2 \quad \text{or} \quad x + 3 = -2$$

$$x = -1 \quad \text{or} \quad x = -5$$

Therefore:

$$\boxed{x = -1 \text{ or } x = -5}$$

3.3 Solving quadratic equations using the quadratic formula

The quadratic formula is a mathematical formula used to find the roots of any quadratic equation expressed in standard form.

It is particularly useful when:

- The quadratic expression cannot easily be factorised.
- The roots are fractions or irrational numbers.
- The coefficients are large or complicated.
- Factorisation is not straightforward.

For a quadratic equation in standard form, identify the coefficients a, b and c carefully before substituting them.

Steps followed when using the quadratic formula

1. Arrange the equation in standard form.
2. Identify the values of a, b and c, including their signs.
3. Substitute the values into the quadratic formula.
4. Simplify the expression under the square root.
5. Evaluate the square root where possible.
6. Calculate the two possible values of x.
7. State both roots clearly.
8. Check the answers by substitution where necessary.

Worked Example 7

Solve using the quadratic formula:

$$2x^2 - 5x - 3 = 0$$

Solution

Step 1: Identify the coefficients.

$$a = 2, \quad b = -5, \quad c = -3$$

Step 2: Substitute the values into the quadratic formula.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$$

Step 3: Simplify.

$$x = \frac{5 \pm \sqrt{25 + 24}}{4}$$

$$x = \frac{5 \pm \sqrt{49}}{4}$$

Step 4: Find the square root.

$$x = \frac{5 \pm 7}{4}$$

Step 5: Calculate the two roots.

First root:

$$x = \frac{5+7}{4} = \frac{12}{4} = 3$$

Second root:

$$x = \frac{5-7}{4} = \frac{-2}{4} = -\frac{1}{2}$$

Therefore:

$$\boxed{x = 3 \text{ or } x = -\frac{1}{2}}$$

Worked Example 8: Quadratic equation with irrational roots

Solve:

$$x^2 - 4x + 1 = 0$$

Solution

Identify the coefficients:

$$a = 1, \quad b = -4, \quad c = 1$$

Substitute:

$$x = \frac{4 \pm \sqrt{16-4}}{2}$$

$$x = \frac{4 \pm \sqrt{12}}{2}$$

Since:

$$\sqrt{12} = 2\sqrt{3}$$

Then:

$$x = \frac{4 \pm 2\sqrt{3}}{2}$$

Therefore:

$$\boxed{x = 2 \pm \sqrt{3}}$$

The two roots are:

$$x = 2 + \sqrt{3}$$

and

$$x = 2 - \sqrt{3}$$

3.4 The discriminant of a quadratic equation

The discriminant is the expression under the square root in the quadratic formula.

It helps determine the nature and number of roots of a quadratic equation.

$$D = b^2 - 4ac$$

Nature of roots

Discriminant	Nature of roots
$D > 0$	Two distinct real roots
$D = 0$	Two equal real roots
$D < 0$	No real roots; two complex conjugate roots

Worked Example 9

Determine the nature of the roots of:

$$x^2 - 6x + 9 = 0$$

Solution

$$a = 1, \quad b = -6, \quad c = 9$$

Calculate the discriminant:

$$D = b^2 - 4ac$$

$$D = (-6)^2 - 4(1)(9)$$

$$D = 36 - 36 = 0$$

Since $D = 0$, the equation has two equal real roots.

3.5 Solving quadratic equations graphically

A quadratic equation can be solved by drawing the graph of the corresponding quadratic function.

The general quadratic function is:

$$y = ax^2 + bx + c$$

Its graph is called a **parabola**.

The roots of the quadratic equation are the points where the graph crosses or touches the horizontal x-axis.

Important features of a quadratic graph

1. The graph is U-shaped when $a > 0$.
2. The graph is inverted U-shaped when $a < 0$.
3. The turning point is called the vertex.
4. The line passing through the vertex and dividing the parabola into two equal halves is the axis of symmetry.
5. The points where the graph meets the x-axis represent the real roots.
6. The point where the graph meets the y-axis is the y-intercept.

Example 10: Drawing a quadratic graph

Draw the graph of:

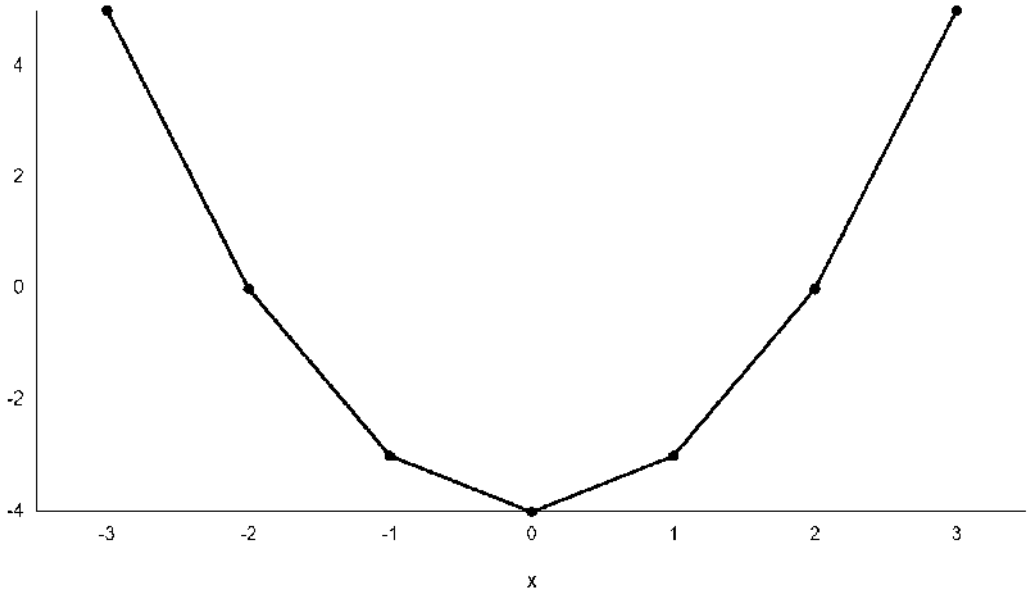
$$y = x^2 - 4$$

Complete the following table:

x	-3	-2	-1	0	1	2	3
$y = x^2 - 4$	5	0	-3	-4	-3	0	5

Graph of $y = x^2 - 4$

The roots occur where the parabola intersects the x-axis.



x	$y = x^2 - 4$
-3	5
-2	0
-1	-3
0	-4
1	-3
2	0
3	5

Interpretation of the graph

- The graph cuts the x-axis at $(-2,0)$ and $(2,0)$.
- The roots are $x = -2$ and $x = 2$.
- The turning point is $(0, -4)$.
- The axis of symmetry is $x = 0$.
- The graph opens upwards because the coefficient of x^2 is positive.

Advantages of graphical solutions

1. They provide a visual understanding of quadratic equations.
2. They help identify the roots of equations.
3. They show the turning points of quadratic functions.
4. They help compare different quadratic functions.
5. They are useful in modelling real-life situations.

Limitation: Graphical solutions may give approximate roots when the intersections do not fall exactly on readable grid points.

4.0 Applications of quadratic equations in real-life situations

Quadratic equations have many applications in everyday activities, engineering, agriculture, business and construction.

4.1 Application in agriculture and land measurement

Quadratic equations can be used to determine unknown dimensions of rectangular farms, gardens and plots of land.

Worked Example 11

A farmer in Narok County has a rectangular piece of land whose length is 8 metres longer than its width. The area is 240 square metres. Determine the dimensions of the land.

Solution

Let the width be x metres.

Length:

$$x + 8$$

Area:

$$x(x + 8) = 240$$

Expanding:

$$x^2 + 8x - 240 = 0$$

Factorising:

$$(x + 20)(x - 12) = 0$$

Therefore:

$$x = -20 \quad \text{or} \quad x = 12$$

Since a length cannot be negative:

$$x = 12$$

Width = 12 metres.

Length:

$$12 + 8 = 20$$

Answer:

- Width = 12 metres.
- Length = 20 metres.

4.2 Application in construction

Quadratic equations help builders and engineers determine unknown dimensions and areas when designing structures.

Worked Example 12

A rectangular classroom floor has a length 3 metres greater than its width. If its area is 54 square metres, determine its dimensions.

Solution

Let the width be x.

Length = $x + 3$.

$$x(x + 3) = 54$$

$$x^2 + 3x - 54 = 0$$

Factorising:

$$(x + 9)(x - 6) = 0$$

$$x = -9 \quad \text{or} \quad x = 6$$

Reject the negative value.

Width = 6 metres.

Length:

$$6 + 3 = 9$$

Therefore, the classroom floor measures 6 metres by 9 metres.

4.3 Application in projectile motion

A projectile is an object thrown or launched into the air and allowed to move under the influence of gravity.

Examples include:

- A football kicked into the air.
- A basketball thrown towards a basket.
- A stone thrown upwards.
- A javelin launched during athletics competitions.

The height of a projectile at a particular time can be modelled using a quadratic function.

Worked Example 13

The height of a ball thrown upwards from the ground is represented by:

$$h = 20t - 5t^2$$

Where:

- h is the height in metres.
- t is time in seconds.

Determine the time taken for the ball to return to the ground.

Solution

When the ball reaches the ground:

$$h = 0$$

Therefore:

$$20t - 5t^2 = 0$$

Factorise:

$$5t(4 - t) = 0$$

Equating each factor to zero:

$$5t = 0 \quad \text{or} \quad 4 - t = 0$$

$$t = 0 \quad \text{or} \quad t = 4$$

The value $t = 0$ represents the starting time.

Therefore, the ball returns to the ground after:

$$\boxed{4 \text{ seconds}}$$

4.4 Application in business and economics

Quadratic equations can be used to determine selling prices, production levels, revenue and profit in business situations.

Worked Example 14

A small business produces and sells x items daily. Its profit in Kenya shillings is represented by:

$$P = -2x^2 + 80x - 600$$

Determine the number of items at which the business makes zero profit.

Solution

At zero profit:

$$P = 0$$

Therefore:

$$-2x^2 + 80x - 600 = 0$$

Divide by -2:

$$x^2 - 40x + 300 = 0$$

Factorise:

$$(x - 10)(x - 30) = 0$$

Therefore:

$$x = 10 \quad \text{or} \quad x = 30$$

The business makes zero profit when it produces and sells either 10 or 30 items.

4.5 Application in sports

Quadratic equations are used to model the curved path followed by balls during sporting activities.

Examples include:

- Football.
- Basketball.
- Volleyball.
- Javelin.
- Shot put.

They can help estimate maximum heights and the points at which objects land.

4.6 Application in engineering and architecture

Quadratic equations are useful when:

1. Designing parabolic arches.
2. Modelling curved bridges.
3. Determining dimensions of structures.
4. Analysing the shape of reflectors.
5. Designing satellite dishes and headlights.

5.0 Key mathematical concepts and summary

Concept	Meaning
Quadratic equation	Equation whose highest power is 2
Root	A value that satisfies the equation
Factorisation	Expressing an expression as a product of factors
Perfect square	An expression written as the square of a binomial
Discriminant	Expression used to determine the nature of roots
Parabola	U-shaped or inverted U-shaped quadratic graph
Vertex	Turning point of a parabola
Axis of symmetry	Line dividing the parabola into two equal halves
x-intercept	Point where a graph meets the x-axis

6.0 Common errors to avoid

1. Forgetting to rearrange equations into standard form before solving.
2. Ignoring negative signs when identifying coefficients.
3. Making mistakes when expanding brackets.
4. Forgetting the positive and negative square roots when solving by completing the square.
5. Using an incorrect value for the discriminant.
6. Rejecting negative roots without considering whether the context permits them.
7. Confusing the roots with the vertex of a parabola.
8. Forgetting to state the correct units in real-life applications.

PART TWO: REVISION EXERCISES

SECTION A: MULTIPLE-CHOICE QUESTIONS (30 MARKS)

Instructions: Answer all questions. Choose the correct answer from the alternatives A, B, C and D. Each question carries 1 mark.

1. Which of the following is a quadratic equation?

A. $3x + 5 = 0$

B. $x^2 + 4x + 3 = 0$

C. $x^3 - 2x = 0$

D. $2x^4 + 3 = 0$

2. What is the coefficient of x^2 in the equation $5x^2 - 7x + 9 = 0$?

A. 5 B. -7 C. 9 D. -5

3. Which of the following is the standard form of a quadratic equation?

A. $ax + b = 0$

B. $ax^3 + bx + c = 0$

C. $ax^2 + bx + c = 0$

D. $ax^4 + bx + c = 0$

4. What is the value of a in $3x^2 - 8x + 5 = 0$?

A. -8

B. 5

C. 8

D. 3

5. Which condition must be satisfied for an equation to be quadratic?

A. $a = 0$

B. $a \neq 0$

C. $b = 0$

D. $c = 0$

6. Factorise $x^2 + 7x + 12$.

A. $(x + 2)(x + 6)$

B. $(x + 3)(x + 4)$

C. $(x - 3)(x - 4)$

D. $(x + 1)(x + 12)$

7. Solve $x^2 - 9 = 0$.

A. $x = 3$ only

B. $x = -3$ only

C. $x = 3$ or $x = -3$

D. $x = 9$ or $x = -9$

8. What are the roots of $x^2 + 5x + 6 = 0$?

A. 2 and 3

B. -2 and -3

C. 1 and 6

D. -1 and -6

9. Solve $x^2 - 5x + 6 = 0$.

A. 1 and 6

B. -2 and -3

C. 2 and 3

D. -1 and -6

10. Which method is particularly useful when a quadratic equation cannot easily be factorised?

A. Addition

B. Subtraction

C. Quadratic formula

D. Division only

11. What is the discriminant of $x^2 - 4x + 4 = 0$?

A. 0

B. 4

C. 8

D. 16

12. If the discriminant of a quadratic equation is negative, the equation has:

A. Two equal real roots

B. Two distinct real roots

C. No real roots

D. One real root

13. The equation $x^2 - 6x + 9 = 0$ has:

A. Two distinct real roots

B. Two equal real roots

C. No real roots

D. Three roots

14. What is the value of b in $4x^2 + 9x - 7 = 0$?

A. 4

B. -7

C. 9

D. -9

15. Which of the following is a perfect square?

A. $x^2 + 6x + 9$

B. $x^2 + 5x + 6$

C. $x^2 + 2x + 3$

D. $x^2 - 3x + 1$

16. The product of two consecutive positive integers is 56. Which equation represents the situation?

A. $x^2 + 56 = 0$

B. $x(x + 1) = 56$

C. $x + 56 = 0$

D. $x(x - 2) = 56$

17. A rectangle has a width of x metres and length of $x + 4$ metres. Which expression represents its area?

A. $2x + 4$

B. $x^2 + 4x$

C. $x^2 + 4$

D. $4x$

18. Solve $2x^2 - 8 = 0$.

A. $x = 2$ or $x = -2$

B. $x = 4$ or $x = -4$

C. $x = 2$ only

D. $x = -2$ only

19. What is the value of the discriminant of $x^2 + 2x - 3 = 0$?

A. 4

B. 8

C. 12

D. 16

20. A graph of a quadratic function is called a:

A. Straight line

B. Circle

C. Parabola

D. Triangle

21. When a is positive in $y = ax^2 + bx + c$, the parabola opens:

A. Downwards

B. Upwards

C. Sideways

D. Horizontally

22. The roots of a quadratic equation are the points where its graph intersects the:

A. y-axis

B. x-axis

C. Origin only

D. Vertex

23. Solve $x^2 - 2x - 8 = 0$.

A. 4 and -2

B. 2 and -4

C. 4 and 2

D. -4 and -2

24. What is the value of c in $7x^2 - 3x + 11 = 0$?

A. 7

B. -3

C. 11

D. -11

25. Which of the following equations has roots 2 and 5?

A. $x^2 + 7x + 10 = 0$

B. $x^2 - 7x + 10 = 0$

C. $x^2 + 3x - 10 = 0$

D. $x^2 - 10x + 7 = 0$

26. A ball is thrown vertically upwards. Its height over time can be modelled using:

A. A linear equation only

B. A quadratic equation

C. A constant equation only

D. A cubic equation only

27. Complete the square for $x^2 + 8x$.

A. $(x + 4)^2 - 16$

B. $(x + 8)^2 - 64$

C. $(x + 4)^2 + 16$

D. $(x + 2)^2 - 4$

28. Solve $x^2 + 4x = 0$.

A. 0 and 4

B. 0 and -4

C. 2 and -2

D. 4 and -4

29. A rectangular farm has width x metres and length $x + 6$ metres. If its area is 40 square metres, which equation represents the situation?

A. $x^2 + 6x - 40 = 0$

B. $x^2 - 6x + 40 = 0$

C. $x^2 + 40 = 0$

D. $x^2 + 6 = 40$

30. Which of the following is a real-life application of quadratic equations?

A. Counting individual books only

B. Reading a calendar

C. Determining the maximum height of a projectile

D. Naming geometrical shapes

SECTION B: STRUCTURED QUESTIONS (50 MARKS)

Instructions:

1. Answer all questions.
2. Show all necessary working.
3. Give answers to the required degree of accuracy.
4. Include appropriate units in real-life problems.

QUESTION 1: FORMATION OF QUADRATIC EQUATIONS (8 MARKS)

A farmer in Narok County owns a rectangular piece of land. The length is 7 metres longer than the width. The area of the land is 120 square metres.

a) Let the width of the land be x metres. Express its length in terms of x. (1 mark)

b) Write an expression for the area of the land. (2 marks)

c) Form a quadratic equation representing the situation in standard form. (2 marks)

d) Factorise the quadratic equation. (2 marks)

e) State the possible width of the land, giving a reason for rejecting the other root. (1 mark)

QUESTION 2: SOLVING BY FACTORISATION (8 MARKS)

Solve the following quadratic equations by factorisation.

a) $x^2 + 9x + 20 = 0$. (2 marks)

b) $x^2 - 7x + 12 = 0$. (2 marks)

c) $2x^2 + 7x + 3 = 0$. (2 marks)

d) $3x^2 - 12x = 0$. (2 marks)

QUESTION 3: COMPLETING THE SQUARE (8 MARKS)

a) Explain what is meant by completing the square. (2 marks)

b) Solve the equation below by completing the square:

$$x^2 + 8x + 7 = 0$$

(4 marks)

c) Express $x^2 + 10x + 25$ as a perfect square. (2 marks)

QUESTION 4: QUADRATIC FORMULA AND DISCRIMINANT (8 MARKS)

a) State the quadratic formula. (2 marks)

b) Solve the equation below using the quadratic formula:

$$2x^2 - 3x - 2 = 0$$

(4 marks)

c) Determine the nature of the roots of the equation:

$$x^2 + 4x + 8 = 0$$

(2 marks)

QUESTION 5: GRAPHICAL REPRESENTATION OF QUADRATIC EQUATIONS (9 MARKS)

Consider the quadratic function:

$$y = x^2 - 2x - 3$$

a) Complete the following table. (4 marks)

x	-2	-1	0	1	2	3	4
y							

b) Using the table, draw the graph of the quadratic function on graph paper. (2 marks)

c) Use the graph to determine the roots of the equation. (1 mark)

d) State the coordinates of the turning point. (1 mark)

e) State the equation of the axis of symmetry. (1 mark)

QUESTION 6: APPLICATIONS OF QUADRATIC EQUATIONS (9 MARKS)

A ball is thrown upwards during a school sports activity. Its height above the ground after t seconds is modelled by:

$$h = 24t - 4t^2$$

Where h is the height in metres and t is the time in seconds.

a) Express the equation in standard quadratic form when the ball is at ground level. (2 marks)

b) Determine the time taken for the ball to return to the ground using factorisation. (3 marks)

c) Calculate the maximum height reached by the ball. (3 marks)

d) State one sporting activity in which quadratic equations can be applied. (1 mark)

PART THREE: ANSWER KEY AND TEACHER'S MARKING GUIDE

A. Multiple-choice answer key

Question	Answer	Question	Answer	Question	Answer
1	B	11	A	21	B
2	A	12	C	22	B
3	C	13	B	23	A
4	D	14	C	24	C
5	B	15	A	25	B
6	B	16	B	26	B
7	C	17	B	27	A
8	B	18	A	28	B
9	C	19	D	29	A
10	C	20	C	30	C

B. Structured questions: Expected answers

Question 1

a) $x + 7$

b) $x(x + 7)$

c) $x^2 + 7x - 120 = 0$

d) $(x + 15)(x - 8) = 0$

e) Width = 8 metres. Reject -15 because a dimension cannot be negative.

Question 2

a) $x = -4, -5$

b) $x = 3,4$

c) $x = -3, -\frac{1}{2}$

d) $x = 0,4$

Question 3

a) Expressing a quadratic expression as a perfect square by adding the appropriate value.

$$x^2 + 8x + 7 = 0$$

$$(x + 4)^2 = 9$$

$$x = -1 \text{ or } x = -7$$

$$(x + 5)^2$$

Question 4

a) Quadratic formula as given in the notes.

$$x = \frac{3 \pm \sqrt{9 + 16}}{4}$$

$$x = \frac{3 \pm 5}{4}$$

$$x = 2 \text{ or } x = -\frac{1}{2}$$

$$D = 16 - 32 = -16$$

No real roots.

Question 5

x	-2	-1	0	1	2	3	4
y	5	0	-3	-4	-3	0	5

b) Correct smooth U-shaped parabola through the seven points.

c) Roots: -1 and 3.

d) Turning point: $(1, -4)$.

e) Axis of symmetry: $x = 1$.

Question 6

$$4t^2 - 24t = 0$$

$$4t(t - 6) = 0$$

$$t = 0 \text{ or } t = 6$$

The ball returns to the ground after 6 seconds.

c) Maximum height occurs at $t = 3$.

$$h = 24(3) - 4(3)^2$$

$$h = 72 - 36 = 36$$

Maximum height = 36 metres.

d) Football, basketball, javelin or shot put.

STRAND 1.0: NUMBERS AND ALGEBRA

SUB-STRAND 1.2: MATRICES

1.0 Introduction to matrices

In everyday life, information and objects are often organised systematically in rows and columns to make them easier to identify, record, compare and analyse.

For example:

- Eggs are arranged in rows and columns in egg trays.
- Learners are arranged in rows and columns in classrooms.
- Goods are displayed in rows and columns on supermarket shelves.
- Seats are arranged in rows and columns in examination halls.
- Agricultural products are packed in rows and columns in storage containers.
- Sports results are recorded in tables showing teams, matches and scores.

In mathematics, such rectangular arrangements of numbers or other mathematical objects can be represented using **matrices**.

Matrices provide an organised way of displaying and manipulating information.

They are widely used in:

1. Business and accounting.
2. Agriculture and food production.
3. Computer programming.
4. Engineering and construction.
5. Science and technology.
6. Transport and logistics.
7. Educational data management.

1.1 Meaning and definition of a matrix

Definition

A matrix is a rectangular arrangement of numbers or mathematical elements in horizontal rows and vertical columns, usually enclosed in brackets or parentheses.

The plural of matrix is **matrices**.

For example:

$$A = \begin{pmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{pmatrix}$$

This is a matrix containing six elements arranged in two rows and three columns.

Another example:

$$B = \begin{pmatrix} 5 & 8 \\ 3 & 6 \\ 7 & 9 \end{pmatrix}$$

This matrix contains three rows and two columns.

Important characteristics of matrices

1. Elements are arranged in horizontal rows and vertical columns.
2. Elements are enclosed in brackets or parentheses.

3. Every matrix has a definite number of rows and columns.
4. A matrix is usually represented by a capital letter such as A, B, C or M.
5. Elements can be positive numbers, negative numbers, fractions, decimals or zero.
6. Matrices can be used to organise numerical information.

2.0 Elements of a matrix and order of a matrix

2.1 Elements of a matrix

The individual numbers or entries in a matrix are called its **elements** or **entries**.

Consider the matrix:

$$A = \begin{pmatrix} 4 & 7 & 9 \\ 2 & 5 & 8 \end{pmatrix}$$

The elements are:

$$4,\ 7,\ 9,\ 2,\ 5,\ 8$$

Each element occupies a specific position determined by its row and column.

2.2 Position of elements in a matrix

The position of an element is identified using two subscripts.

The general notation is:

$$a_{ij}$$

Where:

- a represents the matrix.
- i represents the row number.
- j represents the column number.

For example, a_{23} represents the element in the second row and third column.

Worked Example 1: Identifying elements

Given:

$$A = \begin{pmatrix} 3 & 5 & 7 \\ 2 & 4 & 6 \\ 8 & 9 & 1 \end{pmatrix}$$

Identify the following elements:

a) a_{11}

b) a_{12}

c) a_{23}

d) a_{31}

e) a_{33}

Solution

a) a_{11}: First row, first column.

$$\boxed{3}$$

b) a_{12}: First row, second column.

$\boxed{5}$

c) a_{23}: Second row, third column.

$\boxed{6}$

d) a_{31}: Third row, first column.

$\boxed{8}$

e) a_{33}: Third row, third column.

$\boxed{1}$

Remember: Always read the first subscript as the row and the second subscript as the column.

2.3 Order of a matrix

The order of a matrix refers to the number of rows and columns it contains.

It is written in the form:

$$\boxed{m \times n}$$

Where:

- m represents the number of rows.
- n represents the number of columns.

Therefore:

$$\text{Order of a matrix} = \text{Number of rows} \times \text{Number of columns}$$

Worked Example 2: Determining the order of a matrix

Determine the order of the following matrix:

$$P = \begin{pmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{pmatrix}$$

Solution

The matrix has:

- 2 horizontal rows.
- 3 vertical columns.

Therefore:

$$\boxed{2 \times 3}$$

The order of matrix P is 2×3.

Worked Example 3

Determine the order of:

$$Q = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{pmatrix}$$

Solution

Number of rows = 4.

Number of columns = 2.

Therefore:

$$\boxed{4 \times 2}$$

2.4 Types of matrices according to their order

Matrices can be classified according to the number of rows and columns they contain.

a) Row matrix

A row matrix is a matrix containing only one row.

Example:

$$A = (2 \quad 4 \quad 6 \quad 8)$$

Its order is:

$$1 \times 4$$

b) Column matrix

A column matrix is a matrix containing only one column.

Example:

$$B = \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix}$$

Its order is:

$$3 \times 1$$

c) Square matrix

A square matrix is a matrix that has an equal number of rows and columns.

Example:

$$C = \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$$

Its order is 2×2.

Another example:

$$D = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

Its order is 3×3.

d) Rectangular matrix

A rectangular matrix is a matrix whose number of rows is different from its number of columns.

Example:

$$E = \begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{pmatrix}$$

Its order is 2×3.

e) Zero matrix (null matrix)

A zero matrix is a matrix in which all the elements are zero.

Example:

$$F = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

f) Diagonal matrix

A diagonal matrix is a square matrix in which all elements outside the main diagonal are zero.

Example:

$$G = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{pmatrix}$$

The main diagonal runs from the top-left corner to the bottom-right corner.

g) Identity matrix

An identity matrix is a square matrix in which all the elements along the main diagonal are 1 and all other elements are zero.

Example:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Another example:

$$J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Summary of matrix types

Type	Description	Example order
Row matrix	One row	1×3
Column matrix	One column	3×1
Square matrix	Equal rows and columns	2×2
Rectangular matrix	Unequal rows and columns	2×3
Zero matrix	All elements are zero	2×2
Diagonal matrix	Off-diagonal elements are zero	3×3
Identity matrix	Main diagonal elements are 1; others zero	2×2

2.5 Equal matrices

Two matrices are said to be equal if:

1. They have the same order.
2. Their corresponding elements are equal.

For example:

$$A = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$$

Therefore:

$$A = B$$

However:

$$C = \begin{pmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{pmatrix}$$

is not equal to A, because the matrices have different orders.

Worked Example 4: Finding an unknown element

Given that:

$$\begin{pmatrix} x & 4 \\ 3 & y \end{pmatrix} = \begin{pmatrix} 7 & 4 \\ 3 & 9 \end{pmatrix}$$

Find the values of x and y.

Solution

Since equal matrices have corresponding elements equal:

Comparing the first elements:

$$x = 7$$

Comparing the last elements:

$$y = 9$$

Therefore:

$$\boxed{x = 7, \quad y = 9}$$

3.0 Addition of matrices

3.1 Meaning of addition of matrices

Addition of matrices is the process of combining two or more matrices by adding their corresponding elements.

Two matrices can only be added if they have the same order.

This means that they must have:

- The same number of rows.
- The same number of columns.

Matrices that have the same order are said to be **compatible for addition**.

3.2 Rules for adding matrices

1. Arrange the matrices according to their corresponding positions.
2. Confirm that the matrices have the same order.
3. Add elements occupying corresponding positions.
4. Write the resulting elements in their respective positions.
5. State the order of the resulting matrix.

The general representation is:

If

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

Then:

$$A + B = \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$$

3.3 Worked examples on addition of matrices

Worked Example 5: Addition of two matrices

Given:

$$A = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 1 & 6 \\ 7 & 2 \end{pmatrix}$$

Find $A + B$.

Solution

$$A + B = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix} + \begin{pmatrix} 1 & 6 \\ 7 & 2 \end{pmatrix}$$

Add corresponding elements:

$$= \begin{pmatrix} 2+1 & 4+6 \\ 3+7 & 5+2 \end{pmatrix}$$

Therefore:

$$\boxed{A + B = \begin{pmatrix} 3 & 10 \\ 10 & 7 \end{pmatrix}}$$

Worked Example 6: Addition involving negative numbers

Given:

$$P = \begin{pmatrix} 5 & -3 \\ -2 & 7 \end{pmatrix}$$

and

$$Q = \begin{pmatrix} -1 & 4 \\ 6 & -5 \end{pmatrix}$$

Find $P + Q$.

Solution

$$P + Q = \begin{pmatrix} 5 + (-1) & -3 + 4 \\ -2 + 6 & 7 + (-5) \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 1 \\ 4 & 2 \end{pmatrix}$$

Therefore:

$$\boxed{P+Q=\begin{pmatrix}4 & 1\\ 4 & 2\end{pmatrix}}$$

Worked Example 7: Addition of matrices of order 2×3

Given:

$$A=\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{pmatrix}$$

and

$$B=\begin{pmatrix}6 & 5 & 4\\ 3 & 2 & 1\end{pmatrix}$$

Find $A+B$.

Solution

$$A+B=\begin{pmatrix}1+6 & 2+5 & 3+4\\ 4+3 & 5+2 & 6+1\end{pmatrix}$$

$$\boxed{A+B=\begin{pmatrix}7 & 7 & 7\\ 7 & 7 & 7\end{pmatrix}}$$

3.4 Addition of three matrices

More than two matrices can be added provided they all have the same order.

Worked Example 8

Given:

$$A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$$

$$B=\begin{pmatrix}2 & 1\\ 4 & 3\end{pmatrix}$$

$$C=\begin{pmatrix}3 & 2\\ 1 & 5\end{pmatrix}$$

Find $A+B+C$.

Solution

$$A+B+C=\begin{pmatrix}1+2+3 & 2+1+2\\ 3+4+1 & 4+3+5\end{pmatrix}$$

Therefore:

$$\boxed{A+B+C=\begin{pmatrix}6 & 5\\ 8 & 12\end{pmatrix}}$$

3.5 Properties of matrix addition

Matrix addition has the following properties.

a) Commutative property

Changing the order of the matrices does not change the sum.

$$A+B=B+A$$

b) Associative property

The grouping of matrices does not change the final sum.

$$(A + B) + C = A + (B + C)$$

c) Additive identity

Adding a zero matrix to another matrix gives the original matrix.

$$A + O = A$$

d) Additive inverse

A matrix added to its negative gives a zero matrix.

$$A + (-A) = O$$

For example:

$$A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$

Its additive inverse is:

$$-A = \begin{pmatrix} -2 & -3 \\ -4 & -5 \end{pmatrix}$$

Therefore:

$$A + (-A) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

3.6 Incompatible matrices for addition

Two matrices cannot be added if they have different orders.

Worked Example 9

Given:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

Can $A + B$ be calculated?

Solution

Order of $A = 2 \times 2$.

Order of $B = 2 \times 3$.

Since the orders are different, the matrices are incompatible for addition.

Therefore, $A + B$ is not defined.

Remember

Matrices can only be added when they have exactly the same order. Never add matrices of different orders.

4.0 Subtraction of matrices

4.1 Meaning of subtraction of matrices

Subtraction of matrices is the process of subtracting corresponding elements of one matrix from another matrix.

Just like addition, subtraction can only be performed when the matrices have the same order.

4.2 Rules for subtracting matrices

1. Confirm that the matrices have the same order.
2. Identify corresponding elements.
3. Subtract the second matrix's element from the first matrix's corresponding element.
4. Observe the signs of positive and negative numbers.
5. Arrange the answers in the correct positions.
6. State the resulting matrix.

If:

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

and

$$B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

Then:

$$A - B = \begin{pmatrix} a - e & b - f \\ c - g & d - h \end{pmatrix}$$

4.3 Worked examples on subtraction of matrices

Worked Example 10

Given:

$$A = \begin{pmatrix} 8 & 6 \\ 9 & 7 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 3 & 2 \\ 4 & 5 \end{pmatrix}$$

Find $A - B$.

Solution

$$A - B = \begin{pmatrix} 8 - 3 & 6 - 2 \\ 9 - 4 & 7 - 5 \end{pmatrix}$$

$$\boxed{A - B = \begin{pmatrix} 5 & 4 \\ 5 & 2 \end{pmatrix}}$$

Worked Example 11: Subtraction involving negative numbers

Given:

$$P = \begin{pmatrix} 7 & -4 \\ -3 & 8 \end{pmatrix}$$

and

$$Q = \begin{pmatrix} 2 & -6 \\ 5 & -2 \end{pmatrix}$$

Find $P - Q$.

Solution

$$P - Q = \begin{pmatrix} 7-2 & -4-(-6) \\ -3-5 & 8-(-2) \end{pmatrix}$$

Remember that subtracting a negative number is equivalent to adding a positive number.

$$= \begin{pmatrix} 5 & 2 \\ -8 & 10 \end{pmatrix}$$

Therefore:

$$\boxed{P - Q = \begin{pmatrix} 5 & 2 \\ -8 & 10 \end{pmatrix}}$$

Worked Example 12: Subtraction of rectangular matrices

Given:

$$A = \begin{pmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix}$$

Find $A - B$.

Solution

$$A - B = \begin{pmatrix} 9-3 & 8-2 & 7-1 \\ 6-1 & 5-2 & 4-3 \end{pmatrix}$$

$$\boxed{A - B = \begin{pmatrix} 6 & 6 & 6 \\ 5 & 3 & 1 \end{pmatrix}}$$

4.4 Important properties of subtraction

1. Matrix subtraction is not commutative.

$$A - B \neq B - A$$

in general.

2. Matrix subtraction is not associative.

$$(A - B) - C \neq A - (B - C)$$

in general.

3. Subtracting a matrix from itself gives a zero matrix.

$$A - A = O$$

4. Subtracting a zero matrix leaves the original matrix unchanged.

$$A - O = A$$

5.0 Multiplication of matrices by a scalar

5.1 Meaning of a scalar

A scalar is a single numerical quantity that has magnitude but no matrix arrangement.

Examples include:

$$2, \quad 5, \quad -3, \quad \frac{1}{2}, \quad 0.5$$

5.2 Meaning of scalar multiplication of matrices

Scalar multiplication is the process of multiplying every element of a matrix by the same numerical value.

The resulting matrix has the same order as the original matrix.

For example, if:

$$A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$

Then:

$$2A = 2\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 2(2) & 2(3) \\ 2(4) & 2(5) \end{pmatrix}$$

$$\boxed{2A = \begin{pmatrix} 4 & 6 \\ 8 & 10 \end{pmatrix}}$$

5.3 Rules for multiplying a matrix by a scalar

1. Identify the scalar quantity.
2. Identify the elements of the matrix.
3. Multiply every element by the scalar.
4. Observe the signs of the elements.
5. Arrange the products in their corresponding positions.
6. Retain the original order of the matrix.

If:

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Then:

$$kA = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$$

Where k is a scalar.

5.4 Worked examples on scalar multiplication

Worked Example 13: Multiplication by a positive scalar

Given:

$$A = \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}$$

Find $3A$.

Solution

$$3A = 3\begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}$$

Multiply each element by 3.

$$= \begin{pmatrix} 3(3) & 3(5) \\ 3(2) & 3(4) \end{pmatrix}$$

Therefore:

$$\boxed{3A = \begin{pmatrix} 9 & 15 \\ 6 & 12 \end{pmatrix}}$$

Worked Example 14: Multiplication by a negative scalar

Given:

$$B = \begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix}$$

Find $-2B$.

Solution

$$-2B = -2\begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} -2(2) & -2(-3) \\ -2(4) & -2(-5) \end{pmatrix}$$

$$\boxed{-2B = \begin{pmatrix} -4 & 6 \\ -8 & 10 \end{pmatrix}}$$

Worked Example 15: Multiplication by a fraction

Given:

$$C = \begin{pmatrix} 4 & 8 \\ 6 & 10 \end{pmatrix}$$

Find:

$$\frac{1}{2}C$$

Solution

$$\frac{1}{2}C = \frac{1}{2}\begin{pmatrix} 4 & 8 \\ 6 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2}(4) & \frac{1}{2}(8) \\ \frac{1}{2}(6) & \frac{1}{2}(10) \end{pmatrix}$$

$$\boxed{\frac{1}{2}C = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}}$$

Worked Example 16: Multiplication of a rectangular matrix by a scalar

Given:

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

Find $4P$.

Solution

$$4P = \begin{pmatrix} 4 & 8 & 12 \\ 16 & 20 & 24 \end{pmatrix}$$

The resulting matrix still has order 2×3.

5.5 Properties of scalar multiplication

Scalar multiplication follows several mathematical properties.

a) Distributive property over matrix addition

$$k(A + B) = kA + kB$$

b) Distributive property over scalar addition

$$(k + m)A = kA + mA$$

c) Associative property of scalar multiplication

$$k(mA) = (km)A$$

d) Multiplication by one

$$1A = A$$

e) Multiplication by zero

$$0A = O$$

f) Multiplication by negative one

$$(-1)A = -A$$

6.0 Combined operations on matrices

6.1 Meaning of combined operations

Combined operations on matrices involve performing two or more mathematical operations on matrices within the same expression.

These operations may include:

- Addition of matrices.
- Subtraction of matrices.
- Multiplication of matrices by scalars.

Learners should be able to perform these operations systematically while observing the correct order of operations.

6.2 Order of operations involving matrices

When solving expressions involving addition, subtraction and scalar multiplication, observe the following guidelines:

1. Perform operations inside brackets first.

2. Perform scalar multiplication.
3. Carry out addition and subtraction.
4. Observe the order in which subtraction is written.
5. Check that matrices being added or subtracted have the same order.

Important: Matrix multiplication by matrix and matrix inversion are not part of this Grade 11 sub-strand as specified in the supplied curriculum extract.

6.3 Worked examples on combined operations

Worked Example 17

Given:

$$A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Find $2A + B$.

Solution

Step 1: Multiply matrix A by 2.

$$2A = \begin{pmatrix} 4 & 6 \\ 8 & 10 \end{pmatrix}$$

Step 2: Add matrix B.

$$2A + B = \begin{pmatrix} 4 & 6 \\ 8 & 10 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Step 3: Add corresponding elements.

$$= \begin{pmatrix} 4+1 & 6+2 \\ 8+3 & 10+4 \end{pmatrix}$$

Therefore:

$$\boxed{2A + B = \begin{pmatrix} 5 & 8 \\ 11 & 14 \end{pmatrix}}$$

Worked Example 18

Given:

$$P = \begin{pmatrix} 5 & 4 \\ 3 & 2 \end{pmatrix}$$

and

$$Q = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

Find $3P - 2Q$.

Solution

Step 1: Calculate $3P$.

$$3P = \begin{pmatrix} 15 & 12 \\ 9 & 6 \end{pmatrix}$$

Step 2: Calculate $2Q$.

$$2Q = \begin{pmatrix} 2 & 4 \\ 4 & 2 \end{pmatrix}$$

Step 3: Subtract the resulting matrices.

$$3P - 2Q = \begin{pmatrix} 15-2 & 12-4 \\ 9-4 & 6-2 \end{pmatrix}$$

Therefore:

$$\boxed{3P - 2Q = \begin{pmatrix} 13 & 8 \\ 5 & 4 \end{pmatrix}}$$

Worked Example 19

Given:

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$$

Find $2(A + B) - B$.

Solution

Step 1: Add the matrices inside the brackets.

$$A + B = \begin{pmatrix} 5 & 3 \\ 4 & 6 \end{pmatrix}$$

Step 2: Multiply the result by 2.

$$2(A + B) = \begin{pmatrix} 10 & 6 \\ 8 & 12 \end{pmatrix}$$

Step 3: Subtract matrix B.

$$= \begin{pmatrix} 10-2 & 6-1 \\ 8-3 & 12-2 \end{pmatrix}$$

Therefore:

$$\boxed{2(A + B) - B = \begin{pmatrix} 8 & 5 \\ 5 & 10 \end{pmatrix}}$$

Worked Example 20: Combined operations involving negative numbers

Given:

$$X = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$$

and

$$Y = \begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix}$$

Find $2X + 3Y$.

Solution

Calculate $2X$:

$$2X = \begin{pmatrix} 4 & -2 \\ 6 & 8 \end{pmatrix}$$

Calculate $3Y$:

$$3Y = \begin{pmatrix} -6 & 9 \\ 3 & -3 \end{pmatrix}$$

Add:

$$2X + 3Y = \begin{pmatrix} 4-6 & -2+9 \\ 6+3 & 8-3 \end{pmatrix}$$

Therefore:

$$\boxed{2X + 3Y = \begin{pmatrix} -2 & 7 \\ 9 & 5 \end{pmatrix}}$$

7.0 Applications of matrices in real-life situations

7.1 Introduction

Matrices are important mathematical tools for organising, analysing and comparing information.

They simplify the handling of large amounts of numerical data by arranging information in rows and columns.

In Kenya, matrices can be applied in many sectors, including education, agriculture, business, transport and technology.

7.2 Application of matrices in education

Schools use matrices to organise information about learners, examination results, attendance and subject performance.

Example 21: Examination performance

A school in Nakuru County records the Mathematics and English marks of three learners as follows:

Learner	Mathematics	English
Amina	75	68
Brian	82	74
Carol	65	80

The information can be represented using a matrix:

$$A = \begin{pmatrix} 75 & 68 \\ 82 & 74 \\ 65 & 80 \end{pmatrix}$$

Where:

- The first column represents Mathematics marks.
- The second column represents English marks.
- The rows represent individual learners.

Application: The school can use the matrix to compare performance in the two subjects.

7.3 Application of matrices in agriculture

Matrices are useful in recording agricultural production, crop distribution, farm inputs and yields.

Example 22: Farm production

A farmer grows maize and beans on two different plots. The quantities harvested in bags are recorded below.

Plot	Maize	Beans
Plot A	30	15
Plot B	25	20

Represent the information using a matrix.

Solution

$$A = \begin{pmatrix} 30 & 15 \\ 25 & 20 \end{pmatrix}$$

Where:

- First column represents maize.
- Second column represents beans.
- First row represents Plot A.
- Second row represents Plot B.

Application: The farmer can compare crop yields and plan storage and marketing.

7.4 Application of matrices in business

Businesses use matrices to record sales, purchases, expenditure and stock levels.

Example 23: Supermarket sales

A supermarket records the number of packets of maize flour and sugar sold on two different days.

Day	Maize flour	Sugar
Monday	40	25
Tuesday	35	30

Represent the information using a matrix.

Solution

$$S = \begin{pmatrix} 40 & 25 \\ 35 & 30 \end{pmatrix}$$

Suppose sales increase by the same quantities on the next two days.

The increase is represented by:

$$I = \begin{pmatrix} 10 & 5 \\ 5 & 10 \end{pmatrix}$$

The new sales figures are:

$$S + I = \begin{pmatrix} 40 & 25 \\ 35 & 30 \end{pmatrix} + \begin{pmatrix} 10 & 5 \\ 5 & 10 \end{pmatrix}$$

$$\boxed{S + I = \begin{pmatrix} 50 & 30 \\ 40 & 40 \end{pmatrix}}$$

The new figures are:

- Monday: 50 packets of maize flour and 30 packets of sugar.
- Tuesday: 40 packets of maize flour and 40 packets of sugar.

7.5 Application of matrices in transport

Transport companies use matrices to organise information about passengers, routes, vehicles and transport costs.

Example 24: Passenger transport

A bus company records the number of passengers travelling along two routes during morning and afternoon trips.

Route	Morning	Afternoon
Kilgoris–Narok	40	35
Kilgoris–Nakuru	30	25

Represent the information in matrix form.

Solution

$$T = \begin{pmatrix} 40 & 35 \\ 30 & 25 \end{pmatrix}$$

If the number of passengers increases by 5 on each trip, the increase is represented by:

$$I = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}$$

New passenger numbers:

$$T + I = \begin{pmatrix} 45 & 40 \\ 35 & 30 \end{pmatrix}$$

7.6 Application of matrices in technology and computer science

Matrices are used in computer systems for:

1. Storing numerical information.
2. Processing data.
3. Representing digital images.
4. Organising information in computer databases.
5. Supporting computer graphics.
6. Representing information used in artificial intelligence.

For example, a simple black-and-white digital image can be represented by a matrix in which 0 represents black and 1 represents white.

$$P = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Each number represents the colour value of a pixel in the image.

7.7 Application of matrices in construction and engineering

Matrices help engineers and construction professionals to:

- Organise measurements.
- Record material requirements.
- Compare quantities of construction materials.
- Organise structural data.
- Represent numerical information in engineering calculations.

Example 25: Construction materials

A contractor records the number of cement bags and iron sheets required for two building projects.

Project	Cement bags	Iron sheets
Project A	80	50
Project B	100	60

The information is represented as:

$$M = \begin{pmatrix} 80 & 50 \\ 100 & 60 \end{pmatrix}$$

If both projects require an additional quantity represented by:

$$N = \begin{pmatrix} 10 & 5 \\ 20 & 10 \end{pmatrix}$$

The revised requirements are:

$$M + N = \begin{pmatrix} 90 & 55 \\ 120 & 70 \end{pmatrix}$$

7.8 Application of matrices in health services

Health facilities use matrices to organise information about:

- Patient attendance.
- Available medicines.
- Medical supplies.
- Daily clinic visits.
- Vaccination records.

For example, a health centre may record the number of children vaccinated against different diseases over several days.

Such information can be represented in matrix form to simplify comparison and reporting.

7.9 Application of matrices in sports

Matrices are useful in recording:

- Team scores.
- Match results.
- Player statistics.
- Tournament performance.
- Goals scored and conceded.

For example, a football tournament may record the number of goals scored by different teams in different matches.

$$F = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{pmatrix}$$

Each row can represent a team while each column represents a match.

8.0 Practical activities and projects

The following activities are designed to help learners connect matrices with their immediate environment.

Activity 1: Arranging objects in rows and columns

Materials required

- Egg tray.
- Eggs or small balls.
- Exercise book.
- Pen.

Procedure

1. Obtain an egg tray from the local environment.
2. Observe how the eggs are arranged.
3. Count the number of horizontal rows.
4. Count the number of vertical columns.
5. Record the arrangement as a matrix.
6. Identify the position of selected eggs.
7. Determine the order of the matrix.

Questions

a) How many rows are in the tray?

b) How many columns are in the tray?

c) What is the order of the arrangement?

d) Identify the position of the egg in the second row and third column.

e) Explain why the arrangement can be represented as a matrix.

Activity 2: Classroom seating arrangement

Materials required

- Classroom desks.
- Learner register.
- Exercise books.

Procedure

1. Observe how desks are arranged in your classroom.
2. Identify the number of rows and columns.
3. Represent the seating arrangement using a matrix.

4. Identify the position of a learner sitting in a specified row and column.
5. Discuss how matrices can help teachers organise classroom information.

Activity 3: School agricultural production project

Materials required

- Farm production records.
- Exercise books.
- Calculator.

A school farm produces maize and beans in two terms.

The production is recorded as follows:

Term	Maize (bags)	Beans (bags)
Term 1	40	20
Term 2	50	30

Represent the information in matrix form.

If production increases by:

$$B = \begin{pmatrix} 5 & 10 \\ 10 & 5 \end{pmatrix}$$

Calculate the new production using matrix addition.

Expected outcome

Learners should be able to:

- Organise agricultural information in matrices.
- Identify the order of a matrix.
- Add compatible matrices.
- Interpret the results in the context of agricultural production.

Activity 4: Group project – Matrix operations in the community

Learners should work in groups to identify a real-life situation in their community where information is arranged in rows and columns.

Possible areas include:

- School examination results.
- Agricultural production.
- Shop sales.
- Transport records.
- Sports scores.

Each group should:

1. Collect relevant numerical information.
2. Organise the information in a table.
3. Represent the information using a matrix.
4. Identify the order of the matrix.
5. Construct another compatible matrix.
6. Perform addition or subtraction.

7. Multiply a matrix by a scalar.
8. Present and explain the findings to the class.

9.0 Summary of the sub-strand

9.1 Important definitions

Term	Meaning
Matrix	Rectangular arrangement of elements in rows and columns
Matrices	Plural of matrix
Element	Individual entry in a matrix
Row	Horizontal arrangement of elements
Column	Vertical arrangement of elements
Order	Number of rows by number of columns
Scalar	A single numerical quantity
Compatible matrices	Matrices having the same order for addition or subtraction
Zero matrix	Matrix whose elements are all zero
Square matrix	Matrix with equal numbers of rows and columns
Row matrix	Matrix containing one row
Column matrix	Matrix containing one column
Identity matrix	Square matrix with ones on the main diagonal and zeros elsewhere

9.2 Important mathematical rules

Operation	Rule
Addition	Add corresponding elements
Subtraction	Subtract corresponding elements
Scalar multiplication	Multiply every element by the scalar
Addition compatibility	Same order required
Subtraction compatibility	Same order required
Scalar multiplication result	Retains the original matrix order

9.3 Important points to remember

1. Matrices are written using capital letters.
2. The order of a matrix is written as rows × columns.
3. The first subscript identifies the row.
4. The second subscript identifies the column.
5. Addition requires matrices of the same order.
6. Subtraction requires matrices of the same order.
7. Scalar multiplication involves multiplying every element by the same number.
8. A zero matrix contains only zeros.
9. A square matrix has an equal number of rows and columns.
10. A rectangular matrix has unequal numbers of rows and columns.
11. Combined operations require careful observation of mathematical signs and brackets.

12. Matrix multiplication by another matrix, determinants and inverse matrices are excluded from this particular Grade 11 sub-strand according to the supplied curriculum extract.

PART TWO: REVISION EXERCISES

SECTION A: MULTIPLE-CHOICE QUESTIONS (30 MARKS)

Instructions: Answer all questions. Choose the correct answer from the alternatives A, B, C and D. Each question carries 1 mark.

1. What is a matrix?

A. A circular arrangement of numbers.

B. A rectangular arrangement of numbers in rows and columns.

C. A straight-line arrangement of numbers only.

D. A collection of unrelated numbers.

2. What is the plural of matrix?

A. Matrixes

B. Matrices

C. Matricies

D. Matrixs

3. How many rows are in the matrix below?

$$A = \begin{pmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{pmatrix}$$

A. 1

B. 2

C. 3

D. 6

4. Determine the order of the matrix:

$$B = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{pmatrix}$$

A. 4×2

B. 2×4

C. 2×2

D. 4×4

5. Which element is found in the second row and third column of the matrix below?

$$A = \begin{pmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{pmatrix}$$

A. 4

B. 5

C. 6

D. 7

6. A matrix containing only one row is called a:

A. Column matrix

B. Square matrix

C. Row matrix

D. Zero matrix

7. Which of the following is a square matrix?

A. $(1 \quad 2 \quad 3)$

B. $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

C. $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$

D. $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$

8. Which of the following is a zero matrix?

A. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

B. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

C. $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

D. $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

9. What is the order of a column matrix containing five elements?

A. 1×5

B. 5×1

C. 5×5

D. 1×1

10. Two matrices can be added if they:

A. Have different orders.

B. Have the same order.

C. Contain only positive numbers.

D. Are both square matrices.

11. Given:

$$A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Find $A + B$.

A. $\begin{pmatrix} 3 & 5 \\ 7 & 9 \end{pmatrix}$

B. $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

C. $\begin{pmatrix} 2 & 6 \\ 12 & 20 \end{pmatrix}$

D. $\begin{pmatrix} 3 & 6 \\ 8 & 10 \end{pmatrix}$

12. Find the sum of:

$$\begin{pmatrix} 5 & 7 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 6 & 3 \end{pmatrix}$$

A. $\begin{pmatrix} 7 & 8 \\ 9 & 7 \end{pmatrix}$

B. $\begin{pmatrix} 3 & 6 \\ -3 & 1 \end{pmatrix}$

C. $\begin{pmatrix} 10 & 7 \\ 18 & 12 \end{pmatrix}$

D. $\begin{pmatrix} 7 & 7 \\ 8 & 8 \end{pmatrix}$

13. Which statement about matrix addition is correct?

A. It is always undefined.

B. It is commutative.

C. It changes the order of a matrix.

D. It requires multiplication first.

14. Find:

$$\begin{pmatrix} 8 & 6 \\ 9 & 7 \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ 4 & 5 \end{pmatrix}$$

A. $\begin{pmatrix} 11 & 8 \\ 13 & 12 \end{pmatrix}$

B. $\begin{pmatrix} 5 & 4 \\ 5 & 2 \end{pmatrix}$

C. $\begin{pmatrix} 5 & 8 \\ 13 & 2 \end{pmatrix}$

D. $\begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix}$

15. What is the result of subtracting a matrix from itself?

A. Identity matrix

B. Square matrix

C. Zero matrix

D. Column matrix

16. Given:

$$A = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$$

Find $2A$.

A. $\begin{pmatrix} 3 & 8 \\ 10 & 12 \end{pmatrix}$

B. $\begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$

C. $\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$

D. $\begin{pmatrix} 6 & 4 \\ 5 & 12 \end{pmatrix}$

17. Find $-2A$ if:

$$A = \begin{pmatrix} 2 & -3 \\ 4 & 5 \end{pmatrix}$$

A. $\begin{pmatrix} -4 & 6 \\ -8 & -10 \end{pmatrix}$

B. $\begin{pmatrix} 4 & -6 \\ 8 & 10 \end{pmatrix}$

C. $\begin{pmatrix} -4 & -6 \\ -8 & 10 \end{pmatrix}$

D. $\begin{pmatrix} 2 & -6 \\ 4 & -10 \end{pmatrix}$

18. Find $\frac{1}{2}A$ if:

$$A = \begin{pmatrix} 6 & 8 \\ 4 & 10 \end{pmatrix}$$

A. $\begin{pmatrix} 12 & 16 \\ 8 & 20 \end{pmatrix}$

B. $\begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix}$

C. $\begin{pmatrix} 6 & 4 \\ 2 & 5 \end{pmatrix}$

D. $\begin{pmatrix} 3 & 8 \\ 4 & 5 \end{pmatrix}$

19. What happens to the order of a matrix when it is multiplied by a scalar?

A. It doubles.

B. It becomes a square matrix.

C. It remains unchanged.

D. It becomes a zero matrix.

20. What is the result of multiplying any matrix by zero?

A. Identity matrix

B. Zero matrix

C. Original matrix

D. Column matrix

21. Which of the following represents the additive inverse of:

$$A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$

A. $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$

B. $\begin{pmatrix} -2 & -3 \\ -4 & -5 \end{pmatrix}$

C. $\begin{pmatrix} 5 & 4 \\ 3 & 2 \end{pmatrix}$

D. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

22. If A and B are compatible matrices, which statement is correct?

A. $A + B = B + A$

B. $A + B = AB$

C. $A - B = B - A$

D. $A + B = 0$

23. Given:

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$$

Find $3A$.

A. $\begin{pmatrix} 6 & 3 \\ 9 & 12 \end{pmatrix}$

B. $\begin{pmatrix} 5 & 4 \\ 6 & 7 \end{pmatrix}$

C. $\begin{pmatrix} 2 & 3 \\ 9 & 12 \end{pmatrix}$

D. $\begin{pmatrix} 6 & 1 \\ 3 & 12 \end{pmatrix}$

24. Given:

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$$

Find $2A + B$.

A. $\begin{pmatrix} 5 & 8 \\ 5 & 9 \end{pmatrix}$

B. $\begin{pmatrix} 3 & 5 \\ 4 & 5 \end{pmatrix}$

C. $\begin{pmatrix} 4 & 6 \\ 2 & 8 \end{pmatrix}$

D. $\begin{pmatrix} 5 & 6 \\ 3 & 9 \end{pmatrix}$

25. Find $2A - B$ given:

$$A = \begin{pmatrix} 4 & 3 \\ 2 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$$

A. $\begin{pmatrix} 7 & 4 \\ 1 & 9 \end{pmatrix}$

B. $\begin{pmatrix} 9 & 8 \\ 5 & 11 \end{pmatrix}$

C. $\begin{pmatrix} 3 & 1 \\ -1 & 4 \end{pmatrix}$

D. $\begin{pmatrix} 8 & 6 \\ 4 & 10 \end{pmatrix}$

26. A school records the Mathematics and English marks of four learners using a matrix. What is the order of the matrix?

A. 2×4

B. 4×2

C. 4×4

D. 2×2

27. Which of the following is an application of matrices in agriculture?

A. Measuring rainfall using a thermometer.

B. Organising crop production records.

C. Predicting the colour of flowers.

D. Naming farm animals.

28. A matrix is used in computer technology to represent:

A. Digital image pixels.

B. Sound only.

C. Electricity only.

D. Words without numerical values only.

29. If $A + B = O$, which statement is true?

A. $B = A$

B. $B = -A$

C. $B = 2A$

D. $B = O$ always

30. Which operation is included in the supplied Grade 11 matrices sub-strand?

A. Finding the inverse of a matrix.

B. Calculating the determinant of a matrix.

C. Multiplying a matrix by a scalar.

D. Multiplying one matrix by another matrix.

SECTION B: STRUCTURED QUESTIONS (50 MARKS)

Instructions

1. Answer all questions.
2. Show all necessary working.
3. Write mathematical expressions clearly.
4. Observe the signs of positive and negative numbers.
5. State the order of matrices where necessary.

QUESTION 1: ELEMENTS AND ORDER OF MATRICES (8 MARKS)

Consider the matrix:

$$A = \begin{pmatrix} 3 & 5 & 7 \\ 2 & 4 & 6 \\ 8 & 1 & 9 \end{pmatrix}$$

a) Define a matrix. (1 mark)

b) State the order of matrix A. (1 mark)

c) Identify the following elements:

d) a_{11}

ii) a_{23}

iii) a_{32} (3 marks)

iv) State the number of rows and columns in the matrix. (1 mark)

e) Classify matrix A according to its order. (1 mark)

f) Explain why matrix A is called a square matrix. (1 mark)

QUESTION 2: ADDITION OF MATRICES (8 MARKS)

Given:

$$P = \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}$$

and

$$Q = \begin{pmatrix} 6 & 1 \\ 7 & 3 \end{pmatrix}$$

a) State the order of matrices P and Q. (2 marks)

b) Calculate $P + Q$. (3 marks)

c) Given:

$$R = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

and

$$S = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$

Explain why $R + S$ is not defined. (2 marks)

d) State one property of matrix addition. (1 mark)

QUESTION 3: SUBTRACTION OF MATRICES (8 MARKS)

Given:

$$A = \begin{pmatrix} 9 & 7 \\ 5 & 8 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$$

a) Calculate $A - B$. (3 marks)

b) Calculate $B - A$. (3 marks)

c) Explain why $A - B$ is generally not equal to $B - A$. (2 marks)

QUESTION 4: MULTIPLICATION OF MATRICES BY A SCALAR (8 MARKS)

Given:

$$P = \begin{pmatrix} 2 & -3 \\ 4 & 5 \end{pmatrix}$$

a) Calculate $3P$. (3 marks)

b) Calculate $-2P$. (3 marks)

c) State the order of the resulting matrix in part (a). (1 mark)

d) What happens when a matrix is multiplied by zero? (1 mark)

QUESTION 5: COMBINED OPERATIONS ON MATRICES (9 MARKS)

Given:

$$A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Calculate:

a) $2A + B$. (3 marks)

b) $3A - 2B$. (3 marks)

c) $2(A + B) - B$. (3 marks)

QUESTION 6: APPLICATIONS OF MATRICES IN REAL-LIFE SITUATIONS (9 MARKS)

A school in Nakuru County records the number of packets of maize flour and packets of rice sold by its school canteen over two days.

The records are shown below.

Day	Maize flour	Rice
Monday	40	30
Tuesday	35	25

a) Represent the information in matrix form. (2 marks)

b) State the order of the matrix formed. (1 mark)

c) During the following two days, the canteen records an increase in sales represented by:

$$B = \begin{pmatrix} 10 & 5 \\ 5 & 10 \end{pmatrix}$$

Calculate the new sales figures using matrix addition. (3 marks)

d) If the original sales matrix is multiplied by 2, calculate the resulting matrix. (2 marks)

e) State one advantage of using matrices to organise business records. (1 mark)

PART THREE: ANSWER KEY AND TEACHER'S MARKING GUIDE

A. Multiple-choice answer key

Question	Answer	Question	Answer	Question	Answer
1	B	11	A	21	B
2	B	12	A	22	A
3	B	13	B	23	A
4	B	14	B	24	A
5	D	15	C	25	A
6	C	16	B	26	B
7	C	17	A	27	B
8	B	18	B	28	A
9	B	19	C	29	B
10	B	20	B	30	C

B. Structured questions: Marking scheme

QUESTION 1: ELEMENTS AND ORDER OF MATRICES (8 MARKS)

a) A matrix is a rectangular arrangement of numbers or elements in rows and columns. (1)

b) Order:

$$\boxed{3 \times 3}$$

c) $a_{11} = 3$ (1)

ii) $a_{23} = 6$ (1)

iii) $a_{32} = 1$ (1)

iv) Rows = 3; columns = 3. (1)

e) Square matrix. (1)

f) It has an equal number of rows and columns. (1)

QUESTION 2: ADDITION OF MATRICES (8 MARKS)

a) Both matrices have order 2×2. (2)

$$P + Q = \begin{pmatrix} 3+6 & 5+1 \\ 2+7 & 4+3 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} 9 & 6 \\ 9 & 7 \end{pmatrix}}$$

c) Matrix R has order 2×3, while matrix S has order 2×2. They have different orders; hence, addition is not defined. (2)

d) Any one:

- Commutative property.
- Associative property.
- Additive identity.
- Additive inverse. (1)

QUESTION 3: SUBTRACTION OF MATRICES (8 MARKS)

$$A - B = \begin{pmatrix} 9-3 & 7-2 \\ 5-1 & 8-4 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} 6 & 5 \\ 4 & 4 \end{pmatrix}}$$

$$B - A = \begin{pmatrix} 3-9 & 2-7 \\ 1-5 & 4-8 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} -6 & -5 \\ -4 & -4 \end{pmatrix}}$$

c) Matrix subtraction is not commutative because changing the order of subtraction changes the signs of the corresponding elements. (2)

QUESTION 4: SCALAR MULTIPLICATION (8 MARKS)

$$3P = \begin{pmatrix} 6 & -9 \\ 12 & 15 \end{pmatrix}$$

$$-2P = \begin{pmatrix} -4 & 6 \\ -8 & -10 \end{pmatrix}$$

c) 2×2. (1)

d) It produces a zero matrix. (1)

QUESTION 5: COMBINED OPERATIONS (9 MARKS)

$$2A + B = \begin{pmatrix} 4 & 6 \\ 8 & 10 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} 5 & 8 \\ 11 & 14 \end{pmatrix}}$$

$$3A = \begin{pmatrix} 6 & 9 \\ 12 & 15 \end{pmatrix}$$

$$2B = \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$$

$$\boxed{3A - 2B = \begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix}}$$

$$A + B = \begin{pmatrix} 3 & 5 \\ 7 & 9 \end{pmatrix}$$

$$2(A + B) = \begin{pmatrix} 6 & 10 \\ 14 & 18 \end{pmatrix}$$

$$2(A + B) - B = \begin{pmatrix} 6 - 1 & 10 - 2 \\ 14 - 3 & 18 - 4 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} 5 & 8 \\ 11 & 14 \end{pmatrix}}$$

QUESTION 6: APPLICATIONS OF MATRICES (9 MARKS)

$$A = \begin{pmatrix} 40 & 30 \\ 35 & 25 \end{pmatrix}$$

$$\boxed{2 \times 2}$$

$$A + B = \begin{pmatrix} 40 & 30 \\ 35 & 25 \end{pmatrix} + \begin{pmatrix} 10 & 5 \\ 5 & 10 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} 50 & 35 \\ 40 & 35 \end{pmatrix}}$$

$$2A = 2\begin{pmatrix} 40 & 30 \\ 35 & 25 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} 80 & 60 \\ 70 & 50 \end{pmatrix}}$$

e) Any one:

- Matrices organise information systematically.
- They make comparison of records easier.
- They simplify numerical calculations.
- They make it easier to analyse large amounts of data.

STRAND 1.0: NUMBERS AND ALGEBRA

SUB-STRAND 1.3: VARIATIONS

1.0 Introduction to variations

In everyday life, different quantities are related to one another. When one quantity changes, another quantity may also change depending on the nature of their relationship.

For example:

1. When the number of items purchased increases, the total amount of money paid also increases, provided the price per item remains constant.
2. When the speed of a vehicle increases, the time taken to cover a fixed distance decreases.
3. When more workers are employed to complete a particular task, the time required to finish the task may decrease, provided they work at the same rate.
4. When the radius of a cylinder increases while its height remains constant, its volume increases.
5. When the number of machines working together increases, the time taken to produce a fixed quantity of goods may decrease.

These relationships between changing quantities are studied under a mathematical concept known as **variations**.

Variations help us understand, represent, calculate and predict how one quantity changes in relation to another.

1.1 Meaning and definition of variation

Definition

Variation is a mathematical relationship between two or more quantities in which a change in one quantity is associated with a corresponding change in another quantity.

The quantities involved in a variation are called *variables* because their values can change.

A constant is a fixed numerical value that does not change within a given relationship.

For example, if the cost of one exercise book is Ksh 80, the total cost depends on the number of exercise books purchased.

If C represents the total cost and n represents the number of exercise books, then:

$$C = 80n$$

The number 80 is the constant of variation.

1.2 Important terms used in variations

a) Variable

A variable is a quantity whose value can change.

Examples:

- Distance travelled by a vehicle.
- Time taken to complete a journey.
- Number of workers on a construction site.
- Quantity of goods purchased.
- Radius of a cylindrical container.

Variables are commonly represented by letters such as x, y, z, a, b, t.

b) Constant

A constant is a fixed numerical value in a mathematical relationship.

Examples:

- Price of one kilogram of sugar, if the price remains unchanged.
- Speed of a vehicle, if it travels at a constant speed.
- Number of working hours per day, if the working schedule is fixed.

c) Constant of variation

The constant of variation is the fixed numerical value that connects quantities in a variation relationship.

It is commonly represented by the letter k.

For example:

$$y = 5x$$

Here, $k = 5$.

d) Proportionality

Proportionality describes the relationship between two or more quantities where their ratio or product remains constant.

For example:

$$\frac{y}{x} = k$$

This represents direct proportionality.

e) Direct variation

A relationship in which two quantities increase or decrease together in the same proportion.

f) Inverse variation

A relationship in which one quantity increases as the other decreases, such that their product remains constant.

g) Joint variation

A relationship in which one quantity varies directly as the product of two or more other quantities.

h) Combined variation

A relationship involving both direct and inverse variation with different quantities.

2.0 DIRECT VARIATION

2.1 Meaning of direct variation

Definition

Direct variation is a relationship between two quantities in which one quantity varies directly as the other quantity.

This means that when one quantity increases, the other increases in the same proportion, and when one decreases, the other also decreases in the same proportion.

For example:

- If the number of exercise books bought doubles, the total cost doubles.
- If the distance travelled at constant speed doubles, the time taken doubles.
- If the number of identical bags of maize purchased triples, the total cost triples.
- If the number of litres of petrol bought doubles, the total cost doubles, provided the price per litre remains constant.

The symbol used for direct variation is:

$$\propto$$

It is read as *is directly proportional to* or *varies directly as*.

If y varies directly as x, we write:

$$\boxed{y \propto x}$$

Removing the proportionality sign requires introducing a constant of variation k.

Therefore:

$$\boxed{y = kx}$$

Where:

- y is the dependent variable.
- x is the independent variable.
- k is the constant of variation.

2.2 Characteristics of direct variation

The following characteristics help identify direct variation:

1. Both quantities increase together.
2. Both quantities decrease together.
3. The ratio of the corresponding values remains constant.
4. The graph is a straight line passing through the origin.
5. The relationship can be represented by $y = kx$.
6. The constant of variation is obtained by dividing y by x.

Mathematically:

$$\boxed{k = \frac{y}{x}}$$

2.3 Real-life examples of direct variation in Kenya

Example 1: Purchasing exercise books

A learner buys exercise books at Ksh 80 each.

The total cost depends on the number of books purchased.

Number of books	Total cost (Ksh)
1	80
2	160
3	240
4	320
5	400

From the table, we observe that:

- 1 book costs Ksh 80.
- 2 books cost Ksh 160.
- 3 books cost Ksh 240.
- 4 books cost Ksh 320.

If C represents the total cost and n represents the number of books:

$$C = 80n$$

Therefore:

$$\boxed{C \propto n}$$

The total cost varies directly as the number of books purchased.

Example 2: Purchasing maize flour

A family buys maize flour at Ksh 150 per packet.

Let:

- n represent the number of packets.
- C represent the total cost.

Then:

$$C = 150n$$

If the family buys 6 packets:

$$C = 150 \times 6$$

$$\boxed{C = \text{Ksh } 900}$$

Example 3: Distance and time

A matatu travels at a constant speed of 60 km/h.

The distance travelled depends on the time taken.

We know that:

$$\text{Distance} = \text{Speed} \times \text{Time}$$

Therefore:

$$d = 60t$$

Where:

- d is distance in kilometres.
- t is time in hours.

Time (hours)	Distance (km)
1	60
2	120
3	180
4	240
5	300

At constant speed, distance varies directly as time.

$$\boxed{d \propto t}$$

2.4 Finding the constant of variation in direct variation

When two quantities vary directly, the constant of variation can be determined using:

$$y = kx$$

Rearranging:

$$\boxed{k = \frac{y}{x}}$$

Worked Example 1

Given that y varies directly as x, and $y = 24$ when $x = 6$, find the constant of variation.

Solution

We know that:

$$y = kx$$

Substitute the given values:

$$24 = k(6)$$

$$24 = 6k$$

Divide both sides by 6:

$$k = \frac{24}{6}$$

$$\boxed{k = 4}$$

Worked Example 2

The cost C of buying oranges varies directly as the number n of oranges purchased. If 5 oranges cost Ksh 100, find:

a) The constant of variation.

b) The cost of 12 oranges.

Solution

Since cost varies directly as the number of oranges:

$$C = kn$$

a) Finding the constant:

$$100 = 5k$$

$$k = \frac{100}{5}$$

$$\boxed{k = 20}$$

Therefore, each orange costs Ksh 20.

b) Finding the cost of 12 oranges:

$$C = 20 \times 12$$

$$\boxed{C = \text{Ksh } 240}$$

Worked Example 3

The distance travelled by a vehicle varies directly as time. A vehicle covers 150 km in 3 hours.

Find:

a) The constant of variation.

b) The distance covered in 7 hours.

Solution

Let d be distance and t be time.

$$d = kt$$

a) Given:

$$d = 150, \quad t = 3$$

$$150 = 3k$$

$$k = 50$$

The constant of variation is 50.

b) When $t = 7$:

$$d = 50 \times 7$$

$$\boxed{d = 350 \text{ km}}$$

2.5 Finding an unknown quantity in direct variation

Worked Example 4

If y varies directly as x, and $y = 35$ when $x = 7$, find the value of y when $x = 12$.

Solution

First, determine the constant of variation.

$$y = kx$$

$$35 = 7k$$

$$k = 5$$

Therefore:

$$y = 5x$$

When $x = 12$:

$$y = 5 \times 12$$

$$\boxed{y = 60}$$

Worked Example 5

The mass of maize harvested varies directly as the area of land cultivated, assuming the same yield per unit area.

A farmer harvests 600 kg of maize from 2 acres.

Find the mass of maize expected from 7 acres.

Solution

Let:

- M represent mass of maize.
- A represent area in acres.

Since mass varies directly as area:

$$M = kA$$

Substituting:

$$600 = 2k$$

$$k = 300$$

Therefore:

$$M = 300A$$

For 7 acres:

$$M = 300 \times 7$$

$$\boxed{M = 2100 \text{ kg}}$$

The farmer expects to harvest 2,100 kg, assuming the same yield per acre.

2.6 Direct variation using ratios

Direct variation can also be solved using ratios.

If y varies directly as x, then:

$$\frac{y_1}{x_1} = \frac{y_2}{x_2}$$

Where:

- x_1, y_1 are the first pair of values.
- x_2, y_2 are the second pair of values.

Worked Example 6

If 4 kilograms of sugar cost Ksh 720, calculate the cost of 9 kilograms.

Solution

Let the required cost be C.

Since cost varies directly as mass:

$$\frac{720}{4} = \frac{C}{9}$$

Cross-multiply:

$$4C = 720 \times 9$$

$$4C = 6480$$

$$C = \frac{6480}{4}$$

$$\boxed{C = \text{Ksh } 1620}$$

2.7 Graphs of direct variation

A direct variation relationship is represented graphically by a straight line passing through the origin.

Consider:

$$y = 2x$$

Construct a table of values.

x	0	1	2	3	4	5
$y = 2x$	0	2	4	6	8	10

The coordinates are:

$$(0,0), (1,2), (2,4), (3,6), (4,8), (5,10)$$

Important features of the graph

1. It is a straight line.
2. It passes through the origin $(0,0)$.
3. It slopes upwards from left to right when k is positive.
4. The gradient is equal to the constant of variation k.

For the equation:

$$y = 2x$$

The gradient is:

$$m = 2$$

Worked Example 7: Determining the equation from a graph relationship

A direct variation graph passes through the point $(4,20)$. Find its equation.

Solution

Since:

$$y = kx$$

Substitute $x = 4$ and $y = 20$:

$$20 = 4k$$

$$k = 5$$

Therefore:

$$\boxed{y = 5x}$$

2.8 Applications of direct variation

Direct variation is applied in the following situations:

Situation	Relationship
Total cost and number of identical items	$C = kn$
Distance and time at constant speed	$d = kt$
Wages and hours worked at a fixed hourly rate	$W = kh$
Mass and number of identical objects	$M = kn$
Fuel cost and litres purchased at a fixed price	$C = kl$
Harvest and area cultivated at a constant yield	$M = kA$

Important note: Direct variation only applies when the relevant rate, price, yield or other conditions remain constant.

3.0 INVERSE VARIATION

3.1 Meaning of inverse variation

Definition

Inverse variation is a relationship between two quantities in which one quantity varies inversely as the other, such that their product remains constant.

This means that when one quantity increases, the other decreases proportionally, and when one decreases, the other increases proportionally.

For example:

- When the speed of a vehicle increases, the time taken to cover a fixed distance decreases.
- When the number of workers increases, the time taken to complete a fixed task decreases, assuming equal working rates.
- When more taps of equal capacity are used to fill a tank, the time taken to fill the tank decreases.
- When the number of machines increases, the time taken to manufacture a fixed quantity of products decreases, assuming equal machine output.

If y varies inversely as x, we write:

$$\boxed{y \propto \frac{1}{x}}$$

Removing the proportionality sign:

$$\boxed{y = \frac{k}{x}}$$

Where:

- y is the dependent variable.
- x is the independent variable.
- k is the constant of variation.

Multiplying both sides by x:

$$\boxed{xy = k}$$

This means that the product of the two variables remains constant.

3.2 Characteristics of inverse variation

1. As one quantity increases, the other decreases.
2. As one quantity decreases, the other increases.
3. The product of the two quantities remains constant.
4. The graph is a curve known as a rectangular hyperbola.
5. The relationship can be represented by:

$$y = \frac{k}{x}$$

6. The constant of variation is obtained using:

$$\boxed{k = xy}$$

3.3 Real-life examples of inverse variation in Kenya

Example 1: Speed and time

A vehicle covers a fixed distance of 240 km.

The time taken depends on the speed of the vehicle.

Using:

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

We obtain:

$$t = \frac{240}{v}$$

Where:

- t is time in hours.
- v is speed in km/h.

Speed (km/h)	Time (hours)
20	12
30	8
40	6
60	4
80	3
120	2

Observation:

- At 20 km/h, the journey takes 12 hours.
- At 40 km/h, the journey takes 6 hours.
- At 80 km/h, the journey takes 3 hours.

As speed increases, time decreases.

Therefore:

$$\boxed{t \propto \frac{1}{v}}$$

Example 2: Number of workers and time taken

Eight workers can complete a construction task in 15 days.

Assuming all workers work at the same rate and the working conditions remain constant, determine the time required if 12 workers are employed.

Let:

- W represent the number of workers.
- T represent time in days.

Since time varies inversely as the number of workers:

$$T = \frac{k}{W}$$

Therefore:

$$WT = k$$

Substitute the given values:

$$8 \times 15 = k$$

$$k = 120$$

When $W = 12$:

$$12T = 120$$

$$T = \frac{120}{12}$$

$$\boxed{T = 10 \text{ days}}$$

The 12 workers will complete the task in 10 days.

Example 3: Number of taps and time taken to fill a tank

Four identical taps can fill a water tank in 12 hours.

Determine the time taken by six identical taps to fill the same tank, assuming a constant water supply.

Let:

- n represent the number of taps.
- t represent time in hours.

Since time varies inversely as the number of taps:

$$nt = k$$

Substitute:

$$4 \times 12 = k$$

$$k = 48$$

For six taps:

$$6t = 48$$

$$t = \frac{48}{6}$$

$$\boxed{t = 8 \text{ hours}}$$

3.4 Finding the constant of variation in inverse variation

The general formula is:

$$y = \frac{k}{x}$$

Rearranging:

$$\boxed{k = xy}$$

Worked Example 8

Given that y varies inversely as x, and $y = 12$ when $x = 5$, determine the constant of variation.

Solution

$$y = \frac{k}{x}$$

Substitute:

$$12 = \frac{k}{5}$$

Multiply both sides by 5:

$$k = 12 \times 5$$

$$\boxed{k = 60}$$

Worked Example 9

If y varies inversely as x, and $y = 8$ when $x = 6$, find y when $x = 12$.

Solution

First determine the constant:

$$k = xy$$

$$k = 6 \times 8$$

$$k = 48$$

The variation equation is:

$$y = \frac{48}{x}$$

When $x = 12$:

$$y = \frac{48}{12}$$

$$\boxed{y = 4}$$

3.5 Inverse variation using ratios

The relationship can also be expressed as:

$$x_1 y_1 = x_2 y_2$$

This is because the product remains constant.

Worked Example 10

Ten workers can construct a wall in 18 days. How many days will 15 workers take to construct a similar wall at the same rate?

Solution

Let the required time be t.

$$10 \times 18 = 15 \times t$$

$$180 = 15t$$

$$t = \frac{180}{15}$$

$$\boxed{t = 12 \text{ days}}$$

3.6 Graphs of inverse variation

The graph of inverse variation is a curve known as a rectangular hyperbola.

Consider:

$$y = \frac{12}{x}$$

Construct a table of values.

x	1	2	3	4	6	12
y	12	6	4	3	2	1

Important features of the graph

1. It is a curved graph.
2. It does not pass through the origin.
3. As x increases, y decreases.
4. The product xy remains constant.
5. The curve approaches the coordinate axes but does not touch them for a positive non-zero constant.

3.7 Applications of inverse variation

Situation	Relationship
Speed and time for a fixed distance	$t = \frac{k}{v}$
Workers and time for a fixed task	$t = \frac{k}{n}$
Taps and time to fill a fixed tank	$t = \frac{k}{n}$
Machines and production time for a fixed output	$t = \frac{k}{m}$
Number of people sharing a fixed amount of food	Portion per person $= \frac{k}{n}$

Important note: Inverse variation applies when the total work, distance, volume, quantity or other relevant condition remains fixed.

4.0 JOINT VARIATION

4.1 Meaning of joint variation

Definition

Joint variation is a relationship in which one quantity varies directly as the product of two or more other quantities.

This means that a change in any of the related quantities affects the value of the dependent quantity.

For example:

- The volume of a cylinder varies jointly as its base area and height.
- The area of a rectangle varies jointly as its length and width.
- The total wages paid to workers may vary jointly as the number of workers and the number of hours worked, if the hourly wage rate is fixed.
- The volume of a cuboid varies jointly as its length, width and height.

If y varies jointly as x and z, we write:

$$y \propto xz$$

Introducing the constant of variation:

$$\boxed{y = kxz}$$

Where:

- y is the dependent variable.
- x and z are independent variables.
- k is the constant of variation.

For three independent variables:

$$\boxed{y = kxyz}$$

4.2 Characteristics of joint variation

1. One quantity depends directly on two or more other quantities.
2. Increasing any one independent quantity increases the dependent quantity, provided the other quantities remain constant and k is positive.
3. Decreasing any independent quantity decreases the dependent quantity under the same conditions.
4. The ratio of the dependent quantity to the product of the independent quantities remains constant.
5. The relationship is represented by a formula involving multiplication.

The constant of variation is:

$$\boxed{k = \frac{y}{xz}}$$

For three independent quantities:

$$\boxed{k = \frac{y}{xyz}}$$

4.3 Real-life examples of joint variation in Kenya

Example 1: Area of a rectangular farm

The area of a rectangular farm depends on its length and width.

We know that:

$$\text{Area} = \text{Length} \times \text{Width}$$

Therefore:

$$A = lw$$

The area varies jointly as the length and width.

$$\boxed{A \propto lw}$$

Suppose a rectangular farm has:

- Length = 80 metres.
- Width = 50 metres.

Its area is:

$$A = 80 \times 50$$

$$\boxed{A = 4000 \text{ m}^2}$$

If the length is increased to 100 metres while the width remains 50 metres:

$$A = 100 \times 50$$

$$\boxed{A = 5000 \text{ m}^2}$$

The area increases because it depends on both length and width.

Example 2: Volume of a cylinder

The volume of a cylinder is given by:

$$V = \pi r^2 h$$

Where:

- V is the volume.
- r is the radius.
- h is the height.
- π is a mathematical constant approximately equal to 3.142.

Therefore:

$$V \propto r^2 h$$

The volume varies jointly as the square of the radius and the height.

Worked Example 11

The volume V of a cylindrical water tank varies jointly as the square of its radius r and its height h.

If $V = 6160 \text{ cm}^3$ when $r = 7 \text{ cm}$ and $h = 40 \text{ cm}$, determine the constant of variation.

Solution

Since:

$$V = kr^2 h$$

Substitute the given values:

$$6160 = k \times 7^2 \times 40$$

$$6160 = k \times 49 \times 40$$

$$6160 = 1960k$$

$$k = \frac{6160}{1960}$$

$$\boxed{k = \frac{22}{7}}$$

Worked Example 12

The volume V of a cylinder varies jointly as the square of its radius r and its height h.

If the constant of variation is $\frac{22}{7}$, find the volume when the radius is 7 cm and the height is 10 cm.

Solution

$$V = kr^2h$$

Substitute:

$$V = \frac{22}{7} \times 7^2 \times 10$$

$$V = \frac{22}{7} \times 49 \times 10$$

$$V = 22 \times 7 \times 10$$

$$\boxed{V = 1540 \text{ cm}^3}$$

Example 3: Wages and working hours

The total wages earned by workers vary jointly as the number of workers and the number of hours worked, provided the hourly rate is fixed.

Let:

- W represent total wages.
- n represent the number of workers.
- h represent working hours.
- k represent the hourly wage rate.

Then:

$$\boxed{W = knh}$$

Suppose five workers each earn Ksh 200 per hour and work for eight hours.

$$W = 200 \times 5 \times 8$$

$$\boxed{W = \text{Ksh } 8000}$$

Example 4: Volume of a cuboid

The volume of a cuboid is:

$$V = lwh$$

Where:

- l is length.
- w is width.
- h is height.

Therefore, the volume varies jointly as its length, width and height.

4.4 Finding the constant of variation in joint variation

The general formula is:

$$y = kxz$$

Rearranging:

$$\boxed{k = \frac{y}{xz}}$$

Worked Example 13

Given that y varies jointly as x and z, and $y = 60$ when $x = 3$ and $z = 4$, find the constant of variation.

Solution

$$y = kxz$$

Substitute:

$$60 = k \times 3 \times 4$$

$$60 = 12k$$

$$k = \frac{60}{12}$$

$$\boxed{k = 5}$$

Worked Example 14

The quantity P varies jointly as x and the square of y.

If $P = 180$ when $x = 5$ and $y = 3$, find P when $x = 4$ and $y = 6$.

Solution

Since P varies jointly as x and y^2:

$$P = kxy^2$$

First, find k:

$$180 = k \times 5 \times 3^2$$

$$180 = 45k$$

$$k = 4$$

The equation becomes:

$$P = 4xy^2$$

When $x = 4$ and $y = 6$:

$$P = 4 \times 4 \times 6^2$$

$$P = 16 \times 36$$

$$\boxed{P = 576}$$

5.0 COMBINED VARIATION

5.1 Meaning of combined variation

Definition

Combined variation is a relationship in which one quantity varies directly as one or more quantities and inversely as one or more other quantities.

It combines direct and inverse variation in one mathematical relationship.

For example:

- The time taken to complete a task may vary directly as the amount of work and inversely as the number of workers.
- The time taken to travel a fixed distance varies inversely as speed.
- The cost of transporting goods may vary directly as the mass of goods and the distance travelled, and inversely as the number of trips if the same total work is shared equally.

If y varies directly as x and inversely as z, we write:

$$y \propto \frac{x}{z}$$

Introducing the constant of variation:

$$\boxed{y = \frac{kx}{z}}$$

Where:

- y is the dependent variable.
- x is a quantity with direct variation.
- z is a quantity with inverse variation.
- k is the constant of variation.

If y varies directly as x and z, but inversely as w, then:

$$\boxed{y = \frac{kxz}{w}}$$

If y varies directly as x^2 and inversely as z, then:

$$\boxed{y = \frac{kx^2}{z}}$$

5.2 Characteristics of combined variation

1. It involves both direct and inverse relationships.
2. Some quantities appear in the numerator.
3. Quantities with inverse relationships appear in the denominator.
4. The relationship includes a constant of variation.
5. It can involve squares, cubes or other powers of variables.
6. The correct formula must be established before substituting values.

5.3 Real-life examples of combined variation in Kenya

Example 1: Time taken to complete a construction project

The time T taken to complete a construction project varies directly as the amount of work W and inversely as the number of workers n.

Therefore:

$$T \propto \frac{W}{n}$$

Introducing the constant:

$$\boxed{T = \frac{kW}{n}}$$

This means:

- More work requires more time if the number of workers remains constant.
- More workers reduce the time required if the amount of work remains constant.

Worked Example 15

The time T taken to complete a task varies directly as the amount of work W and inversely as the number of workers n.

If 10 workers complete 120 units of work in 6 days, find the time required by 15 workers to complete 200 units of similar work.

Solution

The variation relationship is:

$$T = \frac{kW}{n}$$

First, find k.

Given:

$$T = 6, \quad W = 120, \quad n = 10$$

Substitute:

$$6 = \frac{k \times 120}{10}$$

$$6 = 12k$$

$$k = \frac{1}{2}$$

The equation becomes:

$$T = \frac{W}{2n}$$

For 200 units of work and 15 workers:

$$T = \frac{200}{2 \times 15}$$

$$T = \frac{200}{30}$$

$$\boxed{T = 6\frac{2}{3} \text{ days}}$$

Example 2: Speed, distance and time

The time taken to travel a distance varies directly as distance and inversely as speed.

Therefore:

$$t \propto \frac{d}{v}$$

Introducing the constant:

$$t = \frac{kd}{v}$$

From the formula for motion:

$$t = \frac{d}{v}$$

Hence, in consistent units, $k = 1$.

Worked Example 16

The time taken by a vehicle to travel a certain route varies directly as the distance and inversely as its speed.

A vehicle takes 4 hours to cover 240 km at a constant speed.

Find the time it will take to cover 360 km at a speed of 80 km/h, assuming the same conditions.

Solution

We use:

$$t = \frac{d}{v}$$

Substitute:

$$t = \frac{360}{80}$$

$$\boxed{t = 4.5 \text{ hours}}$$

The vehicle takes 4 hours 30 minutes.

Example 3: Volume of a container

The volume of a cylindrical container varies directly as the square of its radius and its height.

Therefore:

$$V \propto r^2 h$$

This is a joint variation relationship, rather than a combined direct-and-inverse relationship.

It is important to distinguish between joint and combined variation when forming equations.

5.4 Finding the constant of variation in combined variation

If:

$$y = \frac{kx}{z}$$

Then:

$$yz = kx$$

Therefore:

$$\boxed{k = \frac{yz}{x}}$$

Worked Example 17

Given that y varies directly as x and inversely as z, and $y = 12$ when $x = 8$ and $z = 4$, find:

a) The constant of variation.

b) The value of y when $x = 15$ and $z = 5$.

Solution

The equation is:

$$y = \frac{kx}{z}$$

a) Substitute:

$$12 = \frac{k \times 8}{4}$$

$$12 = 2k$$

$$\boxed{k = 6}$$

b) The equation becomes:

$$y = \frac{6x}{z}$$

When $x = 15$ and $z = 5$:

$$y = \frac{6 \times 15}{5}$$

$$y = \frac{90}{5}$$

$$\boxed{y = 18}$$

Worked Example 18

The quantity P varies directly as the square of x and inversely as y.

If $P = 48$ when $x = 4$ and $y = 2$, find P when $x = 6$ and $y = 3$.

Solution

Form the equation:

$$P = \frac{kx^2}{y}$$

Substitute the first set of values:

$$48 = \frac{k \times 4^2}{2}$$

$$48 = \frac{16k}{2}$$

$$48 = 8k$$

$$k = 6$$

The equation becomes:

$$P = \frac{6x^2}{y}$$

Substitute $x = 6$ and $y = 3$:

$$P = \frac{6 \times 6^2}{3}$$

$$P = \frac{6 \times 36}{3}$$

$$\boxed{P = 72}$$

6.0 COMPARISON OF THE DIFFERENT TYPES OF VARIATION

Learners should understand the differences between direct, inverse, joint and combined variations.

Type	Relationship	General equation	Main feature
Direct	$y \propto x$	$y = kx$	Both change in the same proportion
Inverse			Product remains constant
Joint	$y \propto xz$	$y = kxz$	Depends directly on the product of variables
Combined			Direct and inverse relationships occur together

6.1 How to identify the type of variation

When solving variation problems, learners should follow these steps:

Step 1: Read the question carefully.

Identify the quantities involved.

Step 2: Identify the relationship.

Look for words such as:

- Directly.
- Inversely.
- Jointly.
- Directly as the square of.

- Inversely as the cube of.

Step 3: Form the variation equation.

For example:

- Direct: $y = kx$.
- Inverse: .
- Joint: $y = kxz$.
- Combined: .

Step 4: Substitute the known values.

Use the information given to determine the constant of variation.

Step 5: Form the complete equation.

Replace k with the calculated constant.

Step 6: Calculate the unknown quantity.

Substitute the new values into the equation.

Step 7: State the answer with appropriate units where necessary.

7.0 APPLICATIONS OF VARIATIONS IN EVERYDAY LIFE

7.1 Application in agriculture

Variations are used in agriculture to understand relationships involving:

- Land area and crop production.
- Number of workers and time taken to harvest.
- Water flow rate and irrigation time.
- Fertiliser quantity and area cultivated.
- Number of seedlings and planting area.

Example 19: Farm production

A farmer harvests 900 kg of maize from 3 acres of land.

Assuming constant yield per acre, determine the expected harvest from 8 acres.

Solution

Harvest varies directly as land area.

$$M = kA$$

$$900 = 3k$$

$$k = 300$$

When $A = 8$:

$$M = 300 \times 8$$

$$\boxed{M = 2400 \text{ kg}}$$

7.2 Application in business and trade

Variations are used to determine:

1. Total cost of goods.
2. Total revenue.
3. Profit in relation to sales.
4. Wages in relation to working hours.
5. Transport costs in relation to distance and quantity transported.

Example 20: Shopping at a local market

A trader buys 4 kg of potatoes for Ksh 240.

Find the cost of 10 kg, assuming a constant price per kilogram.

Solution

$$C = kn$$

$$240 = 4k$$

$$k = 60$$

Therefore:

$$C = 60 \times 10$$

$$\boxed{C = \text{Ksh } 600}$$

7.3 Application in transport

Variations help in understanding:

- Speed and travelling time.
- Distance and travelling time.
- Fuel consumption and distance.
- Transport costs and quantity of goods.
- Number of vehicles and delivery time.

Example 21: Travel time

A bus travels 300 km in 5 hours at a constant speed.

Find the time required to cover 420 km at the same speed.

Solution

First determine the speed:

$$v = \frac{300}{5}$$

Then:

$$t = \frac{420}{60}$$

$$\boxed{t = 7 \text{ hours}}$$

7.4 Application in construction

Variations are used in:

- Estimating construction time.
- Determining the number of workers needed.
- Calculating material requirements.
- Comparing labour costs.
- Estimating project expenditure.

Example 22: Construction workers

Twelve workers complete a task in 20 days. How long will 15 workers take to complete the same task, assuming equal productivity?

Solution

Time varies inversely as the number of workers.

$$12 \times 20 = 15t$$

$$240 = 15t$$

$$t = \frac{240}{15}$$

$$\boxed{t = 16 \text{ days}}$$

7.5 Application in geometry and measurement

Variations are used in calculating:

- Areas of rectangular fields.
- Volumes of water tanks.
- Surface areas of containers.
- Perimeters of different shapes.
- Capacity of storage facilities.

Example 23: Volume of a cylindrical tank

A cylindrical water tank has a radius of 3 metres and a height of 2 metres.

Calculate its volume using:

$$V = \pi r^2 h$$

Solution

$$V = \frac{22}{7} \times 3^2 \times 2$$

$$V = \frac{22}{7} \times 9 \times 2$$

$$V = \frac{396}{7}$$

$$\boxed{V \approx 56.57 \text{ m}^3}$$

7.6 Application in digital technology

Variations can be used to explain relationships involving:

- File size and number of files of equal size.
- Downloaded data and downloading time at a constant speed.
- Storage capacity and number of stored files of equal size.
- Data transfer rates and time taken to transfer files.

For example, if downloading 100 MB takes 20 seconds at a constant speed, downloading 500 MB takes:

$$t = \frac{500 \times 20}{100}$$

$$\boxed{t = 100 \text{ seconds}}$$

The downloading time varies directly as the file size when the transfer speed remains constant.

7.7 Application in water supply

Variations are useful in understanding:

- Number of taps and time taken to fill a tank.
- Water flow rate and filling time.
- Number of water containers and total volume.
- Number of households and total water demand.

Example 24

Three identical pipes fill a tank in 18 hours. How long will six identical pipes take to fill the same tank?

Solution

$$3 \times 18 = 6t$$

$$54 = 6t$$

$$\boxed{t = 9 \text{ hours}}$$

8.0 PRACTICAL LEARNING ACTIVITIES AND PROJECTS

These activities enable learners to investigate variation relationships using locally available materials and real-life situations.

Activity 1: Investigating direct variation through shopping

Learning objective: To establish the relationship between the number of items purchased and their total cost.

Materials

- Exercise books.
- Pens.
- Calculator.
- Price list from a local shop.

Procedure

1. Visit a local shop or use a sample price list.
2. Identify an item with a fixed price.
3. Record the cost of purchasing different quantities.
4. Organise the information in a table.

5. Calculate the ratio of total cost to quantity.
6. Establish whether the relationship is direct variation.
7. Form an equation representing the relationship.

Learner task

A shop sells packets of milk at Ksh 65 each.

Complete the table.

Number of packets	1	2	3	4	5	6
Total cost (Ksh)	65					

Questions:

a) What happens to the total cost when the number of packets doubles?

b) Determine the constant of variation.

c) Form an equation connecting cost and quantity.

d) Calculate the cost of 10 packets.

Activity 2: Investigating inverse variation using workers

Learning objective: To establish the relationship between the number of workers and the time taken to complete a task.

Procedure

1. Discuss a task that can be completed by a group of workers.
2. Assume that all workers have equal working ability.
3. Record different numbers of workers and their corresponding completion times.
4. Calculate the product of workers and time.
5. Determine whether the product remains constant.
6. Explain the relationship observed.

Sample investigation

Suppose a task requires 120 worker-days.

Number of workers	2	3	4	5	6
Time (days)	60	40	30	24	20
Workers × time	120	120	120	120	120

Questions:

a) What happens to the time when the number of workers increases?

b) What is the constant of variation?

c) Form the equation connecting workers and time.

d) Find the time required by 10 workers.

Activity 3: Investigating joint variation using a rectangular field

Learning objective: To establish the relationship between the area, length and width of a rectangular field.

Procedure

1. Draw a rectangular field on graph paper.
2. Measure its length and width.
3. Calculate its area.
4. Change the length and width.
5. Calculate the new area.
6. Discuss how the area depends on both dimensions.

Learner task

A rectangular school farm has a length of 60 metres and width of 40 metres.

a) Calculate its area.

b) If the length is increased to 80 metres while the width remains unchanged, calculate the new area.

c) If both dimensions are doubled, what happens to the area?

d) Write the mathematical relationship between area, length and width.

Activity 4: Investigating combined variation through transport

Learning objective: To investigate the relationship between distance, speed and time.

Procedure

1. Identify a familiar journey, such as Kilgoris to Narok.
2. Record an assumed travelling distance.
3. Calculate the time taken at different constant speeds.
4. Compare the results.
5. Establish the relationship between speed, distance and time.
6. Form the appropriate mathematical equation.

Learner task

A vehicle covers 180 km.

Complete the table:

Speed (km/h)	30	45	60	90
Time (hours)				

Questions:

a) What happens to travelling time as speed increases?

b) Is the relationship direct or inverse when distance is fixed?

c) Determine the time taken at 90 km/h.

d) Explain one practical application of this relationship.

Activity 5: Group project – Variations in the local community

Project title: Investigating mathematical variations in our community.

Learners should work in groups to identify at least four situations in their community that demonstrate different types of variations.

Possible areas include:

- Local markets.
- School canteens.
- Transport services.
- Agricultural activities.
- Construction projects.
- Water supply systems.
- Digital technology.

Each group should:

1. Identify a real-life situation.
2. Identify the quantities involved.
3. State the type of variation.
4. Collect or construct relevant numerical data.
5. Form a mathematical equation.
6. Determine the constant of variation.
7. Solve a practical problem using the equation.
8. Present findings to the class.

Expected project outcome: A written report containing tables, mathematical equations, calculations and explanations of the applications of variations.

9.0 SUMMARY OF IMPORTANT FORMULAE

9.1 Direct variation

$$\boxed{y = kx}$$

Constant:

$$\boxed{k = \frac{y}{x}}$$

Ratio:

$$\boxed{\frac{y_1}{x_1} = \frac{y_2}{x_2}}$$

9.2 Inverse variation

$$\boxed{y = \frac{k}{x}}$$

Constant:

$$\boxed{k = xy}$$

Ratio:

$$\boxed{x_1 y_1 = x_2 y_2}$$

9.3 Joint variation

If y varies jointly as x and z:

$$\boxed{y = kxz}$$

Constant:

$$\boxed{k = \frac{y}{xz}}$$

If y varies jointly as x^2 and z:

$$\boxed{y = kx^2z}$$

If y varies jointly as x, y and z:

$$\boxed{w = kxyz}$$

9.4 Combined variation

If y varies directly as x and inversely as z:

$$\boxed{y = \frac{kx}{z}}$$

Constant:

$$\boxed{k = \frac{yz}{x}}$$

If y varies directly as x^2 and inversely as z:

$$\boxed{y = \frac{kx^2}{z}}$$

If y varies directly as x and z, and inversely as w:

$$\boxed{y = \frac{kxz}{w}}$$

9.5 Graphical representation

Type of variation	Nature of graph
Direct variation	Straight line through the origin
Inverse variation	Rectangular hyperbola
Joint variation	Depends on the number of variables and their powers
Combined variation	Depends on the variables and their powers

PART TWO: REVISION EXERCISES

SECTION A: MULTIPLE-CHOICE QUESTIONS (30 MARKS)

Instructions: Answer all questions. Choose the correct answer from alternatives A, B, C and D. Each question carries 1 mark.

1. What is meant by variation in mathematics?

A. A relationship between changing quantities.

B. A method of adding numbers.

C. A process of dividing fractions.

D. A method of drawing geometrical shapes.

2. Which symbol is used to represent proportionality?

A. $=$

B. $\propto$

C. $\div$

D. $\neq$

3. If y varies directly as x, which equation represents the relationship?

A. $y = \frac{k}{x}$

B. $y = k + x$

C. $y = kx$

D. $y = x - k$

4. If y varies directly as x, and $y = 20$ when $x = 4$, find the constant of variation.

A. 4

B. 5

C. 16

D. 24

5. Given that y varies directly as x, and $y = 18$ when $x = 6$, find y when $x = 10$.

A. 24

B. 30

C. 36

D. 60

6. The cost of identical exercise books varies directly as the number purchased. If one book costs Ksh 75, find the cost of 8 books.

A. Ksh 300

B. Ksh 450

C. Ksh 600

D. Ksh 750

7. Which of the following is an example of direct variation?

A. Speed and time for a fixed distance.

B. Number of workers and time for a fixed task.

C. Total cost and number of identical items purchased at a fixed price.

D. Number of taps and time taken to fill a tank.

8. If y varies inversely as x, which equation represents the relationship?

A. $y = kx$

B. $y = k + x$

C. $y = \frac{k}{x}$

D. $y = x^2$

9. If y varies inversely as x, and $y = 8$ when $x = 5$, find the constant of variation.

A. 13

B. 20

C. 40

D. 45

10. Given that y varies inversely as x, and $y = 12$ when $x = 4$, find y when $x = 8$.

A. 3

B. 4

C. 6

D. 24

11. Twelve workers complete a task in 10 days. How many days will 20 workers take to complete the same task at the same rate?

A. 4 days

B. 6 days

C. 8 days

D. 12 days

12. Which graph represents direct variation?

A. A straight line passing through the origin.

B. A circle.

C. A rectangular hyperbola.

D. A horizontal curve.

13. Which graph represents inverse variation for a positive constant?

A. A straight line through the origin.

B. A rectangular hyperbola.

C. A vertical straight line.

D. A circle.

14. The relationship $xy = 48$ represents:

A. Direct variation.

B. Joint variation.

C. Inverse variation.

D. Combined variation.

15. If y varies jointly as x and z, which equation is correct?

A. $y = kx$

B. $y = \frac{k}{xz}$

C. $y = kxz$

D. $y = \frac{kx}{z}$

16. If y varies jointly as x and z, and $y = 60$ when $x = 3$ and $z = 4$, find k.

A. 3

B. 4

C. 5

D. 12

17. The area of a rectangle varies jointly as:

A. Length and width.

B. Length and time.

C. Width and speed.

D. Length and mass.

18. The volume of a cylinder varies jointly as:

A. Radius and height.

B. Square of radius and height.

C. Radius and square of height.

D. Square of height only.

19. If $V = kr^2h$, and $k = 3$, $r = 2$, $h = 5$, find V.

A. 30

B. 40

C. 50

D. 60

20. If y varies directly as x and inversely as z, which equation represents the relationship?

A. $y = kxz$

B. $y = \frac{kx}{z}$

C. $y = \frac{kz}{x}$

D. $y = k(x + z)$

21. If y varies directly as x and inversely as z, and $y = 12$ when $x = 4$ and $z = 2$, find k.

A. 3

B. 4

C. 6

D. 8

22. Given that $y = \frac{6x}{z}$, find y when $x = 5$ and $z = 3$.

A. 5

B. 10

C. 15

D. 30

23. The time taken to complete a fixed task varies inversely as:

A. The amount of work.

B. The number of workers.

C. The distance travelled.

D. The size of the building.

24. Which of the following represents combined variation?

A. $y = kx$

B. $y = \frac{k}{x}$

C. $y = \frac{kxz}{w}$

D. $y = k + x$

25. If y varies directly as the square of x and inversely as z, the relationship is:

A. $y = kx^2z$

B. $y = \frac{kx^2}{z}$

C. $y = \frac{kz}{x^2}$

D. $y = kxz^2$

26. A vehicle covers 240 km at a constant speed of 60 km/h. How long does it take?

A. 2 hours

B. 3 hours

C. 4 hours

D. 5 hours

27. If a farmer harvests 600 kg of maize from 2 acres, how much will be harvested from 5 acres at the same yield?

A. 1,000 kg

B. 1,200 kg

C. 1,500 kg

D. 2,000 kg

28. Four identical taps fill a tank in 15 hours. How long will six identical taps take to fill the same tank?

A. 5 hours

B. 10 hours

C. 12 hours

D. 20 hours

29. Which of the following is an application of variation in digital technology?

A. Organising school desks.

B. Relating file size to downloading time at a constant transfer speed.

C. Measuring the height of a tree without a measuring instrument.

D. Naming computer components.

30. What is the main purpose of studying variations?

A. To understand relationships between changing quantities.

B. To memorise mathematical symbols only.

C. To avoid using mathematical equations.

D. To study geometrical shapes only.

SECTION B: STRUCTURED QUESTIONS (50 MARKS)

Instructions

1. Answer all questions.
2. Show all necessary working.
3. Use appropriate mathematical formulae.
4. Give answers in the correct units.
5. Write equations clearly.

QUESTION 1: DIRECT VARIATION (8 MARKS)

The cost C of purchasing exercise books varies directly as the number n of books purchased.

A learner pays Ksh 240 for 4 exercise books.

a) Write down the general equation representing direct variation. (1 mark)

b) Calculate the constant of variation. (2 marks)

c) Form an equation connecting the cost and the number of exercise books. (1 mark)

d) Calculate the cost of purchasing 15 exercise books. (2 marks)

e) Determine the number of exercise books that can be purchased for Ksh 1,080. (2 marks)

QUESTION 2: INVERSE VARIATION (8 MARKS)

The time T taken to complete a task varies inversely as the number n of workers employed.

Eight workers complete the task in 15 days.

a) Write the mathematical relationship between T and n. (1 mark)

b) Determine the constant of variation. (2 marks)

c) Form the equation connecting T and n. (1 mark)

d) Calculate the time required if 12 workers are employed. (2 marks)

e) Determine the number of workers required to complete the task in 6 days. (2 marks)

QUESTION 3: JOINT VARIATION (8 MARKS)

The volume V of a cylindrical container varies jointly as the square of its radius r and its height h.

When the radius is 7 cm and the height is 10 cm, the volume is $1540\ \text{cm}^3$.

a) Write the variation equation connecting V, r and h. (1 mark)

b) Determine the constant of variation. (2 marks)

c) Form the complete equation. (1 mark)

d) Calculate the volume when the radius is 14 cm and the height is 5 cm. (3 marks)

e) State one practical application of joint variation. (1 mark)

QUESTION 4: COMBINED VARIATION (8 MARKS)

The quantity P varies directly as the square of x and inversely as y.

When $x = 4$, $y = 2$ and $P = 48$:

a) Write the variation equation. (1 mark)

b) Determine the constant of variation. (2 marks)

c) Form the complete equation. (1 mark)

d) Calculate P when $x = 6$ and $y = 3$. (2 marks)

e) Find y when $P = 96$ and $x = 4$. (2 marks)

QUESTION 5: APPLICATIONS OF VARIATIONS (9 MARKS)

A transport company operates buses between two towns in Kenya. A bus covers a distance of 360 km at a constant speed of 60 km/h.

a) State the relationship between distance, speed and time. (1 mark)

b) Calculate the time taken to cover the 360 km journey. (2 marks)

c) Determine the time taken to cover the same distance at a speed of 90 km/h. (2 marks)

d) Explain the type of variation between speed and time when the distance is fixed. (2 marks)

e) State two other real-life applications of variations. (2 marks)

QUESTION 6: COMBINED PROBLEMS AND INTERPRETATION (9 MARKS)

The time T taken to complete a construction project varies directly as the amount of work W and inversely as the number of workers n.

A group of 10 workers completes 120 units of work in 6 days.

a) Form the variation equation connecting T, W and n. (1 mark)

b) Calculate the constant of variation. (2 marks)

c) Form the complete equation. (1 mark)

d) Calculate the time taken by 15 workers to complete 200 units of work. (3 marks)

e) Explain why employing more workers can reduce the time taken to complete a fixed amount of work. (2 marks)

PART THREE: ANSWER KEY AND TEACHER'S MARKING GUIDE

A. Multiple-choice answer key

Question	Answer	Question	Answer	Question	Answer
1	A	11	B	21	C
2	B	12	A	22	B
3	C	13	B	23	B
4	B	14	C	24	C
5	B	15	C	25	B
6	C	16	C	26	C
7	C	17	A	27	C
8	C	18	B	28	B
9	C	19	D	29	B
10	C	20	B	30	A

B. Structured questions: Marking scheme

QUESTION 1: DIRECT VARIATION (8 MARKS)

a) General equation:

$$C = kn$$

b) Given:

$$240 = 4k$$

$$k = \frac{240}{4}$$

$$\boxed{k = 60}$$

c) Complete equation:

$$\boxed{C = 60n}$$

d) When $n = 15$:

$$C = 60 \times 15$$

$$\boxed{C = \text{Ksh } 900}$$

$$1080 = 60n$$

$$n = \frac{1080}{60}$$

$$\boxed{n = 18 \text{ books}}$$

QUESTION 2: INVERSE VARIATION (8 MARKS)

$$\boxed{T = \frac{k}{n}}$$

$$k = nT$$

$$k = 8 \times 15$$

$$\boxed{k = 120}$$

$$\boxed{T = \frac{120}{n}}$$

d) When $n = 12$:

$$T = \frac{120}{12}$$

$$\boxed{T = 10 \text{ days}}$$

e) When $T = 6$:

$$6 = \frac{120}{n}$$

$$6n = 120$$

$$n = \frac{120}{6}$$

$$\boxed{n = 20 \text{ workers}}$$

QUESTION 3: JOINT VARIATION (8 MARKS)

$$\boxed{V = kr^2h}$$

$$1540 = k \times 7^2 \times 10$$

$$1540 = 490k$$

$$k = \frac{1540}{490}$$

$$\boxed{k = \frac{22}{7}}$$

$$\boxed{V = \frac{22}{7}r^2h}$$

d) When $r = 14$ and $h = 5$:

$$V = \frac{22}{7} \times 14^2 \times 5$$

$$V = \frac{22}{7} \times 196 \times 5$$

$$V = 22 \times 28 \times 5$$

$$\boxed{V = 3080 \text{ cm}^3}$$

e) Any one:

- Calculating the volume of cylindrical water tanks.
- Determining the volume of cylindrical containers.
- Calculating the volume of industrial storage tanks.

QUESTION 4: COMBINED VARIATION (8 MARKS)

$$\boxed{P = \frac{kx^2}{y}}$$

$$48 = \frac{k \times 4^2}{2}$$

$$48 = 8k$$

$$\boxed{k = 6}$$

$$\boxed{P = \frac{6x^2}{y}}$$

d) When $x = 6$ and $y = 3$:

$$P = \frac{6 \times 6^2}{3}$$

$$P = \frac{216}{3}$$

$$\boxed{P = 72}$$

$$96 = \frac{6 \times 4^2}{y}$$

$$96 = \frac{96}{y}$$

$$96y = 96$$

$$\boxed{y = 1}$$

QUESTION 5: APPLICATIONS OF VARIATIONS (9 MARKS)

$$\boxed{d = vt}$$

$$t = \frac{d}{v}$$

$$t = \frac{360}{60}$$

$$\boxed{t = 6 \text{ hours}}$$

$$t = \frac{360}{90}$$

$$\boxed{t = 4 \text{ hours}}$$

d) Inverse variation. When the distance is fixed, increasing the speed reduces the time taken proportionally. (2)

e) Any two:

- Number of workers and time taken to complete a task.
- Number of items purchased and total cost.
- Distance and time at constant speed.

- Number of taps and time taken to fill a tank.
- Land area and crop production at constant yield.

QUESTION 6: COMBINED PROBLEMS AND INTERPRETATION (9 MARKS)

$$\boxed{T = \frac{kW}{n}}$$

Given:

$$T = 6, \quad W = 120, \quad n = 10$$

$$6 = \frac{k \times 120}{10}$$

$$6 = 12k$$

$$\boxed{k = \frac{1}{2}}$$

$$\boxed{T = \frac{W}{2n}}$$

d) When $W = 200$ and $n = 15$:

$$T = \frac{200}{2 \times 15}$$

$$T = \frac{200}{30}$$

$$T = \frac{20}{3}$$

$$\boxed{T = 6\frac{2}{3} \text{ days}}$$

e) Increasing the number of workers increases the total working capacity, assuming equal productivity. Therefore, a fixed amount of work can be completed in less time. (2)

STRAND 1.0: NUMBERS AND ALGEBRA

SUB-STRAND 1.4: SEQUENCES AND SERIES

2.0 Number sequences

2.1 Meaning of a sequence

A **sequence** is an ordered list of numbers or objects arranged according to a definite rule or pattern.

Each number in a sequence is called a **term**.

The position of a term is important because changing the order may change the sequence.

Examples of sequences:

1. $2,4,6,8,10,12,\ldots$
2. $5,10,15,20,25,\ldots$
3. $3,6,12,24,48,\ldots$
4. $20,17,14,11,8,\ldots$
5. $1,4,9,16,25,\ldots$

Each sequence follows a particular mathematical rule.

2.2 Important terms used in sequences

Mathematical term	Meaning
Sequence	An ordered list of numbers following a rule
Term	Each individual number in a sequence
First term	The number in the first position
Second term	The number in the second position
nth term	The term occupying position n
Missing term	A term whose value is not provided
Pattern	The rule followed by the terms
Finite sequence	A sequence with a definite number of terms
Infinite sequence	A sequence that continues without an indicated final term

Example 1: Identifying terms in a sequence

Consider the sequence:

$$4,\ 8,\ 12,\ 16,\ 20,\ 24,\ldots$$

The terms are:

- First term = 4
- Second term = 8
- Third term = 12
- Fourth term = 16
- Fifth term = 20
- Sixth term = 24

The sequence increases by 4 each time.

2.3 Recognising number patterns

To identify the rule in a sequence, compare consecutive terms.

The following methods are useful:

1. **Addition:** Check whether the same number is added each time.
2. **Subtraction:** Check whether the same number is subtracted each time.
3. **Multiplication:** Check whether each term is multiplied by the same number.
4. **Division:** Check whether each term is divided by the same number.
5. **Other patterns:** Check whether terms are squares, cubes or formed by another mathematical rule.

Example 2: Identifying the pattern

Find the rule in the sequence:

$$7,\ 12,\ 17,\ 22,\ 27,\ldots$$

Solution

Compare consecutive terms:

$$12 - 7 = 5$$

$$17 - 12 = 5$$

$$22 - 17 = 5$$

$$27 - 22 = 5$$

The difference between consecutive terms is always 5.

Rule: Add 5 to the previous term.

Example 3: Identifying a multiplication pattern

Find the rule in:

$$3,\ 6,\ 12,\ 24,\ 48,\ldots$$

Solution

$$\frac{6}{3} = 2, \quad \frac{12}{6} = 2, \quad \frac{24}{12} = 2$$

Each term is obtained by multiplying the preceding term by 2.

Rule: Multiply by 2.

2.4 Determining missing terms in a sequence

A missing term is a number that has been left out of a sequence. To find it, identify the rule followed by the given terms.

Example 4: Finding a missing term

Find the missing number:

$$6,\ 10,\ \square,\ 18,\ 22$$

Solution

The difference between the first two terms is:

$$10 - 6 = 4$$

The sequence increases by 4.

Therefore:

$$10 + 4 = 14$$

The missing number is **14**.

Example 5: Finding two missing terms

Find the missing terms:

$$5,\ 10,\ \square,\ \square,\ 25$$

Solution

The sequence increases by 5.

$$5,\ 10,\ 15,\ 20,\ 25$$

The missing terms are **15 and 20**.

Example 6: Missing terms in a decreasing sequence

Find the missing number:

$$40,\ 34,\ 28,\ \square,\ 16$$

Solution

The difference is:

$$34 - 40 = -6$$

Each term decreases by 6.

$$28 - 6 = 22$$

The missing number is **22**.

2.5 Number sequences in everyday life

Sequences can be observed in many activities in Kenya.

1. Seating arrangements

Learners may be seated in rows where each successive row contains a fixed number of additional learners.

Example: 8, 12, 16, 20, 24 learners.

2. Bus and matatu schedules

Vehicles may depart at regular time intervals.

Example: 6:00 a.m., 6:30 a.m., 7:00 a.m., 7:30 a.m.

3. Farming activities

A farmer may plant crops in rows with a fixed increase in the number of plants in each row.

Example: 10, 15, 20, 25 plants.

4. Stacking objects

Boxes, bricks or other objects can be stacked in layers following a numerical pattern.

5. Saving money

A learner may save a fixed amount each week, creating a sequence of savings totals.

3.0 Arithmetic sequences

3.1 Meaning of an arithmetic sequence

An **arithmetic sequence** is a sequence in which the difference between any two consecutive terms is constant.

This constant difference is called the **common difference**.

For example:

$$3,\ 7,\ 11,\ 15,\ 19, \ldots$$

The difference between consecutive terms is:

$$7 - 3 = 4$$

$$11 - 7 = 4$$

$$15 - 11 = 4$$

Therefore, the common difference is 4.

Other examples include:

- $2,5,8,11,14, \ldots$
- $10,20,30,40,50, \ldots$
- $50,45,40,35,30, \ldots$
- $100,90,80,70,60, \ldots$

An arithmetic sequence may increase, decrease or remain constant.

3.2 Parts of an arithmetic sequence

Consider:

$$6,\ 10,\ 14,\ 18,\ 22, \ldots$$

The important parts are:

- First term: $a = 6$
- Common difference: $d = 4$
- Second term: $a + d = 10$
- Third term: $a + 2d = 14$
- Fourth term: $a + 3d = 18$
- Fifth term: $a + 4d = 22$

The general arrangement is:

$$a,\ a + d,\ a + 2d,\ a + 3d, \ldots$$

Where:

- a represents the first term.
- d represents the common difference.

3.3 Common difference

The **common difference** is the fixed amount added to or subtracted from each term to obtain the next term in an arithmetic sequence.

It is represented by the letter d.

Formula

$$\boxed{d = T_2 - T_1}$$

More generally:

$$\boxed{d = T_{n+1} - T_n}$$

Where:

- T_1 is the first term.
- T_2 is the second term.
- T_n is the nth term.

Example 7: Finding the common difference

Find the common difference of:

$$8,\ 13,\ 18,\ 23,\ 28, \ldots$$

Solution

$$d = 13 - 8$$

$$\boxed{d = 5}$$

Example 8: Finding a negative common difference

Find the common difference of:

$$60,\ 52,\ 44,\ 36,\ 28, \ldots$$

Solution

$$d = 52 - 60$$

$$\boxed{d = -8}$$

The negative sign indicates that the sequence is decreasing.

3.4 Determining the first term and common difference

Sometimes a sequence is given without explicitly identifying its first term and common difference. These can be determined by examining the sequence.

Example 9

Given the arithmetic sequence:

$$15,\ 21,\ 27,\ 33,\ 39, \ldots$$

Determine:

a) The first term.

b) The common difference.

Solution

a) The first term is the first number:

$$\boxed{a = 15}$$

b) The common difference is:

$$d = 21 - 15$$

$$\boxed{d = 6}$$

3.5 Arithmetic sequences with missing terms

A missing term can be determined using the common difference.

Example 10

Find the missing term in:

$$12,\ 17,\ \boxed{\ \ },\ 27,\ 32$$

Solution

First determine the common difference:

$$d = 17 - 12 = 5$$

The missing term is:

$$17 + 5 = 22$$

Therefore:

$$\boxed{22}$$

4.0 The nth term of an arithmetic sequence

4.1 Meaning of the nth term

The **nth term** is the term found at a particular position n in a sequence.

It enables us to calculate any term in an arithmetic sequence without writing down all the preceding terms.

For example, if we are given:

$$4,\ 7,\ 10,\ 13,\ 16, \ldots$$

We can determine the 50th term directly using a mathematical formula rather than listing all 50 terms.

4.2 Deriving the formula for the nth term

Consider the arithmetic sequence:

$$a,\ a+d,\ a+2d,\ a+3d, \ldots$$

The terms can be represented as follows:

Position	Term
1st	a
2nd	$a+d$
3rd	$a+2d$
4th	$a+3d$
5th	$a+4d$
nth	$a+(n-1)d$

Observe that the number of common differences added to the first term is always one less than the position of the term.

Therefore, the formula for the nth term is:

$$T_n = a + (n-1)d$$

Formula for the nth term of an arithmetic sequence

Where:

- T_n = nth term.
- a = first term.
- n = position of the required term.
- d = common difference.

Important: The value of n is the position of the term, not the number of terms already written.

4.3 Finding a particular term using the nth-term formula

Example 11

Find the 20th term of the arithmetic sequence:

$$5,\ 8,\ 11,\ 14, \ldots$$

Solution

Step 1: Identify the first term.

$$a = 5$$

Step 2: Find the common difference.

$$d = 8 - 5 = 3$$

Step 3: Identify the required position.

$$n = 20$$

Step 4: Substitute into the formula.

$$T_n = a + (n-1)d$$

$$T_{20} = 5 + (20-1) \times 3$$

$$= 5 + 19 \times 3$$

$$= 5 + 57$$

$$\boxed{T_{20} = 62}$$

Example 12: Finding a term in a decreasing sequence

Find the 15th term of:

$$80,\ 75,\ 70,\ 65, \ldots$$

Solution

First term:

$$a = 80$$

Common difference:

$$d = 75 - 80 = -5$$

Position:

$$n = 15$$

Using:

$$T_n = a + (n-1)d$$

$$T_{15} = 80 + (15-1)(-5)$$

$$= 80 + 14(-5)$$

$$= 80 - 70$$

$$\boxed{T_{15} = 10}$$

Example 13: Finding a distant term

A sequence begins:

$$7,\ 12,\ 17,\ 22, \ldots$$

Find the 100th term.

Solution

$$a = 7, \quad d = 5, \quad n = 100$$

$$T_{100} = 7 + (100-1) \times 5$$

$$= 7 + 99 \times 5$$

$$= 7 + 495$$

$$\boxed{T_{100} = 502}$$

4.4 Finding the position of a given term

Sometimes the term is known, but its position is unknown.

In such cases, substitute the known term into the nth-term formula and solve for n.

Example 14

Which term of the arithmetic sequence

$$4,\ 9,\ 14,\ 19, \ldots$$

is 79?

Solution

First term:

$$a = 4$$

Common difference:

$$d = 5$$

Given term:

$$T_n = 79$$

Using:

$$T_n = a + (n-1)d$$

$$79 = 4 + (n-1)5$$

$$79 - 4 = 5(n-1)$$

$$75 = 5(n-1)$$

$$15 = n - 1$$

$$\boxed{n = 16}$$

Therefore, 79 is the 16th term.

4.5 Finding the first term when other information is given

Example 15

The 10th term of an arithmetic sequence is 38 and its common difference is 4. Find the first term.

Solution

Given:

$$T_{10} = 38, \quad d = 4$$

Using:

$$T_n = a + (n-1)d$$

$$38 = a + (10-1) \times 4$$

$$38 = a + 36$$

$$a = 38 - 36$$

$$\boxed{a = 2}$$

4.6 Finding the common difference using two terms

Example 16

The first term of an arithmetic sequence is 6 and the 12th term is 50. Determine the common difference.

Solution

Given:

$$a = 6, \quad T_{12} = 50$$

Using:

$$T_n = a + (n-1)d$$

$$50 = 6 + (12-1)d$$

$$50 = 6 + 11d$$

$$44 = 11d$$

$$\boxed{d = 4}$$

5.0 Arithmetic progression (AP)

5.1 Meaning of an arithmetic progression

An **arithmetic progression (AP)** is an ordered arrangement of numbers in which the difference between consecutive terms is constant.

An arithmetic progression is the same type of sequence as an arithmetic sequence. The term *progression* is commonly used when considering the ordered terms and their sum.

Examples:

$$2,\ 4,\ 6,\ 8,\ 10, \ldots$$

$$5,\ 10,\ 15,\ 20,\ 25, \ldots$$

$$30,\ 25,\ 20,\ 15,\ 10, \ldots$$

5.2 Types of arithmetic progressions

1. Increasing arithmetic progression

The common difference is positive.

Example:

$$3,\ 7,\ 11,\ 15,\ 19, \ldots$$

Here, $d = 4$.

2. Decreasing arithmetic progression

The common difference is negative.

Example:

$$50,\ 45,\ 40,\ 35,\ 30,\ldots$$

Here, $d = -5$.

3. Constant arithmetic progression

The common difference is zero.

Example:

$$8,\ 8,\ 8,\ 8,\ 8,\ldots$$

Here, $d = 0$.

5.3 Difference between a sequence and a series

These two mathematical terms are related but have different meanings.

Sequence	Series
An ordered list of numbers	The sum of the terms of a sequence
Terms are separated by commas	Terms are added together
Example: $2,4,6,8$	Example: $2 + 4 + 6 + 8$
Focuses on the arrangement of terms	Focuses on the total of the terms

Example:

Sequence:

$$3,\ 6,\ 9,\ 12,\ 15$$

Corresponding series:

$$3 + 6 + 9 + 12 + 15 = 45$$

The sum of the five terms is 45.

6.0 Sum of an arithmetic progression

6.1 Meaning of the sum of an arithmetic progression

The **sum of an arithmetic progression** is the total obtained when all the terms in a given arithmetic progression are added together.

It is commonly represented by S_n, where n is the number of terms.

For example, consider:

$$2,\ 4,\ 6,\ 8,\ 10$$

The sum is:

$$S_5 = 2 + 4 + 6 + 8 + 10$$

$$\boxed{S_5 = 30}$$

For a progression containing many terms, using a formula makes the calculation easier and faster.

6.2 Deriving the formula for the sum of an AP

Consider the arithmetic progression:

$$a,\ a+d,\ a+2d,\ \ldots,\ l$$

Let the sum of the terms be:

$$S_n = a + (a+d) + (a+2d) + \cdots + l$$

Write the same sum in reverse order:

$$S_n = l + (l-d) + (l-2d) + \cdots + a$$

Add the two equations vertically:

$$2S_n = (a+l) + (a+l) + (a+l) + \cdots + (a+l)$$

There are n pairs, each with a sum of $a + l$.

Therefore:

$$2S_n = n(a+l)$$

Dividing both sides by 2 gives:

$$\boxed{S_n = \frac{n}{2}(a+l)}$$

Where:

- S_n = sum of the first n terms.
- n = number of terms.
- a = first term.
- l = last term.

Since:

$$l = a + (n-1)d$$

The formula can also be written as:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Sum of the first n terms when the common difference is known

6.3 When to use each formula

Information available	Formula to use
First term, last term and number of terms	$S_n = \frac{n}{2}(a+l)$
First term, common difference and number of terms	$S_n = \frac{n}{2}[2a + (n-1)d]$

6.4 Calculating the sum using the first and last terms

Example 17

Find the sum of the first 10 terms of the arithmetic progression:

$$3,\ 6,\ 9,\ 12, \ldots$$

Solution

First term:

$$a = 3$$

Common difference:

$$d = 3$$

Number of terms:

$$n = 10$$

First find the 10th term:

$$l = a + (n-1)d$$

$$l = 3 + (10-1) \times 3$$

$$l = 3 + 27 = 30$$

Now use:

$$S_n = \frac{n}{2}(a + l)$$

$$S_{10} = \frac{10}{2}(3 + 30)$$

$$= 5 \times 33$$

$$\boxed{S_{10} = 165}$$

6.5 Calculating the sum using the common difference

Example 18

Find the sum of the first 20 terms of:

$$5,\ 8,\ 11,\ 14, \ldots$$

Solution

Given:

$$a = 5, \quad d = 3, \quad n = 20$$

Using:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Substitute:

$$S_{20} = \frac{20}{2}[2(5) + (20-1)3]$$

$$= 10[10 + 57]$$

$$= 10 \times 67$$

$$\boxed{S_{20} = 670}$$

6.6 Finding the sum of a decreasing AP

Example 19

Find the sum of the first 12 terms of:

$$60,\ 55,\ 50,\ 45,\ldots$$

Solution

$$a = 60, \quad d = -5, \quad n = 12$$

Using:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}[2(60) + (12-1)(-5)]$$

$$= 6[120 - 55]$$

$$= 6 \times 65$$

$$\boxed{S_{12} = 390}$$

6.7 Finding the number of terms when the sum is known

Example 20

The sum of the first n terms of an AP is 210. If the first term is 2 and the common difference is 2, find n.

Solution

Given:

$$S_n = 210, \quad a = 2, \quad d = 2$$

Using:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Substitute:

$$210 = \frac{n}{2}[2(2) + (n-1)2]$$

$$210 = \frac{n}{2}[4 + 2n - 2]$$

$$210 = \frac{n}{2}(2n + 2)$$

$$210 = n(n+1)$$

$$n^2 + n - 210 = 0$$

Factorise:

$$(n+15)(n-14) = 0$$

$$n = -15 \quad \text{or} \quad n = 14$$

Since the number of terms cannot be negative:

$\boxed{n = 14}$

7.0 Graphical representation of arithmetic sequences

Arithmetic sequences can be represented graphically by plotting the position of each term against its value.

Consider the sequence:

$$2,\ 5,\ 8,\ 11,\ 14,\ 17$$

The common difference is 3.

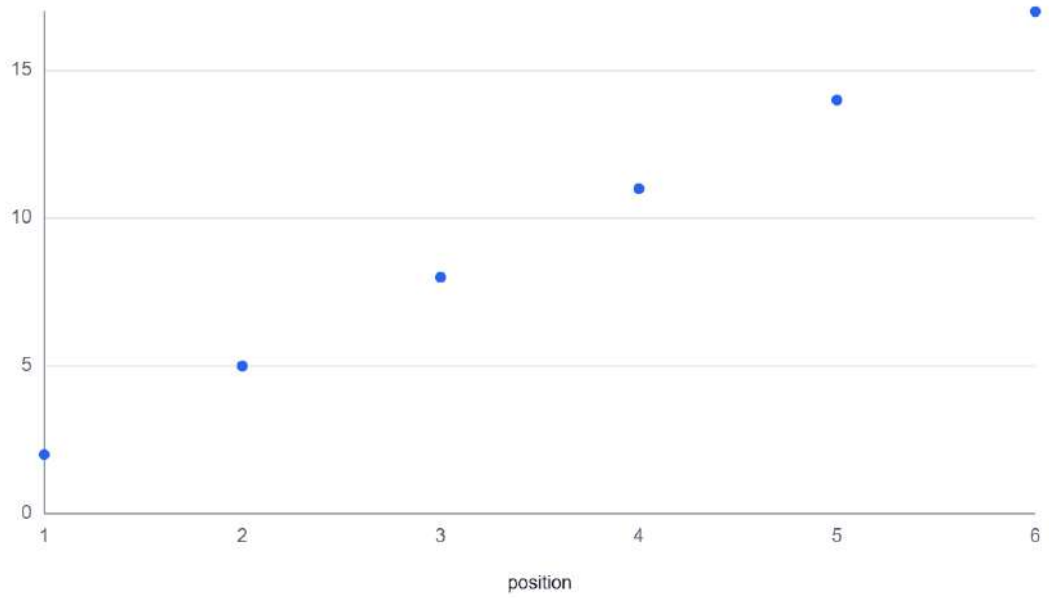
Term position, n	Term value, T_n
1	2
2	5
3	8
4	11
5	14
6	17

The points are:

$$(1,2), (2,5), (3,8), (4,11), (5,14), (6,17)$$

Graph of an arithmetic sequence

Term value plotted against term position for 2, 5, 8, 11, 14 and 17.



position	Term value
1	2
2	5
3	8

position	Term value
4	11
5	14
6	17

Observations

1. The plotted points lie along a straight-line pattern.
2. The term values increase by 3 for every increase of 1 in position.
3. The points are discrete because the term positions are whole numbers.
4. The points should not be interpreted as showing that every value between consecutive terms is itself a term of the sequence.

The nth-term rule is:

$$T_n = 2 + (n-1)3$$

Simplifying:

$$\boxed{T_n = 3n - 1}$$

7.1 Relationship between an arithmetic sequence and a linear expression

The nth-term formula for an arithmetic sequence is:

$$T_n = a + (n-1)d$$

Expanding:

$$T_n = a + nd - d$$

Therefore:

$$\boxed{T_n = dn + (a-d)}$$

This has the form of a linear expression:

$$y = mx + c$$

Where:

- The common difference d corresponds to the coefficient of n.
- The expression $a - d$ is the constant term.

This explains why the plotted terms of an arithmetic sequence form a straight-line pattern.

8.0 Geometric sequences

Although this sub-strand focuses mainly on arithmetic progressions, it is useful to distinguish arithmetic sequences from geometric sequences because both are common number patterns.

8.1 Meaning of a geometric sequence

A **geometric sequence** is a sequence in which each term after the first is obtained by multiplying the preceding term by a fixed non-zero number called the **common ratio**.

Examples:

$$2,\ 4,\ 8,\ 16,\ 32,\ldots$$

$$3,\ 9,\ 27,\ 81,\ldots$$

$$64,\ 32,\ 16,\ 8,\ 4,\ldots$$

The common ratio is represented by r.

8.2 Common ratio

The common ratio is obtained by dividing a term by the term immediately before it.

$$\boxed{r = \frac{\text{second term}}{\text{first term}}}$$

More generally:

$$\boxed{r = \frac{T_{n+1}}{T_n}}$$

Example 21: Finding the common ratio

Find the common ratio of:

$$5,\ 15,\ 45,\ 135,\ldots$$

Solution

$$r = \frac{15}{5}$$

$$\boxed{r = 3}$$

Each term is multiplied by 3 to obtain the next term.

Example 22: A decreasing geometric sequence

Find the common ratio of:

$$80,\ 40,\ 20,\ 10,\ 5,\ldots$$

Solution

$$r = \frac{40}{80}$$

$$\boxed{r = \frac{1}{2}}$$

Each term is multiplied by $\frac{1}{2}$.

8.3 The nth term of a geometric sequence

Consider the geometric sequence:

$$a,\ ar,\ ar^2,\ ar^3,\ldots$$

The terms are:

Position	Term
1	a
2	ar

Position	Term
3	ar^2
4	ar^3
5	ar^4
n	ar^{n-1}

Therefore, the nth-term formula is:

$$T_n = ar^{n-1}$$

Nth term of a geometric sequence

Where:

- T_n = nth term.
- a = first term.
- r = common ratio.
- n = position of the required term.

Example 23: Finding a term in a geometric sequence

Find the 6th term of:

$$2,\ 6,\ 18,\ 54, \ldots$$

Solution

First term:

$$a = 2$$

Common ratio:

$$r = \frac{6}{2} = 3$$

Position:

$$n = 6$$

Using:

$$T_n = ar^{n-1}$$

$$T_6 = 2(3)^{6-1}$$

$$= 2(3^5)$$

$$= 2 \times 243$$

$$\boxed{T_6 = 486}$$

Example 24: Finding the 5th term of a decreasing geometric sequence

Find the 5th term of:

$$160,\ 80,\ 40,\ 20, \ldots$$

Solution

$$a = 160, \quad r = \frac{1}{2}, \quad n = 5$$

$$T_5 = 160\left(\frac{1}{2}\right)^{5-1}$$

$$= 160\left(\frac{1}{16}\right)$$

$$\boxed{T_5 = 10}$$

8.4 Difference between arithmetic and geometric sequences

Arithmetic sequence	Geometric sequence
Has a constant difference	Has a constant ratio
Uses addition or subtraction	Uses multiplication or division
Common difference is d	Common ratio is r
$T_n = a + (n-1)d$	$T_n = ar^{n-1}$
Example: 3, 6, 9, 12	Example: 3, 6, 12, 24

Important: Do not confuse the common difference with the common ratio. An arithmetic sequence is identified by subtracting consecutive terms, while a geometric sequence is identified by dividing consecutive terms.

9.0 Applications of arithmetic progressions in real life

Arithmetic progressions are useful in situations where a quantity increases or decreases by a fixed amount over equal intervals or successive stages.

9.1 Seating arrangements in schools and public gatherings

In a school hall, learners may be arranged in rows. Each successive row may contain a fixed number of additional learners.

Example 25

During a school event in Nakuru County, chairs are arranged so that the first row has 12 chairs. Each following row has 3 more chairs than the previous row.

Find:

a) The number of chairs in the 10th row.

b) The total number of chairs in the first 10 rows.

Solution

Given:

$$a = 12, \quad d = 3, \quad n = 10$$

a) Number of chairs in the 10th row:

$$T_{10} = a + (n-1)d$$

$$= 12 + (10-1)3$$

$$= 12 + 27$$

$$\boxed{39 \text{ chairs}}$$

b) Total number of chairs in the first 10 rows:

$$S_{10} = \frac{10}{2}(12 + 39)$$

$$= 5 \times 51$$

$$\boxed{255 \text{ chairs}}$$

9.2 Saving money regularly

A learner may decide to save a fixed additional amount every week.

Example 26

A learner saves Ksh 100 in the first week and increases the weekly savings by Ksh 50 every subsequent week.

Calculate:

a) The amount saved in the 12th week.

b) The total amount saved over the 12 weeks.

Solution

Given:

$$a = 100, \quad d = 50, \quad n = 12$$

a) Amount saved in the 12th week:

$$T_{12} = 100 + (12 - 1)50$$

$$= 100 + 550$$

$$\boxed{\text{Ksh } 650}$$

b) Total savings:

$$S_{12} = \frac{12}{2}[2(100) + (12 - 1)50]$$

$$= 6[200 + 550]$$

$$= 6 \times 750$$

$$\boxed{\text{Ksh } 4{,}500}$$

9.3 Bus and matatu departure schedules

Public service vehicles may operate according to fixed time intervals.

Example 27

A matatu leaves a stage at 6:00 a.m. and another leaves every 20 minutes.

Determine:

a) The departure time of the 8th matatu.

b) The time at which the 12th matatu leaves.

Solution

The time intervals form an arithmetic sequence:

$$600,\ 620,\ 640,\ 700, \ldots$$

For easier calculation, measure time in minutes after 6:00 a.m.

$$a = 0, \quad d = 20$$

a) For the 8th matatu:

$$T_8 = 0 + (8 - 1)20$$

$$= 140 \text{ minutes}$$

140 minutes = 2 hours 20 minutes.

Therefore, the 8th matatu leaves at:

$$\boxed{820 \text{ a.m.}}$$

b) For the 12th matatu:

$$T_{12} = 0 + (12 - 1)20$$

$$= 220 \text{ minutes}$$

220 minutes = 3 hours 40 minutes.

Therefore, the 12th matatu leaves at:

$$\boxed{940 \text{ a.m.}}$$

9.4 Tracking daily income

A trader may experience a steady increase in daily sales or income.

Example 28

A trader earns Ksh 500 on the first day and increases daily earnings by Ksh 100 each day.

Calculate:

a) The earnings on the 15th day.

b) The total earnings for the 15 days.

Solution

$$a = 500, \quad d = 100, \quad n = 15$$

a) Earnings on the 15th day:

$$T_{15} = 500 + (15 - 1)100$$

$$= 500 + 1{,}400$$

$$\boxed{\text{Ksh } 1{,}900}$$

b) Total earnings:

$$S_{15} = \frac{15}{2}[2(500) + (15 - 1)100]$$

$$= \frac{15}{2}[1{,}000 + 1{,}400]$$

$$= \frac{15}{2} \times 2{,}400$$

$$\boxed{\text{Ksh } 18{,}000}$$

Note: This example assumes the daily income follows the stated constant-increase pattern.

9.5 Stacking objects in layers

Boxes, bricks, tins and other objects can be arranged in layers.

Example 29

A storekeeper stacks cartons in layers. The first layer contains 5 cartons, and each subsequent layer contains 2 more cartons than the preceding layer.

Find:

a) The number of cartons in the 12th layer.

b) The total number of cartons in the first 12 layers.

Solution

$$a = 5, \quad d = 2, \quad n = 12$$

$$T_{12} = 5 + (12 - 1)2$$

$$= 5 + 22$$

$$\boxed{27 \text{ cartons}}$$

$$S_{12} = \frac{12}{2}(5 + 27)$$

$$= 6 \times 32$$

$$\boxed{192 \text{ cartons}}$$

9.6 Farming and planting activities

Farmers may arrange seedlings, trees or other crops in rows with a fixed increase in the number of plants.

Example 30

A farmer plants 8 seedlings in the first row, 11 in the second row, 14 in the third row and continues the pattern.

Find:

a) The number of seedlings in the 20th row.

b) The total number of seedlings planted in the first 20 rows.

Solution

$$a = 8, \quad d = 3, \quad n = 20$$

$$T_{20} = 8 + (20 - 1)3$$

$$= 8 + 57$$

$$\boxed{65 \text{ seedlings}}$$

$$S_{20} = \frac{20}{2}(8 + 65)$$

$$= 10 \times 73$$

$$\boxed{730 \text{ seedlings}}$$

9.7 Construction and building projects

Arithmetic progressions can be used when quantities of materials increase or decrease by a fixed amount from one stage to another.

Examples include:

- Bricks used in successive rows.
- Blocks arranged in layers.
- Materials delivered in increasing quantities.
- Steps in a staircase with a regular pattern.

Example 31

A contractor uses 25 blocks on the first level of a structure. Each successive level uses 5 more blocks than the previous level. Find the total number of blocks used in 8 levels.

Solution

$$a = 25, \quad d = 5, \quad n = 8$$

First calculate the 8th term:

$$T_8 = 25 + (8 - 1)5$$

$$= 25 + 35 = 60$$

Now calculate the sum:

$$S_8 = \frac{8}{2}(25 + 60)$$

$$= 4 \times 85$$

$$\boxed{340 \text{ blocks}}$$

10.0 Practical activities and CBE learning experiences

These activities enable learners to develop mathematical reasoning, collaboration, communication and problem-solving skills.

Activity 1: Investigating classroom seating

Materials: Chairs, exercise books, pens and a calculator.

Procedure:

1. Arrange chairs in rows.
2. Put 6 chairs in the first row.
3. Add 2 chairs to each succeeding row.
4. Record the number of chairs in each row.
5. Identify the first term and common difference.

6. Calculate the number of chairs in the 10th row.
7. Determine the total number of chairs in the first 10 rows.

Expected sequence:

$$6,\ 8,\ 10,\ 12,\ 14,\ \ldots$$

Discussion: How can the nth-term formula help a school plan seating for a large event?

Activity 2: Investigating regular savings

Procedure:

1. Assume a learner saves Ksh 100 in the first week.
2. Increase the weekly savings by Ksh 20.
3. Write the savings for 10 weeks.
4. Identify the common difference.
5. Calculate the amount saved in the 10th week.
6. Calculate the total savings for 10 weeks.

Activity 3: Investigating a transport timetable

Procedure:

1. Choose a hypothetical bus departure time.
2. Assume a bus leaves every 30 minutes.
3. Record the first ten departure times.
4. Express the times as an arithmetic sequence using minutes.
5. Determine the departure time of the 15th bus.

Activity 4: Community project

In groups, identify a situation in the community where quantities increase or decrease by a fixed amount.

Possible areas include:

- Savings schemes.
- School seating arrangements.
- Agricultural planting.
- Transport schedules.
- Stacking of construction materials.
- Daily sales or production.

Prepare a short report showing:

1. The real-life situation.
2. The arithmetic sequence.
3. The first term.
4. The common difference.
5. The nth-term formula.
6. The sum of a specified number of terms.
7. How the mathematics can help solve the identified problem.

11.0 Summary of important formulae

Concept	Formula
Common difference	$d = T_2 - T_1$
General form of an AP	$a,\ a+d,\ a+2d, \ldots$
nth term of an AP	$T_n = a + (n-1)d$
Sum using last term	$S_n = \frac{n}{2}(a+l)$
Sum using common difference	$S_n = \frac{n}{2}[2a + (n-1)d]$
Common ratio of a GP	$r = \frac{T_2}{T_1}$
nth term of a GP	$T_n = ar^{n-1}$

Important points to remember

1. A sequence is an ordered list of numbers following a particular rule.
2. Each number in a sequence is called a term.
3. An arithmetic sequence has a constant difference between consecutive terms.
4. The common difference may be positive, negative or zero.
5. The nth-term formula helps to find a term at any specified position.
6. An arithmetic progression is a sequence whose consecutive terms have a constant difference.
7. A series is the sum of the terms of a sequence.
8. The sum formula can be used to calculate the total of many AP terms.
9. A geometric sequence has a constant ratio rather than a constant difference.
10. Arithmetic progressions are useful in planning, saving, transport, farming, business and construction.

REVISION EXERCISES

Section A: Multiple-choice questions

Instructions: Answer all questions. Choose the correct answer from the four alternatives provided. Each question carries 1 mark.

1. Which of the following best describes a sequence?

   A. A collection of unrelated numbers
   B. An ordered list of numbers following a particular rule
   C. The product of two numbers
   D. The difference between two numbers

2. Find the next term in the sequence:

$$4,\ 8,\ 12,\ 16, \ldots$$

A. 18
B. 19
C. 20
D. 22

3. Find the missing term:

$$7,\ 12,\ \square,\ 22,\ 27$$

A. 15 B. 16 C. 17 D. 18

4. Which of the following is an arithmetic sequence?

   A. 2, 4, 8, 16
   B. 3, 6, 9, 12
   C. 1, 4, 9, 16
   D. 2, 6, 18, 54

5. Find the common difference of:

$$9,\ 14,\ 19,\ 24,\ldots$$

A. 3 B. 4 C. 5 D. 6

6. Find the common difference of:

$$40,\ 35,\ 30,\ 25,\ldots$$

A. 5 B. 10 C. -5 D. -10

7. The first term of an arithmetic sequence is 6 and the common difference is 4. What is the fourth term?

   A. 14 B. 16 C. 18 D. 20

8. Which formula gives the nth term of an arithmetic sequence?

   A. $T_n = ar^{n-1}$
   B. $T_n = a + (n+1)d$
   C. $T_n = a + (n-1)d$
   D. $T_n = nd$

9. Find the 10th term of:

$$2,\ 5,\ 8,\ 11,\ldots$$

A. 26 B. 27 C. 29 D. 30

10. The first term of an AP is 10 and the common difference is 3. Find the 8th term.

    A. 28
    B. 29
    C. 31
    D. 34

11. Which of the following is the common difference of the sequence:

$$100,\ 90,\ 80,\ 70,\ldots$$

A. 10
B. -10
C. 20
D. -20

12. The first term of an AP is 5 and the common difference is 2. Find the 15th term.

    A. 31
    B. 32
    C. 33
    D. 35

13. The 5th term of an AP is 22. If the first term is 6, find the common difference.

A. 3
B. 4
C. 5
D. 6

14. Which term of the AP $3,7,11,15,\ldots$ is 39?

A. 8th
B. 9th
C. 10th
D. 11th

15. Find the sum of the first five terms of:

$$2,\ 4,\ 6,\ 8,\ 10$$

A. 25
B. 28
C. 30
D. 35

16. Which formula is used to calculate the sum of the first n terms of an AP when the first and last terms are known?

A. $S_n = n(a + l)$
B. $S_n = \frac{a+l}{n}$
C. $S_n = \frac{n}{2}(a + l)$
D. $S_n = a + (n - 1)d$

17. Find the sum of the first 10 terms of:

$$1,\ 2,\ 3,\ 4,\ldots$$

A. 45
B. 50
C. 55
D. 60

18. Find the sum of the first 8 terms of:

$$3,\ 6,\ 9,\ 12,\ldots$$

A. 96
B. 108
C. 120
D. 132

19. The first term of an AP is 4 and the common difference is 3. Find the sum of the first 6 terms.

A. 54
B. 57
C. 59
D. 69

20. A sequence has a first term of 12 and a common difference of zero. Which of the following represents the sequence?

A. 12, 14, 16, 18
B. 12, 12, 12, 12
C. 12, 10, 8, 6
D. 12, 24, 36, 48

21. A learner saves Ksh 100 in the first week and increases the weekly savings by Ksh 50. How much does the learner save in the fourth week?

A. Ksh 200
B. Ksh 250
C. Ksh 300
D. Ksh 350

22. A matatu leaves a station every 30 minutes. If the first matatu leaves at 7:00 a.m., at what time does the fifth matatu leave?

A. 8:00 a.m.
B. 8:30 a.m.
C. 9:00 a.m.
D. 9:30 a.m.

23. A school arranges chairs in rows. The first row has 10 chairs and each following row has 2 more chairs. How many chairs are in the sixth row?

A. 18
B. 20
C. 22
D. 24

24. A trader earns Ksh 200 on the first day and increases daily earnings by Ksh 100. What is the total earnings for the first four days?

A. Ksh 1,000
B. Ksh 1,200
C. Ksh 1,400
D. Ksh 1,600

25. Which of the following is a geometric sequence?

A. 2, 4, 6, 8
B. 3, 6, 12, 24
C. 5, 10, 15, 20
D. 7, 14, 21, 28

26. Find the common ratio of:

$$4,\ 12,\ 36,\ 108, \ldots$$

A. 2
B. 3
C. 4
D. 6

27. Which formula gives the nth term of a geometric sequence?

A. $T_n = a + (n-1)d$
B. $T_n = ar^{n-1}$
C. $T_n = a + nd$
D. $T_n = nr$

28. Find the fourth term of the geometric sequence:

$$2,\ 6,\ 18, \ldots$$

A. 36
B. 48
C. 54
D. 72

29. Which statement correctly distinguishes a sequence from a series?

A. A sequence is always decreasing.
B. A series is the sum of the terms of a sequence.
C. A sequence must contain five terms.
D. A series contains only negative numbers.

30. A farmer plants 6 trees in the first row and increases the number by 4 trees in each succeeding row. How many trees are in the 10th row?

A. 38
B. 40
C. 42
D. 46

SECTION B: STRUCTURED QUESTIONS

Instructions: Answer all questions. Show all working clearly. Give answers with appropriate units where necessary.

Question 1: Number sequences and arithmetic progressions (8 marks)

a) Define an arithmetic sequence. (2 marks)

b) Consider the sequence:

$$7,\ 12,\ 17,\ 22,\ 27, \ldots$$

Determine:

i) The first term. (1 mark)

ii) The common difference. (1 mark)

iii) Find the missing terms in the following sequence:

$$15,\ 20,\ __,\ __,\ 35$$

(2 marks)

d) Write down the next two terms of the sequence:

$$50,\ 44,\ 38,\ 32, \ldots$$

(2 marks)

Question 2: The nth term of an arithmetic sequence (8 marks)

a) Find the nth-term formula for the arithmetic sequence:

$$4,\ 9,\ 14,\ 19,\ldots$$

(3 marks)

b) Using the formula obtained in (a), calculate the 25th term. (2 marks)

c) Which term of the sequence is 99? (3 marks)

Question 3: Determining unknown terms and common differences (8 marks)

a) The first term of an arithmetic progression is 8 and the 12th term is 63.

Determine:

i) The common difference. (3 marks)

ii) The 20th term. (2 marks)

b) The 6th term of an arithmetic progression is 23 and its common difference is 4.

Find the first term. (3 marks)

Question 4: Sum of an arithmetic progression (8 marks)

a) Find the sum of the first 15 terms of:

$$3,\ 7,\ 11,\ 15,\ldots$$

(4 marks)

b) Find the sum of the first 20 terms of an arithmetic progression whose first term is 5 and common difference is 3. (4 marks)

Question 5: Application of arithmetic progressions in real life (9 marks)

A school in Kenya is arranging chairs for a parents' meeting. The first row contains 15 chairs. Each succeeding row contains 3 more chairs than the preceding row.

a) Write down the first four terms of the sequence representing the number of chairs in each row. (2 marks)

b) Determine the number of chairs in the 12th row. (3 marks)

c) Calculate the total number of chairs required for the first 12 rows. (4 marks)

Question 6: Application of series and sequences (9 marks)

A learner decides to save money for a school project. The learner saves Ksh 200 in the first week and increases the weekly savings by Ksh 50 every subsequent week.

a) State the first term and common difference. (2 marks)

b) Calculate the amount saved in the 16th week. (3 marks)

c) Calculate the total amount saved during the first 16 weeks. (4 marks)

ANSWERS AND MARKING SCHEME

Section A: Multiple-choice answers

Each correct answer earns 1 mark. Total: 30 marks.

Question	Answer	Question	Answer
1	B	16	C

Question	Answer	Question	Answer
2	C	17	C
3	C	18	B
4	B	19	A
5	C	20	B
6	C	21	B
7	B	22	B
8	C	23	B
9	C	24	B
10	C	25	B
11	B	26	B
12	C	27	B
13	B	28	C
14	C	29	B
15	C	30	A

Section B: Detailed marking scheme

Question 1 (8 marks)

a) An arithmetic sequence is an ordered list of numbers in which the difference between consecutive terms is constant. (2 marks)

b) First term:

$$\boxed{a = 7}$$

(1 mark)

ii) Common difference:

$$d = 12 - 7$$

$$\boxed{d = 5}$$

(1 mark)

$$15,\ 20,\ 25,\ 30,\ 35$$

Missing terms:

$$\boxed{25,\ 30}$$

(2 marks)

$$50,\ 44,\ 38,\ 32,\ 26,\ 20$$

Next two terms:

$$\boxed{26,\ 20}$$

(2 marks)

Question 2 (8 marks)

a) Given:

$$a = 4, \quad d = 9 - 4 = 5$$

Using:

$$T_n = a + (n-1)d$$

$$T_n = 4 + (n-1)5$$

$$= 4 + 5n - 5$$

$$\boxed{T_n = 5n - 1}$$

(3 marks)

$$T_{25} = 5(25) - 1$$

$$= 125 - 1$$

$$\boxed{124}$$

(2 marks)

$$99 = 5n - 1$$

$$100 = 5n$$

$$n = 20$$

$$\boxed{\text{20th term}}$$

(3 marks)

Question 3 (8 marks)

a) Given:

$$a = 8, \quad T_{12} = 63$$

i) Using:

$$T_n = a + (n-1)d$$

$$63 = 8 + (12-1)d$$

$$63 = 8 + 11d$$

$$55 = 11d$$

$$\boxed{d = 5}$$

(3 marks)

$$T_{20} = 8 + (20-1)5$$

$$= 8 + 95$$

$$\boxed{103}$$

(2 marks)

b) Given:

$$T_6 = 23, \quad d = 4$$

$$23 = a + (6-1)4$$

$$23 = a + 20$$

$$\boxed{a = 3}$$

(3 marks)

Question 4 (8 marks)

a) Given:

$$a = 3, \quad d = 4, \quad n = 15$$

Using:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}[2(3) + (15-1)4]$$

$$= \frac{15}{2}[6 + 56]$$

$$= \frac{15}{2} \times 62$$

$$\boxed{465}$$

(4 marks)

b) Given:

$$a = 5, \quad d = 3, \quad n = 20$$

$$S_{20} = \frac{20}{2}[2(5) + (20-1)3]$$

$$= 10[10 + 57]$$

$$= 10 \times 67$$

$$\boxed{670}$$

(4 marks)

Question 5 (9 marks)

Given:

$$a = 15, \quad d = 3, \quad n = 12$$

a) First four terms:

$$15,\ 18,\ 21,\ 24$$

(2 marks)

b) Number of chairs in the 12th row:

$$T_{12} = a + (n-1)d$$

$$= 15 + (12-1)3$$

$$= 15 + 33$$

$$\boxed{48 \text{ chairs}}$$

(3 marks)

c) Total number of chairs:

$$S_{12} = \frac{12}{2}(15 + 48)$$

$$= 6 \times 63$$

$$\boxed{378 \text{ chairs}}$$

(4 marks)

Question 6 (9 marks)

Given:

$$a = 200, \quad d = 50, \quad n = 16$$

First term:

$$\boxed{a = 200}$$

Common difference:

$$\boxed{d = 50}$$

(2 marks)

b) Amount saved in the 16th week:

$$T_{16} = a + (n - 1)d$$

$$= 200 + (16 - 1)50$$

$$= 200 + 750$$

$$\boxed{\text{Ksh } 950}$$

(3 marks)

c) Total savings:

$$S_{16} = \frac{16}{2}[2(200) + (16 - 1)50]$$

$$= 8[400 + 750]$$

$$= 8 \times 1{,}150$$

$$\boxed{\text{Ksh } 9{,}200}$$

(4 marks)

STRAND 2.0: MEASUREMENTS AND GEOMETRY

SUB-STRAND 2.1: APPROXIMATION AND ERRORS

2.1 Meaning of estimation

Estimation is the process of finding an approximate value of a quantity without necessarily measuring it exactly.

An estimate is a value that is close to the actual or expected value.

Estimation is useful when:

- An exact measurement is not necessary.
- Measuring instruments are not available.
- A quick decision is required.
- The quantity is too large or difficult to measure directly.
- We want to check whether a measured answer is reasonable.

Examples of estimation in everyday life

1. Estimating the distance from the school gate to the administration block.
2. Estimating the height of a classroom wall.
3. Estimating the volume of water in a storage tank.
4. Estimating the number of learners in a school assembly.
5. Estimating the mass of a bag of maize.
6. Estimating the time required to walk from home to school.
7. Estimating the amount of money needed to purchase food items.

2.2 Meaning of measurement

Measurement is the process of determining the size, amount or magnitude of a quantity by comparing it with an accepted standard unit.

For example:

- Measuring the length of a desk in centimetres.
- Measuring the mass of a bag of maize in kilograms.
- Measuring the time taken to run 100 metres in seconds.
- Measuring the volume of water in litres.
- Measuring the temperature of water in degrees Celsius.

A measurement consists of two important parts:

1. A numerical value.
2. A unit of measurement.

For example, in 5 metres:

- 5 is the numerical value.
- Metre (m) is the unit.

Important: A measurement should always be accompanied by an appropriate unit.

2.3 Difference between estimation and measurement

Estimation	Measurement
Gives an approximate value	Gives a value obtained using a measuring method or instrument
May not require a measuring instrument	Usually requires a measuring instrument or standard
Is often quick	May require more time
May be less precise	Can be more precise, depending on the instrument and method
Example: The classroom is about 8 m long	Example: The classroom is 8.24 m long

Example 1: Estimating and measuring a book

A learner estimates that a mathematics textbook is 25 cm long. After measuring it using a ruler, the actual length is found to be 27 cm.

- Estimated length = 25 cm.
- Measured length = 27 cm.

The estimate is close to the measured value, but it is not exact.

2.4 Common quantities measured in everyday life

A physical quantity is a property that can be measured and expressed using a number and a unit.

Quantity	Meaning	Common units	Measuring instrument
Length	Distance between two points	mm, cm, m, km	Ruler, tape measure
Mass	Amount of matter in an object	g, kg	Weighing scale
Time	Duration of an event	s, min, h	Clock, stopwatch
Temperature	Degree of hotness or coldness	$^{\circ}$C, K	Thermometer
Volume	Space occupied by a substance	cm^3, m^3, mL, L	Measuring cylinder
Area	Surface covered by a shape	cm^2, m^2, ha	Ruler and calculation
Capacity	Amount a container can hold	mL, L	Measuring jug

2.5 International System of Units (SI)

The International System of Units (SI) is a globally accepted system used to express measurements consistently.

Some important SI base units are:

Physical quantity	SI unit	Symbol
Length	Metre	m
Mass	Kilogram	kg

Physical quantity	SI unit	Symbol
Time	Second	s
Temperature	Kelvin	K
Electric current	Ampere	A
Amount of substance	Mole	mol
Luminous intensity	Candela	cd

Other commonly used units include:

- Centimetre (cm)
- Millimetre (mm)
- Kilometre (km)
- Gram (g)
- Litre (L)
- Millilitre (mL)
- Degree Celsius (°C)

2.6 Conversion of units of measurement

Measurements are sometimes expressed in different units. It is important to convert them into suitable units before carrying out calculations.

A. Conversion of length

Remember:

$$1 \text{ km} = 1{,}000 \text{ m}$$

$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ cm} = 10 \text{ mm}$$

$$1 \text{ m} = 1{,}000 \text{ mm}$$

Example 2

Convert 4.5 kilometres into metres.

Solution

$$1 \text{ km} = 1{,}000 \text{ m}$$

$$4.5 \times 1{,}000 = 4{,}500$$

$$\boxed{4{,}500 \text{ m}}$$

Example 3

Convert 375 centimetres into metres.

Solution

$$100 \text{ cm} = 1 \text{ m}$$

$$375 \div 100 = 3.75$$

$$\boxed{3.75 \text{ m}}$$

B. Conversion of mass

Remember:

$$1 \text{ tonne} = 1{,}000 \text{ kg}$$

$$1 \text{ kg} = 1{,}000 \text{ g}$$

$$1 \text{ g} = 1{,}000 \text{ mg}$$

Example 4

Convert 3.6 kg into grams.

Solution

$$3.6 \times 1{,}000 = 3{,}600$$

$$\boxed{3{,}600 \text{ g}}$$

C. Conversion of volume and capacity

Remember:

$$1 \text{ L} = 1{,}000 \text{ mL}$$

$$1 \text{ m}^3 = 1{,}000 \text{ L}$$

$$1 \text{ cm}^3 = 1 \text{ mL}$$

Example 5

Convert 2.75 litres into millilitres.

Solution

$$2.75 \times 1{,}000 = 2{,}750$$

$$\boxed{2{,}750 \text{ mL}}$$

D. Conversion of area

For area, the conversion factor must be squared.

$$1 \text{ m} = 100 \text{ cm}$$

Therefore:

$$1 \text{ m}^2 = 100 \times 100 \text{ cm}^2$$

$$\boxed{1 \text{ m}^2 = 10{,}000 \text{ cm}^2}$$

Also:

$$1 \text{ km}^2 = 1{,}000{,}000 \text{ m}^2$$

$$1 \text{ hectare} = 10{,}000 \text{ m}^2$$

Example 6

Convert 4.5 m^2 into square centimetres.

Solution

$$1 \text{ m}^2 = 10{,}000 \text{ cm}^2$$

$$4.5 \times 10{,}000 = 45{,}000$$

$$\boxed{45{,}000 \text{ cm}^2}$$

3.0 Estimating measurements of different quantities

3.1 Estimating length

Length is the measurement of the distance between two points.

Length can be estimated by comparing an unknown length with a familiar object or known distance.

Examples:

- The length of a pen.
- The width of a classroom.
- The height of a door.
- The distance from the school gate to the staffroom.
- The length of a football pitch.

Methods of estimating length

1. **Using familiar objects:** Compare an object with a known length, such as a ruler.
2. **Using body measurements:** Use a handspan, footstep or arm length as a rough reference.
3. **Visual estimation:** Observe an object and estimate its length based on experience.
4. **Pacing:** Estimate distance by counting steps and using an approximate step length.
5. **Comparing known distances:** Compare the unknown distance with a familiar measured distance.

Example 7: Estimating the length of a classroom

A learner estimates that a classroom is 9 metres long. When measured using a tape measure, the length is found to be 9.6 metres.

Determine:

a) The estimated length.

b) The measured length.

c) The difference between the two values.

Solution

a) Estimated length = 9 m.

b) Measured length = 9.6 m.

c) Difference:

$$9.6 - 9 = 0.6$$

$$\boxed{0.6 \text{ m}}$$

3.2 Estimating distance

Distance is the length between two points along a specified route or path.

Distance may be estimated by:

- Counting footsteps.
- Using a map scale.

- Comparing with known distances.
- Using a vehicle's odometer.
- Using digital mapping applications.

Example 8: Estimating distance using footsteps

A learner estimates that the distance from the school gate to the administration block is 80 metres.

The learner walks 100 steps. If the average step length is 0.8 metres, calculate the distance obtained using footsteps.

Solution

$$\text{Distance} = \text{Number of steps} \times \text{Length of one step}$$

$$= 100 \times 0.8$$

$$\boxed{80 \text{ m}}$$

The estimated distance and the distance calculated using footsteps are both 80 metres.

3.3 Estimating mass

Mass is the quantity of matter contained in an object.

It is commonly measured in grams and kilograms.

Examples:

- Mass of a packet of sugar.
- Mass of a bag of maize.
- Mass of a learner's school bag.
- Mass of a sack of potatoes.
- Mass of a parcel.

Methods of estimating mass

1. Comparing an object with another object of known mass.
2. Using experience with familiar packaged commodities.
3. Holding an object and judging its heaviness.
4. Comparing several objects before weighing them.

Example 9

A learner estimates that a school bag has a mass of 4 kg. A weighing scale shows that its actual mass is 4.5 kg.

Find the difference between the estimated and measured mass.

Solution

$$4.5 - 4 = 0.5$$

$$\boxed{0.5 \text{ kg}}$$

3.4 Estimating volume

Volume is the amount of three-dimensional space occupied by an object or substance.

Common units include:

- Cubic centimetres (cm^3).

- Cubic metres (m^3).
- Millilitres (mL).
- Litres (L).

Volume is important when measuring:

- Water in tanks.
- Milk in containers.
- Fuel in storage tanks.
- Liquid medicine.
- Cooking oil.
- Sand and other materials.

Methods of estimating volume

1. Comparing the liquid with a container of known capacity.
2. Observing the level of liquid in a marked container.
3. Using the dimensions of a regular solid.
4. Estimating the number of smaller containers needed to fill a larger container.

Example 10: Estimating liquid volume

A learner estimates that a container holds 2 litres of water. When the water is measured using a graduated jug, the volume is found to be 2.4 litres.

Find the difference between the estimated and measured volumes.

Solution

$$2.4 - 2 = 0.4$$

$$\boxed{0.4 \text{ L}}$$

3.5 Estimating time

Time is the duration between two events.

It is measured in seconds, minutes and hours.

Examples:

- Time taken to walk to school.
- Time taken to complete an examination.
- Time taken by a matatu to travel between towns.
- Time taken to fill a water tank.

Example 11

A learner estimates that walking from home to school takes 20 minutes. A stopwatch shows that the actual time is 24 minutes.

Find the difference between the estimate and the actual time.

$$24 - 20 = 4$$

$$\boxed{4 \text{ minutes}}$$

3.6 Estimating area

Area is the amount of surface covered by a two-dimensional shape.

It is expressed in square units, such as:

- Square centimetres (cm^2).
- Square metres (m^2).
- Hectares (ha).
- Square kilometres (km^2).

Area is important in:

- Measuring agricultural land.
- Planning school compounds.
- Estimating floor tiles.
- Determining the amount of paint required.
- Planning construction projects.

Example 12

A farmer estimates that a rectangular farm is 20 metres long and 15 metres wide.

Estimate the area of the farm.

Solution

$$A = \text{Length} \times \text{Width}$$

$$= 20 \times 15$$

$$\boxed{300\ m^2}$$

3.7 Estimating temperature

Temperature is a measure of how hot or cold a substance or environment is.

It is commonly measured in degrees Celsius ($^\circ C$) in everyday situations.

Examples:

- Temperature of the classroom.
- Temperature of drinking water.
- Temperature of a person's body.
- Temperature during food preparation.

A person may estimate the temperature of water by touch, but a thermometer gives a more reliable numerical measurement.

Important: Touching a substance is not a safe or reliable method of measuring temperature, particularly when it may be very hot or very cold.

4.0 Approximation of numbers

4.1 Meaning of approximation

Approximation is the process of expressing a number or measurement by a value that is sufficiently close to the exact value for a particular purpose.

Approximation is useful when:

- The exact value is difficult to obtain.
- A simpler value is sufficient for calculations.
- A measurement contains many decimal places.

- We need to communicate a value quickly.
- We want to avoid reporting unnecessary digits.

For example:

- The value of π may be approximated as 3.14.
- A distance of 4,982 metres may be approximated as 5,000 metres.
- A mass of 2.876 kg may be approximated as 2.9 kg.

4.2 Rounding off numbers

Rounding off is the process of replacing a number with a nearby value that has fewer digits or decimal places.

The most common forms are:

1. Rounding to the nearest whole number.
2. Rounding to a specified number of decimal places.
3. Rounding to a specified number of significant figures.
4. Rounding to a convenient unit.

4.3 Rules for rounding off numbers

The following rules are used:

- If the digit immediately after the required place is 0, 1, 2, 3 or 4, leave the required digit unchanged.
- If the digit immediately after the required place is 5, 6, 7, 8 or 9, increase the required digit by 1.
- Remove the digits that come after the required place when giving the rounded value.
- Where necessary, replace removed whole-number digits with zeros to maintain place value.

Example 13: Rounding to the nearest whole number

Round 46.7 to the nearest whole number.

Solution

The tenths digit is 7.

Since 7 is greater than 4, increase the units digit by 1.

$$46.7 \approx 47$$

$$\boxed{47}$$

Example 14: Rounding to one decimal place

Round 18.376 to one decimal place.

Solution

The first decimal digit is 3.

The next digit is 7.

Since 7 is greater than 4, increase 3 by 1.

$$18.376 \approx 18.4$$

$$\boxed{18.4}$$

Example 15: Rounding to two decimal places

Round 35.7864 to two decimal places.

Solution

The second decimal digit is 8.

The next digit is 6.

Since 6 is greater than 4, increase 8 by 1.

$$35.7864 \approx 35.79$$

$$\boxed{35.79}$$

Example 16: Rounding to the nearest hundred

Round 8,764 to the nearest hundred.

Solution

The hundreds digit is 7.

The tens digit is 6.

Since 6 is greater than 4, increase the hundreds digit by 1.

$$8{,}764 \approx 8{,}800$$

$$\boxed{8{,}800}$$

4.4 Decimal places

A **decimal place** refers to the position of a digit after the decimal point.

Consider:

$$47.5836$$

Digit	Position
5	First decimal place
8	Second decimal place
3	Third decimal place
6	Fourth decimal place

Examples:

- To 1 decimal place: 47.6
- To 2 decimal places: 47.58
- To 3 decimal places: 47.584

Example 17

Round 0.07864 to three decimal places.

Solution

The third decimal digit is 8.

The next digit is 6.

Therefore, increase 8 by 1.

$$0.07864 \approx \boxed{0.079}$$

4.5 Significant figures

Significant figures are the digits in a number that carry meaningful information about its size or precision.

The following rules are important:

1. All non-zero digits are significant.
2. Zeros between non-zero digits are significant.
3. Zeros before the first non-zero digit are not significant.
4. Zeros at the end of a decimal number are significant when they indicate the precision of the measurement.
5. Trailing zeros in a whole number without a decimal point can be ambiguous; scientific notation can make the intended precision clear.

Examples

Number	Significant figures
45	2
678	3
4.56	3
0.0045	2
0.0306	3
2.500	4
7.08	3

Example 18: Rounding to three significant figures

Round 47,826 to three significant figures.

Solution

The first three significant digits are 4, 7 and 8.

The next digit is 2.

Since 2 is less than 5, the 8 remains unchanged.

$$47{,}826 \approx 47{,}800$$

$$\boxed{47{,}800}$$

Example 19: Rounding a decimal to two significant figures

Round 0.006784 to two significant figures.

Solution

The first two significant digits are 6 and 7.

The next digit is 8.

Increase 7 by 1.

$$0.006784 \approx 0.0068$$

$\boxed{0.0068}$

4.6 Difference between decimal places and significant figures

Decimal places	Significant figures
Counted from the decimal point	Counted from the first non-zero digit
Indicates the number of digits after the decimal point	Indicates the number of meaningful digits
Example: 5.678 to 2 decimal places is 5.68	Example: 5.678 to 2 significant figures is 5.7

Remember: Decimal places and significant figures are not interchangeable.

5.0 Errors in measurements and estimations

5.1 Meaning of measurement error

A **measurement error** is the difference between a measured value and the accepted or true value of a quantity.

Measurements may not always give the exact true value because of limitations in instruments, methods or human observation.

For example, if the accepted length of a desk is 120 cm but a learner measures it as 118 cm, the measurement has an error of 2 cm.

5.2 Meaning of estimation error

An **estimation error** is the difference between an estimated value and the actual or accepted value of the quantity.

For example:

- Estimated length of a book = 25 cm.
- Actual measured length = 27 cm.

The estimation error is:

$$27 - 25 = 2 \text{ cm}$$

5.3 Absolute error

Absolute error is the positive magnitude of the difference between an approximate or measured value and the accepted or true value.

It is represented by:

$$\boxed{\text{Absolute error} = |\text{Approximate value} - \text{True value}|}$$

The vertical bars mean that the difference is taken as a positive value.

For estimation problems where the measured value is used as the reference:

$$\boxed{E = |\text{Estimated value} - \text{Measured value}|}$$

Where E represents the absolute error.

Example 20: Calculating absolute error

A learner estimates that a classroom is 8 metres long. When measured using a tape measure, its length is found to be 8.6 metres.

Calculate the absolute error.

Solution

Estimated value = 8 m.

Measured value = 8.6 m.

$$E = |8 - 8.6|$$

$$E = |-0.6|$$

$$\boxed{E = 0.6 \text{ m}}$$

Example 21: Calculating error in mass

A trader estimates that a bag of maize has a mass of 50 kg. The weighing scale gives a reading of 48 kg.

Find the absolute error.

Solution

$$E = |50 - 48|$$

$$\boxed{E = 2 \text{ kg}}$$

5.4 Types of measurement errors

Measurement errors can arise in different ways.

A. Instrumental errors

These are errors caused by faults or limitations in measuring instruments.

Examples:

- A weighing scale that does not read zero when empty.
- A damaged ruler with a broken starting edge.
- A thermometer that gives inaccurate readings.
- A measuring cylinder with unclear graduations.

Ways of reducing instrumental errors:

1. Use suitable instruments.
2. Check instruments before use.
3. Calibrate instruments when necessary.
4. Replace damaged instruments.
5. Read measurements using the correct scale.

B. Human errors

These are errors caused by the person taking or recording the measurement.

Examples:

- Reading a ruler from the wrong starting point.
- Recording 25.6 cm as 26.5 cm.
- Reading a scale from an incorrect viewing angle.
- Forgetting to record the correct unit.
- Making calculation mistakes.

Ways of reducing human errors:

1. Read instruments carefully.
2. Repeat measurements.
3. Record values immediately.
4. Check calculations.
5. Follow the correct measuring procedure.

C. Environmental errors

These are errors caused by environmental conditions that affect measurements.

Examples:

- Temperature changes causing materials to expand or contract.
- Strong winds affecting the reading of a sensitive weighing scale.
- Poor lighting making it difficult to read a scale.
- Vibrations affecting delicate instruments.

Ways of reducing environmental errors:

1. Measure under suitable environmental conditions.
2. Protect instruments from strong winds and vibrations.
3. Use adequate lighting.
4. Allow measuring instruments to stabilise before taking readings.

D. Random errors

These are unpredictable variations in measurements that may cause repeated readings to differ.

For example, when measuring the time taken by a learner to run 100 metres, the recorded time may vary slightly because of reaction time when starting or stopping the stopwatch.

Ways of reducing random errors:

1. Repeat measurements several times.
2. Calculate the average of repeated readings.
3. Use more suitable and reliable instruments.
4. Keep measuring conditions as consistent as possible.

E. Systematic errors

These are errors that consistently affect measurements in the same direction because of a problem with the instrument or measuring method.

For example, a weighing scale that always adds 0.5 kg to every reading has a systematic error.

Ways of reducing systematic errors:

1. Identify the source of the error.
2. Calibrate the instrument.
3. Correct the zero error.
4. Use the correct measuring method.

5.5 Accuracy and precision

These two terms are related but have different meanings.

Accuracy refers to how close a measured value is to the true or accepted value.

Precision refers to how close repeated measurements are to one another.

For example:

A learner measures the length of a table four times and obtains:

$$120,\ 120,\ 121,\ 120 \text{ cm}$$

The readings are close to one another, so they are precise. If the accepted length is 120 cm, they are also close to the accepted value and therefore accurate.

Important points:

- A measurement can be precise but inaccurate.
- A measurement can be accurate on average but have variation between repeated readings.
- Repeating a measurement helps identify variation but does not automatically remove systematic errors.

6.0 Percentage error

6.1 Meaning of percentage error

Percentage error is the absolute error expressed as a percentage of the accepted or actual value.

It shows how large the error is in relation to the true value.

Percentage error is useful because it allows us to compare errors in quantities of different sizes.

Formula

$$\boxed{\text{Percentage error} = \frac{\text{Absolute error}}{\text{Actual value}} \times 100\%}$$

For an estimate compared with an actual measured value:

$$\boxed{\text{Percentage error} = \frac{|\text{Estimated value} - \text{Actual value}|}{\text{Actual value}} \times 100\%}$$

Important: The actual or accepted value is used as the denominator, not the estimated value.

6.2 Calculating percentage error from an estimate

Example 22

A learner estimates that a book is 24 cm long. The actual length measured using a ruler is 25 cm.

Calculate the percentage error.

Solution

Estimated value = 24 cm.

Actual value = 25 cm.

Step 1: Find the absolute error.

$$E = |24 - 25|$$

$$E = 1 \text{ cm}$$

Step 2: Apply the percentage error formula.

$$\text{Percentage error} = \frac{1}{25} \times 100\%$$

$$= 4\%$$

$$\boxed{4\%}$$

Example 23: Estimating the volume of water

A learner estimates that a container holds 18 litres of water. When measured, the actual volume is 20 litres.

Calculate the percentage error.

Solution

Estimated volume = 18 L.

Actual volume = 20 L.

Absolute error:

$$E = |18 - 20| = 2 \text{ L}$$

Percentage error:

$$= \frac{2}{20} \times 100\%$$

$$= \frac{1}{10} \times 100\%$$

$$\boxed{10\%}$$

Example 24: Estimating distance

A learner estimates that the distance from the school gate to the dining hall is 150 metres. The measured distance is 160 metres.

Find the percentage error.

Solution

Absolute error:

$$E = |150 - 160| = 10 \text{ m}$$

Percentage error:

$$= \frac{10}{160} \times 100\%$$

$$= 6.25\%$$

$$\boxed{6.25\%}$$

6.3 Calculating the actual value when percentage error is known

Sometimes the estimated value and percentage error are known, but the actual value is required.

Example 25

A learner estimates the length of a classroom as 18 metres. The percentage error is 10%. The estimate is less than the actual length. Find the actual length.

Solution

Let the actual length be x metres.

Absolute error:

$$10\% \times x = \frac{10}{100}x$$

Since the estimate is less than the actual length:

$$x - 18 = 0.1x$$

$$x - 0.1x = 18$$

$$0.9x = 18$$

$$x = \frac{18}{0.9}$$

$$\boxed{x = 20 \text{ m}}$$

6.4 Finding the estimated value when the actual value and percentage error are known

Example 26

The actual mass of a bag of maize is 80 kg. A learner's estimate has a percentage error of 5%. If the learner overestimates the mass, find the estimated mass.

Solution

Absolute error:

$$E = \frac{5}{100} \times 80$$

$$E = 4 \text{ kg}$$

Since the learner overestimates:

$$\text{Estimated mass} = 80 + 4$$

$$\boxed{84 \text{ kg}}$$

If the learner had underestimated, the estimate would have been:

$$80 - 4 = 76 \text{ kg}$$

6.5 Comparing percentage errors

Percentage error helps us compare the accuracy of different estimates.

Example 27

Two learners estimate the length of two different objects.

- Learner A estimates a length of 48 cm, while the actual length is 50 cm.
- Learner B estimates a length of 96 cm, while the actual length is 100 cm.

Determine which learner made the more accurate estimate.

Solution

For learner A:

$$E_A = |48 - 50| = 2 \text{ cm}$$

$$\text{Percentage error}_A = \frac{2}{50} \times 100\%$$

$$= 4\%$$

For learner B:

$$E_B = |96 - 100| = 4 \text{ cm}$$

$$\text{Percentage error}_B = \frac{4}{100} \times 100\%$$

$$= 4\%$$

Both learners have the same percentage error.

Both estimates are equally accurate in percentage terms.

6.6 Important properties of percentage error

1. Percentage error is expressed as a percentage.
2. Absolute error is always taken as a positive value.
3. The actual or accepted value is used as the denominator.
4. A smaller percentage error generally indicates a closer estimate.
5. A larger percentage error indicates a greater relative difference.
6. Percentage error is undefined when the actual value is zero.
7. The units cancel when calculating percentage error, leaving a percentage.

7.0 Accuracy of measurements and the effect of rounding

7.1 Why measurements should be recorded carefully

A measurement should reflect the capability of the measuring instrument.

For example:

- A ruler marked in centimetres may not support a reading to the nearest 0.01 cm.
- A digital weighing scale displaying two decimal places may give a reading such as 2.35 kg.
- A measuring cylinder with 10 mL graduations does not normally support an exact reading to 1 mL without further suitable scale information.

Recording too many decimal places may give a false impression of accuracy.

7.2 Error caused by rounding

Rounding changes the original value. The difference between the original value and the rounded value is called the rounding error.

Example 28

A measurement is 12.678 metres. It is rounded to two decimal places.

Find the rounding error.

Solution

Rounded value:

$$12.678 \approx 12.68$$

Rounding error:

$$E = |12.68 - 12.678|$$

$$\boxed{0.002 \text{ m}}$$

7.3 Maximum rounding error

When a number is rounded to a specified place, the maximum possible absolute rounding error is half the value of the place to which it is rounded.

For example:

Rounding place	Maximum absolute rounding error
Nearest 10	5
Nearest 100	50
Nearest whole number	0.5
1 decimal place	0.05
2 decimal places	0.005
3 decimal places	0.0005

Example 29

A length is recorded as 8.4 cm to one decimal place.

State the maximum possible rounding error.

Solution

One decimal place has a rounding unit of 0.1 cm.

Maximum rounding error:

$$\frac{0.1}{2} = 0.05$$

$$\boxed{0.05 \text{ cm}}$$

Therefore, the original length lies approximately between 8.35 cm and 8.45 cm, with the exact treatment of boundary values depending on the rounding convention used.

8.0 Applications of approximation and errors in real life

Approximation and error calculations are useful in many sectors of the Kenyan economy.

8.1 Construction and building

Builders estimate and measure:

- Length and width of rooms.
- Height of walls.
- Area of floors.
- Volume of concrete.
- Quantity of building materials.

Errors in construction measurements may lead to:

- Wastage of cement and other materials.

- Incorrect room dimensions.
- Poor fitting of doors and windows.
- Additional construction costs.
- Structural problems.

Example 30

A contractor estimates that a wall is 12 metres long. The actual length is 12.5 metres.

Calculate the percentage error.

Solution

Absolute error:

$$E = |12 - 12.5| = 0.5 \text{ m}$$

Percentage error:

$$= \frac{0.5}{12.5} \times 100\%$$

$$\boxed{4\%}$$

8.2 Agriculture

Farmers use estimates and measurements when:

- Determining the size of farms.
- Measuring distances between planting rows.
- Estimating the volume of water required for irrigation.
- Measuring quantities of fertiliser.
- Estimating crop yields.

Errors may lead to:

- Uneven planting.
- Incorrect application of farm inputs.
- Water shortages or wastage.
- Inaccurate production estimates.

Example 31

A farmer estimates that a field is 50 metres long. The actual length is 52 metres.

Calculate the percentage error.

Solution

$$E = |50 - 52| = 2 \text{ m}$$

$$\text{Percentage error} = \frac{2}{52} \times 100\%$$

$$\boxed{3.85\% \text{ approximately}}$$

8.3 Trade and business

Traders use measurements when selling:

- Sugar.

- Rice.
- Maize.
- Potatoes.
- Fruits.
- Cooking oil.
- Other commodities.

Accurate measurement promotes fairness between buyers and sellers.

Errors in weighing or measuring can cause:

- Financial losses.
- Customer complaints.
- Unfair trading practices.
- Incorrect stock records.

8.4 Transport

Approximation and measurement are used to estimate:

- Distance between towns.
- Fuel consumption.
- Travel time.
- Vehicle loads.
- Road dimensions.

Errors in these estimates may lead to poor travel planning, fuel shortages or delays.

8.5 Healthcare

Healthcare workers measure:

- Body temperature.
- Body mass.
- Height.
- Blood pressure.
- Liquid medicine quantities.

Accurate measurements are important for reliable health assessments and safe care.

8.6 Water supply and storage

Approximation and measurement are used to estimate:

- Water stored in tanks.
- Water consumed daily.
- Water required by households.
- Volume of water supplied to schools.
- Capacity of containers.

Errors in these estimates may result in water shortages or wastage.

8.7 Education and school management

Teachers and school administrators use measurements and estimates when:

- Estimating the number of desks required.

- Measuring classroom dimensions.
- Estimating the amount of paint needed.
- Determining the area of the school compound.
- Planning seating arrangements.
- Estimating the number of learners attending an event.

9.0 Practical learning activities and investigations

The following activities are designed to help learners apply approximation and error calculations in familiar situations.

Activity 1: Estimating and measuring classroom objects

Materials required:

- Ruler.
- Tape measure.
- Exercise books.
- Pens.
- Desks.
- Calculator.

Procedure:

1. Select five objects in the classroom.
2. Estimate the length of each object.
3. Measure each object using an appropriate instrument.
4. Record the estimated and measured values.
5. Calculate the absolute error for each object.
6. Calculate the percentage error.
7. Compare the results with those of other groups.

Suggested recording table:

Object	Estimated length (cm)	Measured length (cm)	Absolute error (cm)	Percentage error
Exercise book				
Pen				
Desk				
Window				
Door				

Activity 2: Estimating distances within the school compound

Procedure:

1. Identify two points within the school compound, such as the school gate and the administration block.
2. Estimate the distance between the two points.
3. Measure the distance using a tape measure or another suitable method.
4. Compare the estimated and measured distances.
5. Calculate the absolute error.
6. Calculate the percentage error.

7. Discuss why the estimates may differ from the measured values.

Activity 3: Estimating volume of liquids

Materials required:

- Water.
- Measuring jug.
- Different containers.
- Calculator.

Procedure:

1. Select a container.
2. Estimate the amount of water it can hold.
3. Fill the container with water.
4. Pour the water into a graduated measuring jug.
5. Record the actual measured volume.
6. Calculate the absolute and percentage errors.
7. Discuss how the shape of a container affects visual estimation.

Activity 4: Investigating the accuracy of repeated measurements

Procedure:

1. Select an object, such as a mathematics textbook.
2. Measure its length five times using the same ruler.
3. Record all five readings.
4. Calculate the average measurement.
5. Compare the readings.
6. Discuss why repeated readings may differ.

The average of repeated readings is calculated using:

$$\text{Average} = \frac{\text{Sum of readings}}{\text{Number of readings}}$$

Example 32

A learner measures a book five times and obtains:

$$25.2,\ 25.3,\ 25.1,\ 25.2,\ 25.2 \text{ cm}$$

Calculate the average length.

Solution

$$\text{Average} = \frac{25.2 + 25.3 + 25.1 + 25.2 + 25.2}{5}$$

$$= \frac{126.0}{5}$$

$$\boxed{25.2 \text{ cm}}$$

Activity 5: Community-based project

In groups, investigate how estimation and measurement are used in a local activity.

Choose one area:

- Building construction.
- Farming.
- Market trading.
- Water storage.
- Transport.
- School administration.

Prepare a report showing:

1. The activity investigated.
2. The quantities being estimated or measured.
3. The instruments used.
4. The estimated values.
5. The measured values.
6. The absolute errors.
7. The percentage errors.
8. The possible effects of measurement errors.
9. Ways of reducing errors.

Present the findings to the class.

10.0 Summary of important concepts and formulae

Concept	Meaning or formula
Estimation	Finding an approximate value
Measurement	Comparing a quantity with an accepted standard unit
Absolute error	$E = \lvert\text{Approximate value} - \text{True value}\rvert$
Estimation error	$E = \lvert\text{Estimated value} - \text{Measured value}\rvert$
Percentage error	$\frac{\text{Absolute error}}{\text{Actual value}} \times 100\%$
Average measurement	$\frac{\text{Sum of readings}}{\text{Number of readings}}$
Maximum rounding error	Half the value of the rounding unit

Key points to remember

1. Estimation gives an approximate value of a quantity.
2. Measurement determines the value of a quantity using a standard unit.
3. A measurement must have a numerical value and a unit.
4. Length, mass, time, volume, area and temperature are common measurable quantities.
5. The International System of Units provides standard units for measurements.
6. Rounding off simplifies numbers while retaining an appropriate level of accuracy.
7. Decimal places are counted from the decimal point.
8. Significant figures begin with the first non-zero digit.
9. Absolute error is the positive difference between an approximate value and the true or accepted value.
10. Percentage error expresses absolute error as a percentage of the actual value.
11. Instrumental, human, environmental, random and systematic errors may affect measurements.
12. Accurate measurements are important in construction, agriculture, trade, healthcare and transport.
13. Repeated measurements can help identify variations in readings.

14. Suitable instruments and correct measuring procedures help minimise errors.

REVISION EXERCISES

SECTION A: MULTIPLE-CHOICE QUESTIONS

Instructions: Answer all questions. Choose the correct answer from the alternatives A, B, C and D. Each question carries 1 mark.

1. What is estimation?

   A. Finding the exact value of a quantity
   B. Finding an approximate value of a quantity
   C. Adding two measurements
   D. Converting a number into a fraction

2. Which of the following is an appropriate instrument for measuring the length of a classroom?

   A. Thermometer
   B. Weighing scale
   C. Tape measure
   D. Stopwatch

3. The SI unit of length is:

   A. Kilogram
   B. Metre
   C. Litre
   D. Second

4. Convert 3.5 kilometres into metres.

   A. 35 m
   B. 350 m
   C. 3,050 m
   D. 3,500 m

5. Convert 450 centimetres into metres.

   A. 0.45 m
   B. 4.5 m
   C. 45 m
   D. 450 m

6. Convert 2.75 litres into millilitres.

   A. 275 mL
   B. 2,075 mL
   C. 2,750 mL
   D. 27,500 mL

7. Convert 4.2 kilograms into grams.

   A. 42 g
   B. 420 g
   C. 4,020 g
   D. 4,200 g

8. Round 46.8 to the nearest whole number.

A. 46
B. 47
C. 48
D. 50

9. Round 12.376 to two decimal places.

A. 12.30
B. 12.37
C. 12.38
D. 12.40

10. Round 8,764 to the nearest hundred.

A. 8,700
B. 8,760
C. 8,800
D. 9,000

11. How many significant figures are in 0.00456?

A. 2
B. 3
C. 4
D. 5

12. How many significant figures are in 5.060?

A. 2
B. 3
C. 4
D. 5

13. Which of the following is the SI unit of mass?

A. Gram
B. Kilogram
C. Tonne
D. Milligram

14. What is a measurement error?

A. The sum of two measurements
B. The difference between a measured value and the true or accepted value
C. The product of two quantities
D. The conversion of metres into centimetres

15. A learner estimates the length of a desk as 100 cm. The actual length is 105 cm. Find the absolute error.

A. 2 cm
B. 3 cm
C. 5 cm
D. 10 cm

16. A learner estimates a mass as 48 kg while the actual mass is 50 kg. What is the absolute error?

A. 1 kg
B. 2 kg

C. 3 kg
D. 4 kg

17. A learner estimates a distance as 90 m while the actual distance is 100 m. Calculate the percentage error.

A. 5%
B. 10%
C. 15%
D. 20%

18. A book is estimated to be 18 cm long, but its actual length is 20 cm. What is the percentage error?

A. 5%
B. 10%
C. 15%
D. 20%

19. Which formula is used to calculate percentage error?

A. $\frac{\text{Actual value}}{\text{Absolute error}} \times 100\%$

B. $\frac{\text{Absolute error}}{\text{Actual value}} \times 100\%$

C. Absolute error × Actual value

D. Actual value − Absolute error

20. Which of the following can cause an instrumental error?

A. A damaged ruler
B. Poor handwriting
C. Incorrect addition
D. Forgetting a measurement

21. A weighing scale that does not read zero when empty may cause:

A. A random error only
B. A systematic error
C. A rounding error only
D. No error

22. Which of the following is a human error?

A. Temperature changes in a metal ruler
B. A damaged thermometer
C. Reading a ruler from the wrong starting point
D. Strong wind affecting an instrument

23. What is accuracy?

A. How close repeated measurements are to one another
B. How close a measurement is to the true or accepted value
C. The number of measurements taken
D. The length of a measuring instrument

24. What is precision?

A. How close repeated measurements are to one another
B. How close a measurement is to zero
C. The difference between two units
D. The total number of significant figures

25. A learner records the following measurements of a book:

    20 cm, 21 cm, 20 cm, 19 cm.

    Find the average measurement.

    A. 19 cm
    B. 19.5 cm
    C. 20 cm
    D. 21 cm

26. What is the maximum possible rounding error when rounding to the nearest whole number?

    A. 0.1
    B. 0.25
    C. 0.5
    D. 1

27. A measurement of 7.86 cm is rounded to one decimal place. What is the rounding error?

    A. 0.01 cm
    B. 0.02 cm
    C. 0.04 cm
    D. 0.06 cm

28. A farmer estimates that a field is 80 m long, while its actual length is 84 m. Find the percentage error.

    A. 2.76%
    B. 4.76%
    C. 5.00%
    D. 6.25%

29. Which of the following is the best way to reduce random measurement errors?

    A. Take only one reading
    B. Ignore differences between readings
    C. Repeat measurements and calculate their average
    D. Use a damaged instrument

30. Why is it important to minimise measurement errors?

    A. To increase wastage of materials
    B. To make measurements unreliable
    C. To improve accuracy and support correct decisions
    D. To avoid using measuring instruments

SECTION B: STRUCTURED QUESTIONS

Instructions: Answer all questions. Show all working clearly and give appropriate units.

Question 1: Estimation and measurement (8 marks)

a) Define the following terms:

i. Estimation. (2 marks)

ii. Measurement. (2 marks)

b) State two quantities that can be measured in a school environment and give an appropriate measuring instrument for each. (2 marks)

c) Convert:

i. 3.6 km into metres. (1 mark)

ii. 2.5 litres into millilitres. (1 mark)

Question 2: Approximation and rounding off (8 marks)

a) Round off the following numbers to the indicated accuracy:

i. 48.763 to two decimal places. (1 mark)

ii. 7,856 to the nearest hundred. (1 mark)

iii. 0.006784 to two significant figures. (2 marks)

b) State the number of significant figures in each of the following:

i. 0.00456. (1 mark)

ii. 3.050. (1 mark)

c) Find the maximum possible rounding error when a measurement is rounded to:

i. The nearest whole number. (1 mark)

ii. Two decimal places. (1 mark)

Question 3: Absolute and percentage errors (8 marks)

A learner estimates that the length of a mathematics textbook is 28 cm. When measured using a ruler, the actual length is found to be 30 cm.

a) State the estimated length and the actual length. (2 marks)

b) Calculate the absolute error. (2 marks)

c) Calculate the percentage error. (3 marks)

d) State one way of reducing errors when measuring the length of an object. (1 mark)

Question 4: Application of percentage errors (8 marks)

A farmer estimates that a rectangular piece of land is 40 metres long. A proper measurement gives the actual length as 42 metres.

a) Calculate the absolute error. (2 marks)

b) Calculate the percentage error. (3 marks)

c) If the width of the land is 25 metres, calculate its actual area. (2 marks)

d) Give one reason why accurate land measurements are important to a farmer. (1 mark)

Question 5: Measurement errors and accuracy (9 marks)

a) Define the following types of errors:

i. Instrumental error. (2 marks)

ii. Human error. (2 marks)

b) State three ways of minimising errors when taking measurements. (3 marks)

c) Explain the difference between accuracy and precision. (2 marks)

Question 6: Practical application of approximation and errors (9 marks)

A water container in a school is estimated to hold 180 litres of water. When measured using a suitable measuring instrument, the actual volume is found to be 200 litres.

a) Calculate the absolute error. (2 marks)

b) Calculate the percentage error. (3 marks)

c) If the school uses 50 litres of water each day, calculate how many complete days the container can supply water at that rate. (2 marks)

d) State two ways in which accurate measurement of water can benefit the school. (2 marks)

ANSWERS AND MARKING SCHEME

Section A: Multiple-choice answers

Each correct answer carries 1 mark. Total: 30 marks.

Question	Answer	Question	Answer
1	B	16	B
2	C	17	B
3	B	18	B
4	D	19	B
5	B	20	A
6	C	21	B
7	D	22	C
8	B	23	B
9	C	24	A
10	C	25	C
11	B	26	C
12	C	27	D
13	B	28	B
14	B	29	C
15	C	30	C

Section B: Detailed marking scheme

Question 1 (8 marks)

i. Estimation is the process of finding an approximate value of a quantity without necessarily measuring it exactly. (2 marks)

ii. Measurement is the process of determining the size or magnitude of a quantity by comparing it with an accepted standard unit. (2 marks)

b) Any two correct pairs:

Quantity	Instrument
Length	Ruler or tape measure
Mass	Weighing scale

Quantity	Instrument
Time	Stopwatch or clock
Temperature	Thermometer
Volume	Measuring cylinder or jug

(1 mark for each correct pair: 2 marks)

$$3.6 \times 1{,}000 = \boxed{3{,}600 \text{ m}}$$

(1 mark)

$$2.5 \times 1{,}000 = \boxed{2{,}500 \text{ mL}}$$

(1 mark)

Question 2 (8 marks)

$$48.763 \approx \boxed{48.76}$$

(1 mark)

$$7{,}856 \approx \boxed{7{,}900}$$

(1 mark)

$$0.006784 \approx \boxed{0.0068}$$

(2 marks)

i. 0.00456 has **3 significant figures.** (1 mark)

ii. 3.050 has **4 significant figures.** (1 mark)

i. Nearest whole number:

$$\boxed{0.5}$$

(1 mark)

ii. Two decimal places:

$$\boxed{0.005}$$

(1 mark)

Question 3 (8 marks)

Estimated length = 28 cm. (1 mark)

Actual length = 30 cm. (1 mark)

b) Absolute error:

$$E = |28 - 30|$$

$$\boxed{2 \text{ cm}}$$

(2 marks)

c) Percentage error:

$$= \frac{2}{30} \times 100\%$$

$$= 6.666 \ldots \%$$

$$\boxed{6.67\%}$$

(3 marks)

d) Any one:

- Use a suitable ruler.
- Start measuring from the zero mark.
- Read the scale carefully.
- Repeat the measurement.
- Avoid reading from an incorrect angle.

(1 mark)

Question 4 (8 marks)

a) Absolute error:

$$E = |40 - 42|$$

$$\boxed{2 \text{ m}}$$

(2 marks)

b) Percentage error:

$$= \frac{2}{42} \times 100\%$$

$$= 4.7619 \ldots \%$$

$$\boxed{4.76\%}$$

(3 marks)

c) Area:

$$A = \text{Length} \times \text{Width}$$

$$= 42 \times 25$$

$$\boxed{1{,}050 \text{ m}^2}$$

(2 marks)

d) Any one:

- To determine the correct size of the farm.
- To plan planting activities.
- To estimate the amount of fertiliser required.
- To avoid disputes over land boundaries.
- To calculate the correct quantity of materials.

(1 mark)

Question 5 (9 marks)

i. Instrumental error is an error caused by a fault, defect or limitation in a measuring instrument. (2 marks)

ii. Human error is an error caused by mistakes made by a person during measurement, reading or recording. (2 marks)

b) Any three:

- Use suitable measuring instruments.
- Calibrate instruments when necessary.
- Check instruments for zero errors.
- Read scales carefully.
- Repeat measurements.
- Record measurements correctly.
- Use proper measuring procedures.
- Avoid unsuitable environmental conditions.

(1 mark each: 3 marks)

- Accuracy refers to how close a measurement is to the true or accepted value. (1 mark)
- Precision refers to how close repeated measurements are to one another. (1 mark)

Question 6 (9 marks)

a) Absolute error:

$$E = |180 - 200|$$

$$\boxed{20 \text{ L}}$$

(2 marks)

b) Percentage error:

$$= \frac{20}{200} \times 100\%$$

$$= \boxed{10\%}$$

(3 marks)

c) Number of complete days:

$$\text{Days} = \frac{\text{Total volume}}{\text{Daily consumption}}$$

$$= \frac{200}{50}$$

$$\boxed{4 \text{ days}}$$

(2 marks)

d) Any two:

- To monitor water consumption.
- To prevent water wastage.
- To plan when to refill the container.
- To ensure sufficient water for learners.
- To prepare an accurate school water budget.

(1 mark each: 2 marks)

STRAND 2.0: MEASUREMENTS AND GEOMETRY

SUB-STRAND 2.2: DISTANCE, TIME AND SPEED

2.1 DISTANCE, TIME AND SPEED

2.1.1 Distance

Definition: Distance is the total length of the actual path travelled by a moving object, regardless of the direction of movement.

Distance tells us how far an object has travelled.

Examples of distance in everyday life

- The distance covered by a learner when running around a school field.
- The distance travelled by a matatu from Kilgoris to Narok.
- The distance covered by a cyclist travelling from one village to another.
- The length of a road between two towns.
- The distance covered by a footballer while moving around a football pitch.

Units of distance

The SI unit of distance is the **metre (m)**.

Other commonly used units include:

Unit	Symbol	Equivalent
Millimetre	mm	0.001 m
Centimetre	cm	0.01 m
Metre	m	1 m
Kilometre	km	1,000 m

Conversion of units of distance

The following conversions are important:

$$1 \text{ km} = 1000 \text{ m}$$

$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ cm} = 10 \text{ mm}$$

$$1 \text{ km} = 100{,}000 \text{ cm}$$

Worked Example 1

Convert 5.6 km into metres.

Solution

Since:

$$1 \text{ km} = 1000 \text{ m}$$

Therefore:

$$5.6 \times 1000 = 5600 \text{ m}$$

Answer: 5,600 m

Worked Example 2

Convert 3,750 metres into kilometres.

Solution

$$1 \text{ km} = 1000 \text{ m}$$

$$3750 \div 1000 = 3.75 \text{ km}$$

Answer: 3.75 km

2.1.2 Types of distance

Distance can be considered in different contexts.

a) Total distance

This is the entire length of the path covered by an object.

Example: A learner runs 200 m around a field, then runs another 100 m. The total distance is:

$$200 + 100 = 300 \text{ m}$$

b) Distance travelled in a straight line

This is the length of a straight path between two points when the object actually travels along that straight path.

c) Distance travelled along a curved path

An object may move along a curved road, circular track or winding path. The distance is measured along the actual path travelled.

Figure 1: Learners participating in an athletics competition. The distance covered can be measured along the running track.

2.1.3 Time

Definition: Time is the measure of the duration or period taken by an event or an object to complete an activity or journey.

Time tells us how long an activity takes.

Examples of time in everyday life

- The time taken by a learner to run 100 metres.
- The duration of a journey from Nairobi to Mombasa.
- The time taken by a boda boda rider to travel between two villages.
- The time taken by a train to travel between stations.
- The time taken by a vehicle to accelerate from rest to a particular speed.

Units of time

The SI unit of time is the **second (s)**.

Other commonly used units include:

Unit	Symbol	Equivalent
Second	s	1 second
Minute	min	60 seconds

Unit	Symbol	Equivalent
Hour	h	60 minutes
Day	d	24 hours

Important conversions:

$$1 \text{ minute} = 60 \text{ seconds}$$

$$1 \text{ hour} = 60 \text{ minutes}$$

$$1 \text{ hour} = 3600 \text{ seconds}$$

$$1 \text{ day} = 24 \text{ hours}$$

Worked Example 3

Convert 3 hours into seconds.

Solution

$$1 \text{ hour} = 3600 \text{ seconds}$$

$$3 \times 3600 = 10800 \text{ seconds}$$

Answer: 10,800 seconds

Worked Example 4

Convert 450 seconds into minutes.

Solution

$$1 \text{ minute} = 60 \text{ seconds}$$

$$450 \div 60 = 7.5 \text{ minutes}$$

Answer: 7.5 minutes

2.1.4 Measuring time

Time can be measured using different instruments.

Instrument	Application
Stopwatch	Measuring the time taken in a race
Wall clock	Measuring time during classroom lessons
Wristwatch	Measuring and keeping track of time
Digital timer	Measuring the duration of an activity
Electronic timing system	Recording athletes' race times

Practical activity: Measuring time during a running competition

1. Organise learners into groups.
2. Mark a straight running distance of 50 metres or 100 metres.
3. Ask each learner to run from the starting point to the finishing point.
4. Use a stopwatch to record the time taken by each learner.
5. Record the results in a table.
6. Compare the times taken by different learners.
7. Discuss why some learners take less time than others.

Example of a record table:

Learner	Distance covered	Time taken
A	50 m	10 s
B	50 m	12 s
C	50 m	9 s
D	50 m	11 s

The learners can use these results to calculate and compare their speeds.

2.2 SPEED

2.2.1 Meaning of speed

Definition: Speed is the distance travelled by an object per unit time.

Speed tells us how fast or slowly an object is moving.

For example:

- A matatu travelling at 80 km/h covers 80 kilometres in one hour if it maintains a constant speed.
- A learner running at 5 m/s covers 5 metres every second if the speed remains constant.
- A cyclist travelling at 10 km/h covers 10 kilometres in one hour at constant speed.

Formula for speed

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Using symbols:

$$v = \frac{d}{t}$$

Where:

- v = speed
- d = distance travelled
- t = time taken

SI unit of speed

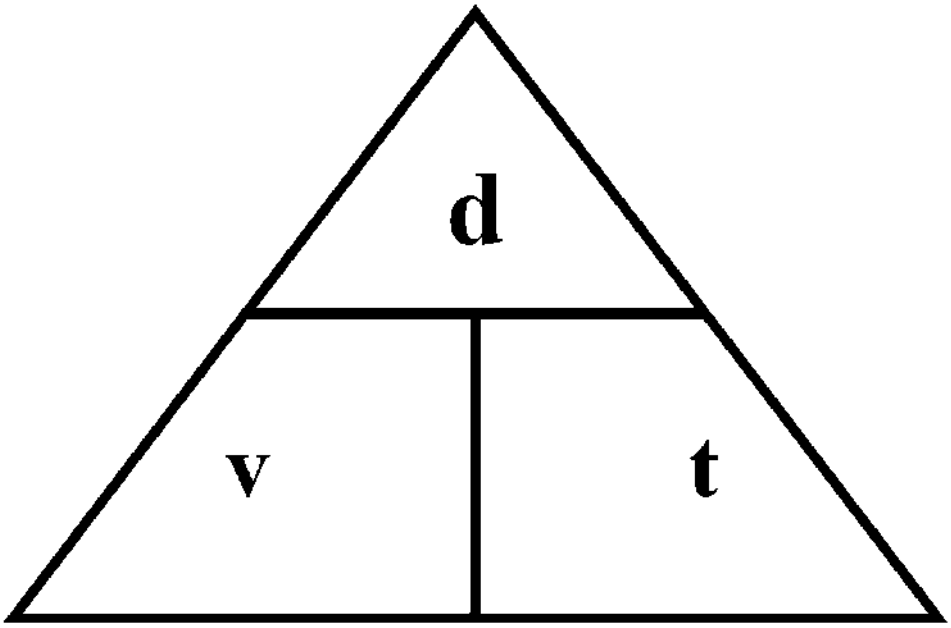
The SI unit of speed is metres per second (m/s).

Other commonly used units include:

- Kilometres per hour (km/h)
- Centimetres per second (cm/s)
- Metres per minute (m/min)

Formula triangle for speed

Speed formula triangle



Cover the quantity you want to calculate.

- To calculate speed: $v = d \div t$
- To calculate distance: $d = v \times t$
- To calculate time: $t = d \div v$

2.2.2 Calculating speed

Worked Example 5

A learner runs a distance of 100 metres in 20 seconds. Calculate the learner's speed.

Solution

Given:

$$d = 100 \text{ m}, \quad t = 20 \text{ s}$$

Formula:

$$v = \frac{d}{t}$$

Substitute:

$$v = \frac{100}{20}$$

Answer: 5 m/s

Worked Example 6

A matatu travels a distance of 180 kilometres in 3 hours. Calculate its average speed.

Solution

Given:

$$d = 180 \text{ km}, \quad t = 3 \text{ h}$$

$$v = \frac{d}{t}$$

$$v = \frac{180}{3}$$

Answer: 60 km/h

2.2.3 Calculating distance

The speed formula can be rearranged to calculate distance.

$$\boxed{d = v \times t}$$

Worked Example 7

A vehicle travels at a constant speed of 72 km/h for 4 hours. Calculate the distance covered.

Solution

Given:

Formula:

$$d = v \times t$$

$$d = 72 \times 4$$

$$d = 288 \text{ km}$$

Answer: 288 km

2.2.4 Calculating time

The speed formula can also be rearranged to calculate time.

$$\boxed{t = \frac{d}{v}}$$

Worked Example 8

A lorry travels a distance of 240 km at an average speed of 80 km/h. Calculate the time taken.

Solution

Given:

$$t = \frac{d}{v}$$

$$t = \frac{240}{80}$$

$$t = 3 \text{ hours}$$

Answer: 3 hours

2.2.5 Average speed

Definition: Average speed is the total distance travelled divided by the total time taken.

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

Average speed is not necessarily the same as the arithmetic average of the speeds recorded during a journey.

Worked Example 9

A vehicle travels 120 km in 2 hours and then travels another 180 km in 3 hours. Calculate its average speed for the whole journey.

Solution

Total distance:

$$120 + 180 = 300 \text{ km}$$

Total time:

$$2 + 3 = 5 \text{ hours}$$

Average speed:

$$= \frac{300}{5}$$

Answer: 60 km/h

Important point: When calculating average speed, always use total distance divided by total time.

2.2.6 Converting speed from km/h to m/s

In mathematics and science, speed may be expressed in kilometres per hour or metres per second.

To convert km/h into m/s, multiply by:

$$\frac{5}{18}$$

Therefore:

To convert m/s into km/h, multiply by:

$$\frac{18}{5}$$

Therefore:

Derivation of the conversion factor

We know that:

$$1 \text{ km} = 1000 \text{ m}$$

and

$$1 \text{ hour} = 3600 \text{ s}$$

Therefore:

Simplifying:

Hence:

Worked Example 10

Convert 90 km/h into m/s.

Solution

$$90 \times \frac{5}{18}$$

$$= 5 \times 5$$

Answer: 25 m/s

Worked Example 11

Convert 15 m/s into km/h.

Solution

$$15 \times \frac{18}{5}$$

$$= 3 \times 18$$

Answer: 54 km/h

2.2.7 Uniform and non-uniform speed

a) Uniform speed

Uniform speed occurs when an object covers equal distances in equal intervals of time.

Example: A vehicle moving at a constant speed of 60 km/h on a straight road.

b) Non-uniform speed

Non-uniform speed occurs when an object covers unequal distances in equal intervals of time, or equal distances in unequal intervals of time.

Examples:

- A matatu slowing down when approaching a bus stop.
- A cyclist moving faster downhill and slower uphill.
- A vehicle changing speed due to traffic.
- A runner accelerating towards the finishing line.

2.2.8 Factors affecting speed

The speed of a moving object may be affected by:

1. **Road condition:** Vehicles may move slowly on rough or muddy roads.
2. **Traffic congestion:** Heavy traffic reduces vehicle speed.
3. **Weather:** Rain, fog and strong winds may affect movement.
4. **Slope of the road:** Vehicles may move faster downhill and slower uphill.
5. **Engine power:** The power of a vehicle's engine affects its ability to move at different speeds.
6. **Load carried:** A heavily loaded vehicle may accelerate more slowly.
7. **Friction:** Friction between moving parts and surfaces affects motion.
8. **Safety regulations:** Speed limits restrict the maximum permitted speed on roads.

2.2.9 Distance-time graphs

A distance-time graph shows how the distance travelled by an object changes with time.

- Time is represented on the horizontal axis (x-axis).
- Distance is represented on the vertical axis (y-axis).
- The gradient (slope) of the graph represents speed.

$$\text{Speed} = \frac{\text{Change in distance}}{\text{Change in time}}$$

Interpreting distance-time graphs

1. Straight sloping line

A straight sloping line indicates uniform speed.

2. Horizontal line

A horizontal line indicates that the object is stationary because the distance does not change as time passes.

3. Curved line

A curved line indicates changing speed.

4. Steeper line

A steeper line represents a greater speed.

Figure 2: Distance-time graph for a moving object

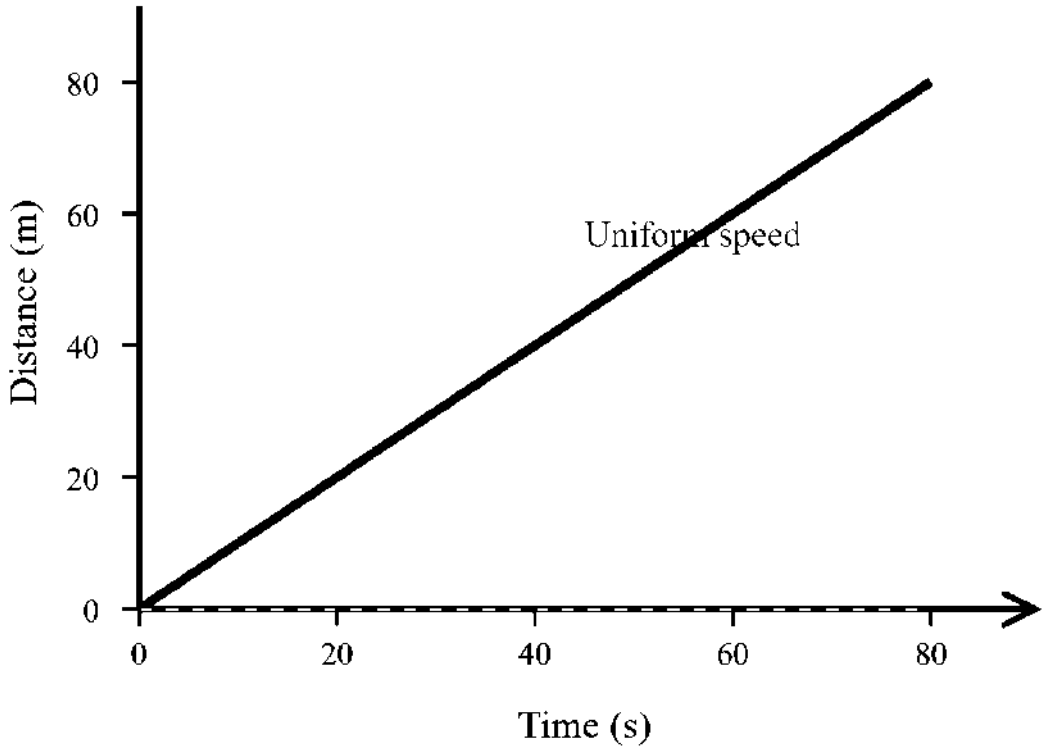


Illustration: The straight sloping line shows that distance increases uniformly with time.

Worked Example 12: Finding speed from a distance-time graph

A learner covers 80 metres in 20 seconds at uniform speed. Calculate the speed represented by the graph.

Solution

$$\text{Speed} = \frac{\text{Change in distance}}{\text{Change in time}}$$

$$= \frac{80 - 0}{20 - 0}$$

$$= \frac{80}{20}$$

Answer: 4 m/s

Important points about distance-time graphs

- A horizontal line means zero speed.
- A straight sloping line means constant speed.
- A curved line means changing speed.
- The gradient is calculated using two points on the line.
- The steeper the line, the greater the speed.

2.3 VELOCITY

2.3.1 Meaning of velocity

Definition: Velocity is the rate of change of displacement with time in a specified direction.

Unlike speed, velocity includes both magnitude and direction.

For example:

- A vehicle travelling at 80 km/h has a speed of 80 km/h.
- A vehicle travelling at 80 km/h towards Nakuru has a velocity of 80 km/h towards Nakuru.

The direction is important when describing velocity.

Difference between speed and velocity

Speed	Velocity
It is distance travelled per unit time.	It is displacement per unit time.
It is a scalar quantity.	It is a vector quantity.
It has magnitude only.	It has magnitude and direction.
It is always non-negative.	It may be positive, negative or zero depending on the chosen direction.
SI unit is m/s.	SI unit is m/s.

2.3.2 Scalar and vector quantities

Scalar quantity: A physical quantity that has magnitude only and no direction.

Examples:

- Distance
- Speed
- Time
- Mass
- Temperature
- Energy

Vector quantity: A physical quantity that has both magnitude and direction.

Examples:

- Displacement
- Velocity
- Acceleration
- Force
- Momentum

2.3.3 Displacement

Definition: Displacement is the straight-line distance from the starting point to the finishing point in a specified direction.

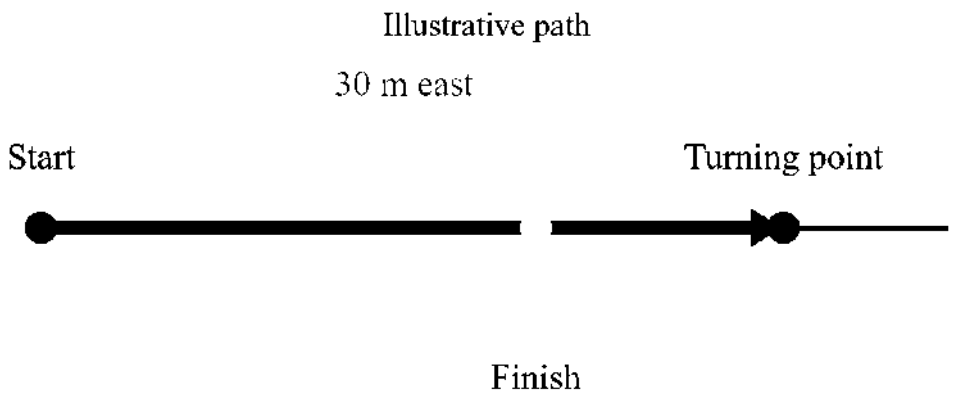
Displacement considers the change in position and direction of movement.

Difference between distance and displacement

Consider a learner who walks 30 metres east and then 10 metres west.

- Total distance = 30 + 10 = 40 m.
- Displacement = 30 − 10 = 20 m east.

Figure 3: Distance and displacement



The learner covers 40 m in total, but finishes 20 m east of the starting point.

Worked Example 13

A learner walks 50 metres east and then 20 metres west. Calculate:

a) The total distance covered.

b) The displacement.

Solution

a) Total distance:

$$50 + 20 = 70 \text{ m}$$

b) Displacement:

$$50 - 20 = 30 \text{ m east}$$

Answers:

- Distance = 70 m.
- Displacement = 30 m east.

2.3.4 Formula for velocity

Average velocity is given by:

$$\boxed{\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}}}$$

Using symbols:

$$\boxed{v = \frac{s}{t}}$$

Where:

- v = average velocity
- s = displacement
- t = time taken

The SI unit is metres per second (m/s).

Worked Example 14

A cyclist moves 120 metres east in 20 seconds. Calculate the average velocity.

Solution

Given:

$$s = 120 \text{ m east}$$

$$t = 20 \text{ s}$$

$$v = \frac{s}{t}$$

$$v = \frac{120}{20}$$

Answer: 6 m/s east.

Worked Example 15

A learner walks 100 metres east and then 100 metres west, returning to the starting point. The journey takes 50 seconds. Calculate:

a) The total distance.

b) The displacement.

c) The average speed.

d) The average velocity.

Solution

a) Total distance:

$$100 + 100 = 200 \text{ m}$$

b) Displacement:

The learner returns to the starting point.

$$\text{Displacement} = 0 \text{ m}$$

c) Average speed:

d) Average velocity:

Answers:

- Distance = 200 m.

- Displacement = 0 m.
- Average speed = 4 m/s.
- Average velocity = 0 m/s.

Important: An object can travel a considerable distance and still have zero displacement if it returns to its starting point.

2.3.5 Direction and sign convention

When solving velocity problems involving opposite directions, it is useful to choose one direction as positive and the opposite direction as negative.

For example, if east is positive:

- 10 m/s east = +10 m/s.
- 10 m/s west = −10 m/s.

If north is positive:

- 5 m/s north = +5 m/s.
- 5 m/s south = −5 m/s.

This makes calculations involving motion in different directions easier.

2.4 ACCELERATION

2.4.1 Meaning of acceleration

Definition: Acceleration is the rate of change of velocity with time.

An object accelerates when its velocity changes. The change may involve:

1. An increase in speed.
2. A decrease in speed.
3. A change in direction.
4. A combination of changes in speed and direction.

Examples in Kenya include:

- A matatu increasing speed after leaving a bus stop.
- A lorry slowing down when approaching a road junction.
- A motorcycle accelerating when moving away from rest.
- A train increasing speed after leaving a station.
- A cyclist slowing down when approaching a steep bend.

2.4.2 Formula for acceleration

$$\boxed{a = \frac{v - u}{t}}$$

Where:

- a = acceleration
- v = final velocity
- u = initial velocity
- t = time taken

The SI unit of acceleration is metres per second squared (m/s^2).

This means the change in velocity, measured in metres per second, for every second.

2.4.3 Types of acceleration

a) Positive acceleration

Positive acceleration occurs when velocity increases in the chosen positive direction.

Example: A vehicle increases its velocity from 10 m/s to 20 m/s in 5 seconds.

b) Negative acceleration

Negative acceleration occurs when acceleration acts in the negative direction of the chosen coordinate system.

For an object moving in the positive direction, negative acceleration may cause it to slow down.

c) Uniform acceleration

Uniform acceleration occurs when velocity changes by equal amounts in equal intervals of time.

d) Non-uniform acceleration

Non-uniform acceleration occurs when velocity changes by unequal amounts in equal intervals of time.

e) Deceleration

Deceleration is a decrease in speed. It is commonly used to describe slowing down, although the sign of acceleration depends on the chosen direction.

2.4.4 Calculating acceleration

Worked Example 16

A vehicle increases its velocity from 10 m/s to 30 m/s in 5 seconds. Calculate its acceleration.

Solution

Given:

$$t = 5 \text{ s}$$

Formula:

$$a = \frac{v - u}{t}$$

Substitute:

$$a = \frac{30 - 10}{5}$$

$$a = \frac{20}{5}$$

Answer: 4 m/s²

Worked Example 17

A motorcycle increases its velocity from 5 m/s to 25 m/s in 4 seconds. Find its acceleration.

Solution

$$a = \frac{v - u}{t}$$

$$a = \frac{25 - 5}{4}$$

$$a = \frac{20}{4}$$

Answer: 5 m/s²

Worked Example 18: Deceleration

A car travelling at 25 m/s slows down to 5 m/s in 4 seconds. Calculate its acceleration.

Solution

Given:

$$t = 4 \text{ s}$$

$$a = \frac{v - u}{t}$$

$$a = \frac{5 - 25}{4}$$

$$a = \frac{-20}{4}$$

Answer: −5 m/s²

The negative sign indicates that the acceleration is opposite to the direction of the initial motion, assuming the vehicle was travelling in the positive direction.

The magnitude of the deceleration is 5 m/s².

Worked Example 19: Finding final velocity

A vehicle starts with a velocity of 4 m/s and accelerates uniformly at 3 m/s² for 6 seconds. Calculate its final velocity.

Solution

Formula:

$$a = \frac{v - u}{t}$$

Rearrange:

$$v = u + at$$

Substitute:

$$v = 4 + (3 \times 6)$$

$$v = 4 + 18$$

Answer: 22 m/s

Worked Example 20: Finding time

A vehicle increases its velocity from 10 m/s to 30 m/s with a uniform acceleration of 4 m/s². Calculate the time taken.

Solution

$$a = \frac{v - u}{t}$$

Rearrange:

$$t = \frac{v - u}{a}$$

$$t = \frac{30 - 10}{4}$$

$$t = \frac{20}{4}$$

$$t = 5 \text{ s}$$

Answer: 5 seconds

2.4.5 Velocity-time graphs

A velocity-time graph shows how the velocity of an object changes with time.

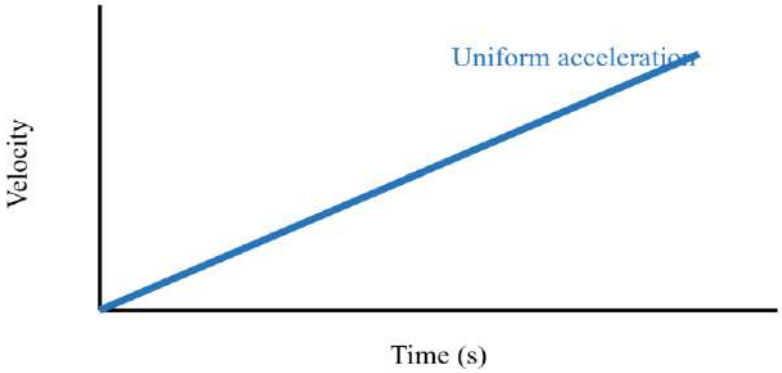
- Time is represented on the horizontal axis.
- Velocity is represented on the vertical axis.
- The gradient represents acceleration.
- The area under the graph represents displacement.

$$\boxed{\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Change in time}}}$$

$$\boxed{\text{Displacement} = \text{Area under velocity-time graph}}$$

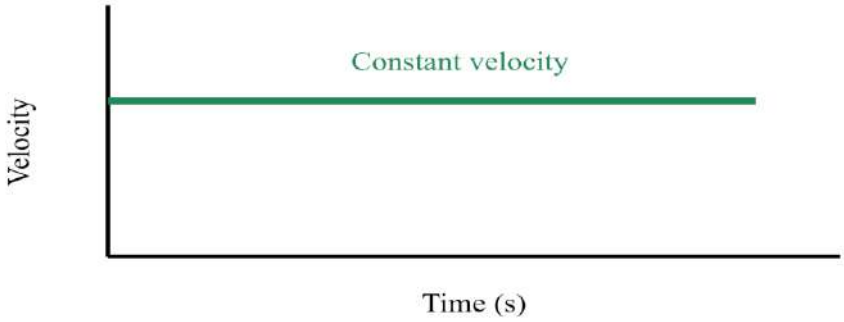
Common shapes of velocity-time graphs

Figure 4: Uniform acceleration



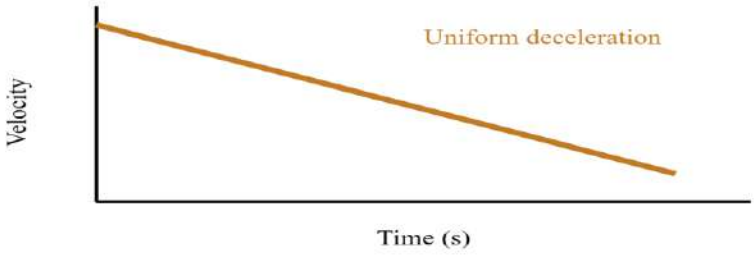
A straight sloping line shows a constant rate of change of velocity.

Figure 5: Constant velocity



A horizontal line shows that velocity is constant and acceleration is zero.

Figure 6: Uniform deceleration



A downward sloping line indicates that velocity is decreasing with time.

Worked Example 21: Acceleration from a velocity-time graph

A vehicle increases its velocity uniformly from 0 m/s to 20 m/s in 5 seconds. Calculate its acceleration.

Solution

$$a = \frac{v - u}{t}$$

$$a = \frac{20 - 0}{5}$$

Answer: 4 m/s²

Worked Example 22: Displacement from a velocity-time graph

A vehicle accelerates uniformly from rest to 20 m/s in 5 seconds. Calculate the displacement.

Solution

The area under the graph is a triangle.

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 5 \times 20$$

$$= 50 \text{ m}$$

Answer: 50 metres.

2.4.6 Importance of understanding acceleration

Understanding acceleration is important because it helps in:

1. **Road safety:** Drivers need to understand how quickly vehicles can accelerate or stop.
2. **Vehicle design:** Engineers use acceleration calculations when designing vehicles.
3. **Sports:** Coaches use acceleration to improve athletes' performance.
4. **Transport planning:** Acceleration affects journey times and vehicle movement.
5. **Machine operation:** Workers need to understand how machines increase or reduce speed.
6. **Traffic control:** Acceleration and deceleration are important when vehicles approach junctions and roundabouts.

2.5 RELATIVE MOTION

2.5.1 Meaning of relative motion

Definition: Relative motion is the movement of one object as observed from another object.

It describes how the position or velocity of one body changes in relation to another body.

For example:

- A passenger sitting inside a moving matatu appears stationary relative to other passengers in the same vehicle.
- The same passenger appears to be moving relative to a person standing beside the road.
- A vehicle overtaking another vehicle is moving faster relative to the vehicle being overtaken.
- Two vehicles approaching each other have a relative speed equal to the sum of their speeds when moving directly towards one another.

Relative motion helps us understand the movement of objects in relation to one another rather than considering each object's motion independently.

2.5.2 Everyday examples of relative motion

1. Two matatus travelling along the Nairobi–Nakuru highway.
2. A motor vehicle overtaking a slower lorry.
3. Two cyclists moving in the same direction.
4. Two vehicles approaching each other from opposite directions.
5. A person walking inside a moving train.
6. A passenger observing buildings from a moving bus.

7. A boat moving downstream while water flows in a river.
8. Two athletes running side by side on a track.

2.5.3 Relative speed

Definition: Relative speed is the speed of one object as observed from another moving object.

Relative speed depends on the direction in which the objects are moving.

There are two main cases:

1. Objects moving in the same direction.
2. Objects moving in opposite directions.

2.5.4 Relative motion of two bodies moving in the same direction

When two bodies move in the same direction, their relative speed is the difference between their speeds.

$$\boxed{\text{Relative speed} = \text{Faster speed} - \text{Slower speed}}$$

If the speeds are v_1 and v_2, where $v_1 > v_2$:

$$\boxed{v_r = v_1 - v_2}$$

Explanation

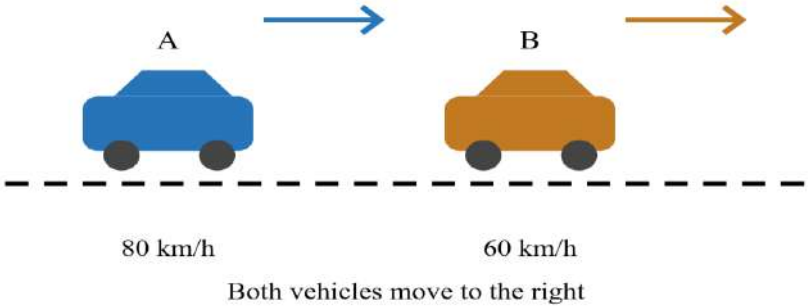
Suppose two vehicles are travelling along the same road:

- Vehicle A travels at 80 km/h.
- Vehicle B travels at 60 km/h.

Vehicle A gains on Vehicle B at a rate of:

Therefore, the relative speed of A with respect to B is 20 km/h.

Figure 7: Two vehicles moving in the same direction



Vehicle A is faster than vehicle B. Their relative speed is 20 km/h.

Worked Example 23

Two matatus are travelling in the same direction along a highway. The first matatu moves at 90 km/h while the second moves at 65 km/h. Calculate their relative speed.

Solution

Given:

Since they are moving in the same direction:

$$v_r = v_1 - v_2$$
$$= 90 - 65$$

Answer: 25 km/h

Worked Example 24: Time taken to overtake

A lorry travelling at 50 km/h is overtaken by a bus travelling at 80 km/h. If the bus needs to gain a relative distance of 150 metres to complete the overtaking manoeuvre, calculate the time taken to gain this distance.

Solution

First, calculate relative speed:

Convert to m/s:

Time:

$$t = \frac{\text{Distance}}{\text{Relative speed}}$$

$$t = 150 \times \frac{3}{25}$$
$$t = 18\ \text{s}$$

Answer: 18 seconds.

Note: This is a simplified mathematical model. Actual overtaking requires additional space and time for safe manoeuvring, and drivers must obey road markings and traffic rules.

2.5.5 Relative motion of two bodies moving in opposite directions

When two bodies move directly towards each other, their relative speed is the sum of their speeds.

$$\boxed{\text{Relative speed} = \text{Speed of A} + \text{Speed of B}}$$

Using symbols:

$$\boxed{v_r = v_1 + v_2}$$

Explanation

Suppose:

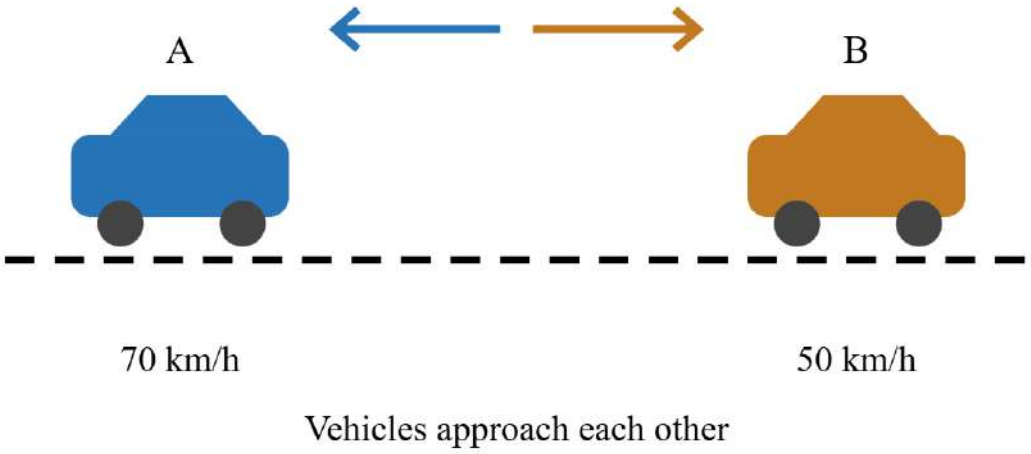
- Vehicle A travels east at 70 km/h.
- Vehicle B travels west at 50 km/h.

The vehicles are approaching each other.

The distance between them reduces at:

Their relative speed is therefore 120 km/h.

Figure 8: Two vehicles moving in opposite directions



The relative speed is the sum of the two speeds.

Worked Example 25

Two vehicles are travelling towards each other on a straight road. One travels at 72 km/h and the other at 54 km/h. Calculate their relative speed.

Solution

Since they are moving in opposite directions:

$$v_r = v_1 + v_2$$
$$= 72 + 54$$

Answer: 126 km/h

Worked Example 26: Time taken for two vehicles to meet

Two vehicles are 150 km apart and travel directly towards each other. Their speeds are 80 km/h and 70 km/h respectively. Calculate the time taken for them to meet.

Solution

Relative speed:

Time:

$$t = \frac{\text{Distance between vehicles}}{\text{Relative speed}}$$

$$t = \frac{150}{150}$$

$$t = 1 \text{ hour}$$

Answer: 1 hour.

Worked Example 27: Finding the distance between two approaching vehicles

Two buses are 300 km apart. They travel towards each other at speeds of 80 km/h and 70 km/h. Calculate the distance between them after 1.5 hours.

Solution

Relative speed:

Distance closed in 1.5 hours:

$$d = 150 \times 1.5$$

$$d = 225 \text{ km}$$

Remaining distance:

$$300 - 225 = 75 \text{ km}$$

Answer: 75 km.

2.5.6 Summary of relative speed

Direction of motion	Relative speed	Mathematical expression
Same direction	Difference of the speeds	$v_r = v_1 - v_2$, where $v_1 > v_2$
Opposite directions	Sum of the speeds	$v_r = v_1 + v_2$

Remember:

- Same direction: subtract the speeds.
- Opposite directions: add the speeds.
- When calculating time, use the relative distance and relative speed.
- Ensure that all units are consistent before calculating.

2.5.7 Relative velocity using direction

Relative velocity can also be calculated using signed velocities.

If the velocity of object A is v_A and that of object B is v_B, then:

Where means the velocity of A relative to B.

For example, take east as positive.

- A moves east at 60 km/h: $v_A = +60$.
- B moves west at 40 km/h: $v_B = -40$.

Therefore:

$$= 60 + 40$$

This signed-velocity method works for both same-direction and opposite-direction motion.

2.6 APPLICATIONS OF RELATIVE MOTION IN REAL LIFE

Relative motion is useful in many everyday activities.

2.6.1 Overtaking on roads

Overtaking occurs when a faster vehicle moves past a slower vehicle travelling in the same direction.

Relative speed determines how quickly the faster vehicle gains on the slower one.

Example:

A car moving at 100 km/h overtakes a lorry moving at 70 km/h.

$$\text{Relative speed} = 100 - 70$$

The car gains on the lorry at a rate of 30 km/h.

Safety considerations:

- Drivers should overtake only when it is safe and permitted.
- They should maintain a safe following distance.
- They should not overtake on blind corners, steep hills or where road markings prohibit it.
- They should consider the speed and distance of oncoming traffic.
- They should return to their lane only when there is sufficient space.

2.6.2 Vehicles approaching each other

When vehicles move in opposite directions, their relative speed is the sum of their speeds.

This helps drivers understand how quickly the distance between approaching vehicles decreases.

2.6.3 Walking inside a moving vehicle

A passenger may walk inside a moving bus.

If the bus moves forward at 50 km/h and the passenger walks forward at 4 km/h relative to the bus, then the passenger's speed relative to the road is:

If the passenger walks backwards at 4 km/h relative to the bus:

This simplified example assumes the passenger walks in the same straight-line direction as the bus.

2.6.4 Boats moving in rivers

A boat's motion relative to the riverbank depends on its speed relative to the water and the speed of the river current.

When a boat moves downstream:

$$\text{Speed relative to bank} = \text{Boat speed in still water} + \text{Current speed}$$

When a boat moves upstream:

$$\text{Speed relative to bank} = \text{Boat speed in still water} - \text{Current speed}$$

Worked Example 28

A boat travels at 12 km/h in still water. The river current flows at 3 km/h. Calculate:

a) The speed downstream.

b) The speed upstream.

Solution

a) Downstream:

b) Upstream:

Answers:

- Downstream speed = 15 km/h.
- Upstream speed = 9 km/h.

2.6.5 Movement of athletes

Relative motion can be used to compare athletes running in the same race.

For example, if one runner moves at 8 m/s and another moves at 7 m/s in the same direction:

The faster runner gains on the slower runner at 1 m/s.

2.6.6 Movement of trains

Relative motion is used to determine the time taken by two trains to pass each other or by one train to pass a stationary object.

When two trains move in opposite directions, their relative speed is the sum of their speeds.

When they move in the same direction, their relative speed is the difference between their speeds.

For train-passing problems, the total distance needed for the trains to pass each other is the sum of their lengths.

Worked Example 29: Two trains passing each other

Train A is 120 m long and travels at 20 m/s. Train B is 80 m long and travels at 15 m/s in the opposite direction. Calculate the time taken for the trains to pass each other completely.

Solution

Total distance to be covered relative to each other:

$$120 + 80 = 200 \text{ m}$$

Relative speed:

Time:

$$t = \frac{200}{35}$$

$$t = \frac{40}{7}$$

$$t \approx 5.71 \text{ s}$$

Answer: Approximately 5.71 seconds.

2.6.7 Importance of relative motion

Relative motion is important because it:

1. Helps drivers estimate the movement of other vehicles.
2. Helps determine the time required for vehicles to meet.
3. Helps calculate the time required for one vehicle to overtake another.
4. Helps in planning safe road transport.
5. Helps in understanding the movement of trains, boats and aircraft.
6. Helps athletes compare their movement during competitions.
7. Helps engineers design transport systems.
8. Helps learners understand motion in everyday situations.

2.7 PRACTICAL ACTIVITIES AND CBE LEARNING EXPERIENCES

The following activities help learners develop mathematical reasoning, problem-solving skills, collaboration and practical application of mathematics.

Activity 1: Investigating distance, time and speed

Learning task: Determine the speed of learners participating in a running competition.

Materials:

- Measuring tape
- Stopwatch
- Cones or markers
- Recording sheets
- Calculator

Procedure:

1. Measure and mark a distance of 50 metres.
2. Organise learners into groups.
3. Let each learner run the marked distance.
4. Record the time taken by each learner.
5. Calculate the speed of each learner.
6. Compare the speeds obtained.

Sample results:

Learner	Distance (m)	Time (s)	Speed (m/s)
A	50	10	5.0
B	50	12	4.17
C	50	8	6.25

Learner	Distance (m)	Time (s)	Speed (m/s)
D	50	10	5.0

Discussion questions:

1. Which learner recorded the highest speed?
2. Which learner took the longest time?
3. Why did the learners record different speeds?
4. What would happen to the calculated speed if the distance were doubled while the time remained the same?

Activity 2: Demonstrating velocity

Materials:

- Toy car
- Measuring tape
- Stopwatch
- Chalk
- Direction markers

Procedure:

1. Mark a straight path on the ground.
2. Identify the starting and finishing points.
3. Move a toy car along the marked path.
4. Measure the displacement and time taken.
5. Record the direction of movement.
6. Calculate the average velocity.
7. Repeat the activity in the opposite direction.

Discussion: Why is it necessary to state the direction when describing velocity?

Activity 3: Investigating acceleration

Materials:

- Toy car
- Stopwatch
- Measuring tape
- Smooth ramp
- Markers

Procedure:

1. Set up a small ramp on a flat surface.
2. Release a toy car from rest.
3. Observe how its speed changes as it moves.
4. Record its velocity at selected points using appropriate measuring equipment.
5. Discuss whether the car is accelerating.
6. Calculate acceleration if the initial velocity, final velocity and time are known.

Discussion questions:

1. What happens to the speed of the toy car as it moves down the ramp?
2. What is meant by acceleration?

3. How would friction affect the motion of the car?

Activity 4: Investigating relative motion using toy cars

Materials:

- Two toy cars
- Straight track
- Measuring tape
- Stopwatch
- Recording sheet

Procedure:

1. Place two toy cars on a straight track.
2. Move both cars in the same direction at different speeds.
3. Observe how the distance between them changes.
4. Calculate their relative speed.
5. Repeat the activity with the cars moving in opposite directions.
6. Compare the two relative speeds.

Discussion questions:

1. Why do we subtract speeds when two objects move in the same direction?
2. Why do we add speeds when two objects move towards each other?
3. How can relative motion help drivers avoid accidents?

Activity 5: Observing relative motion in the community

Learners may observe traffic from a safe location under teacher supervision.

They should identify:

- Vehicles moving in the same direction.
- Vehicles moving in opposite directions.
- Vehicles overtaking other vehicles.
- Vehicles slowing down and accelerating.
- Pedestrians moving alongside vehicles.

Learners should record their observations and explain how relative motion is involved.

2.8 SUMMARY OF IMPORTANT FORMULAE

Quantity	Formula	SI unit
Speed	$v = \frac{d}{t}$	m/s
Distance	$d = vt$	m
Time	$t = \frac{d}{v}$	s
Average speed	$\frac{\text{Total distance}}{\text{Total time}}$	m/s
Average velocity	$\frac{\text{Displacement}}{\text{Time}}$	m/s
Acceleration	$a = \frac{v-u}{t}$	m/s^2
Final velocity	$v = u + at$	m/s
Time using acceleration	$t = \frac{v-u}{a}$	s

Relative speed, same direction	$v_r = v_1 - v_2$	m/s
Relative speed, opposite directions	$v_r = v_1 + v_2$	m/s
Time for two bodies to meet	$t = \frac{\text{Separation}}{\text{Relative speed}}$	s
Distance from velocity-time graph	Area under graph	m

Important conversions

$$1 \text{ km} = 1000 \text{ m}$$

$$1 \text{ h} = 3600 \text{ s}$$

REVISION EXERCISES

Section A: 30 marks

SECTION A: MULTIPLE-CHOICE QUESTIONS

1. What is the meaning of distance?

A. The rate of change of velocity with time.

B. The total length of the actual path travelled by an object.

C. The straight-line change in position in a specified direction.

D. The time taken by an object to move.

2. The SI unit of time is:

A. Minute

B. Hour

C. Second

D. Day

3. Convert 4.5 kilometres into metres.

A. 450 m

B. 4,500 m

C. 45,000 m

D. 450,000 m

4. A learner runs 100 metres in 20 seconds. What is the learner's speed?

A. 2 m/s

B. 4 m/s

C. 5 m/s

D. 10 m/s

5. A vehicle travels at 60 km/h for 3 hours. Calculate the distance covered.

A. 20 km

B. 60 km

C. 120 km

D. 180 km

6. A cyclist covers 90 km at an average speed of 30 km/h. How long does the journey take?

A. 2 hours

B. 3 hours

C. 4 hours

D. 5 hours

7. Convert 72 km/h into metres per second.

A. 10 m/s

B. 15 m/s

C. 20 m/s

D. 25 m/s

8. Convert 10 m/s into kilometres per hour.

A. 18 km/h

B. 24 km/h

C. 36 km/h

D. 60 km/h

9. Which of the following is a scalar quantity?

A. Velocity

B. Displacement

C. Acceleration

D. Speed

10. Which of the following is a vector quantity?

A. Distance

B. Time

C. Velocity

D. Speed

11. A learner walks 40 metres east and then 15 metres west. What is the total distance covered?

A. 15 m

B. 25 m

C. 40 m

D. 55 m

12. From Question 11, what is the displacement of the learner?

A. 15 m east

B. 25 m east

C. 40 m east

D. 55 m west

13. A vehicle increases its velocity from 5 m/s to 25 m/s in 5 seconds. Calculate its acceleration.

A. 2 m/s^2

B. 4 m/s^2

C. 5 m/s^2

D. 10 m/s^2

14. A car slows down from 30 m/s to 10 m/s in 5 seconds. What is its acceleration?

A. -2 m/s^2

B. -4 m/s^2

C. 4 m/s^2

D. 8 m/s^2

15. What is the SI unit of acceleration?

A. m/s

B. km/h

C. m/s^2

D. m^2/s

16. A vehicle starts from rest and accelerates uniformly at 2 m/s^2 for 6 seconds. Calculate its final velocity.

A. 8 m/s

B. 10 m/s

C. 12 m/s

D. 14 m/s

17. Which graph represents uniform velocity on a velocity-time graph?

A. A vertical straight line

B. A horizontal straight line

C. A curved line

D. A downward sloping line

18. The gradient of a distance-time graph represents:

A. Acceleration

B. Distance

C. Speed

D. Time

19. The gradient of a velocity-time graph represents:

A. Distance

B. Acceleration

C. Speed

D. Displacement

20. The area under a velocity-time graph represents:

A. Acceleration

B. Time

C. Displacement

D. Speed

21. Two vehicles travel in the same direction at 90 km/h and 60 km/h. Their relative speed is:

A. 30 km/h

B. 60 km/h

C. 90 km/h

D. 150 km/h

22. Two vehicles approach each other at speeds of 70 km/h and 50 km/h. Their relative speed is:

A. 20 km/h

B. 50 km/h

C. 70 km/h

D. 120 km/h

23. Two buses are 240 km apart and travel towards each other at 50 km/h and 70 km/h. How long will they take to meet?

A. 1 hour

B. 2 hours

C. 3 hours

D. 4 hours

24. A car travels at 100 km/h while a lorry travels at 80 km/h in the same direction. The relative speed of the car with respect to the lorry is:

A. 10 km/h

B. 20 km/h

C. 80 km/h

D. 180 km/h

25. Which of the following best describes relative motion?

A. Motion of an object in a vacuum only.

B. Motion of an object as observed from another object.

C. Motion of an object without changing speed.

D. Motion of an object in a circular path only.

26. A boat travels at 15 km/h in still water. If the river current flows at 4 km/h, what is its downstream speed?

A. 11 km/h

B. 15 km/h

C. 19 km/h

D. 60 km/h

27. A boat travels at 20 km/h in still water against a current of 5 km/h. Calculate its upstream speed.

A. 5 km/h

B. 15 km/h

C. 20 km/h

D. 25 km/h

28. Two trains of lengths 100 m and 150 m travel in opposite directions at 15 m/s and 10 m/s respectively. What is their relative speed?

A. 5 m/s

B. 10 m/s

C. 15 m/s

D. 25 m/s

29. Which of the following situations demonstrates non-uniform speed?

A. A vehicle moving at 60 km/h constantly.

B. A train covering equal distances in equal time intervals.

C. A cyclist changing speed while riding uphill.

D. A car maintaining the same speed on a straight road.

30. Why is understanding relative motion important in road transport?

A. It helps drivers calculate the colour of vehicles.

B. It helps drivers estimate the movement of other vehicles and make safer decisions.

C. It eliminates the need for traffic rules.

D. It allows vehicles to travel without brakes.

SECTION B: STRUCTURED QUESTIONS

50 marks

Instructions: Answer all questions. Show all your working clearly. Give appropriate units in your answers.

QUESTION 1: DISTANCE, TIME AND SPEED (8 MARKS)

A learner participates in a 200-metre athletics competition at school. The learner takes 25 seconds to complete the race.

a) Define the term distance. (1 mark)

b) State the SI unit of distance. (1 mark)

c) Calculate the average speed of the learner during the race. (3 marks)

d) Convert the learner's speed into kilometres per hour. (3 marks)

QUESTION 2: SPEED AND AVERAGE SPEED (8 MARKS)

A matatu travels from Narok to Nairobi, covering a distance of 160 km in 2 hours. It then travels another 80 km in 1.5 hours.

a) Calculate the speed of the matatu during the first part of the journey. (2 marks)

b) Calculate the speed during the second part of the journey. (2 marks)

c) Determine the total distance travelled. (1 mark)

d) Calculate the total time taken. (1 mark)

e) Calculate the average speed for the entire journey. (2 marks)

QUESTION 3: VELOCITY AND DISPLACEMENT (8 MARKS)

A learner walks 60 metres east from a classroom to a school gate. The learner then walks 20 metres west to a water point. The entire journey takes 40 seconds.

a) Define displacement. (1 mark)

b) Calculate the total distance travelled. (2 marks)

c) Determine the displacement of the learner, stating the direction. (2 marks)

d) Calculate the average speed. (1 mark)

e) Calculate the average velocity. (2 marks)

QUESTION 4: ACCELERATION (8 MARKS)

A lorry moving along a highway increases its velocity from 8 m/s to 28 m/s in 5 seconds.

a) Define acceleration. (1 mark)

b) State the SI unit of acceleration. (1 mark)

c) Calculate the acceleration of the lorry. (3 marks)

d) If the lorry continues accelerating uniformly at the same rate for another 3 seconds, calculate its velocity at the end of the additional 3 seconds. (3 marks)

QUESTION 5: RELATIVE MOTION IN THE SAME AND OPPOSITE DIRECTIONS (9 MARKS)

Two vehicles, A and B, are travelling along a straight highway.

Vehicle A travels at 90 km/h while vehicle B travels at 60 km/h.

a) Calculate the relative speed when both vehicles are travelling in the same direction. (2 marks)

b) Calculate the relative speed when the vehicles are travelling in opposite directions. (2 marks)

c) If the vehicles are moving towards each other and are initially 300 km apart, calculate the time taken for them to meet. (3 marks)

d) State two reasons why understanding relative motion is important to drivers. (2 marks)

QUESTION 6: APPLICATION OF RELATIVE MOTION AND GRAPHS (9 MARKS)

Two trains, P and Q, are travelling towards each other on parallel railway tracks. Train P travels at 20 m/s while train Q travels at 15 m/s. Train P is 120 metres long and train Q is 80 metres long.

a) Calculate the relative speed of the two trains. (2 marks)

b) Calculate the total distance the trains must cover relative to each other to pass completely. (2 marks)

c) Calculate the time taken for the two trains to pass each other completely. (3 marks)

d) State two applications of relative motion in everyday life. (2 marks)

TOTAL: 50 MARKS

ANSWER KEY AND MARKING SCHEME

Section A: Multiple-choice answers

Question	Answer	Question	Answer
1	B	16	C
2	C	17	B
3	B	18	C
4	C	19	B

Question	Answer	Question	Answer
5	D	20	C
6	B	21	A
7	C	22	D
8	C	23	B
9	D	24	B
10	C	25	B
11	D	26	C
12	B	27	B
13	B	28	D
14	B	29	C
15	C	30	B

Section B: Suggested marking scheme

Question 1 (8 marks)

a) Distance is the total length of the actual path travelled by an object. (1)

b) Metre (m). (1)

$$v = \frac{d}{t}$$

$$= \frac{200}{25}$$

- Correct formula (1)
- Correct substitution (1)
- Correct answer and unit (1)

$$8 \times \frac{18}{5}$$

- Correct conversion formula (1)
- Correct substitution (1)
- Correct answer and unit (1)

Question 2 (8 marks)

(2 marks)

$$v = \frac{80}{1.5}$$

(2 marks)

$$160 + 80 = 240 \text{ km}$$

(1 mark)

$$2 + 1.5 = 3.5 \text{ hours}$$

(1 mark)

$$\text{Average speed} = \frac{240}{3.5}$$

- Correct formula and substitution (1)
- Correct answer (1)

Question 3 (8 marks)

a) Displacement is the straight-line change in position from the starting point to the finishing point in a specified direction. (1)

$$60 + 20 = 80 \text{ m}$$

(2 marks)

$$60 - 20 = 40 \text{ m east}$$

(2 marks)

(1 mark)

$$\text{Average velocity} = \frac{40}{40}$$

(2 marks)

Question 4 (8 marks)

a) Acceleration is the rate of change of velocity with time. (1)

b) Metres per second squared (m/s²). (1)

$$a = \frac{v - u}{t}$$

$$= \frac{28 - 8}{5}$$

$$= \frac{20}{5}$$

- Correct formula (1)
- Correct substitution (1)
- Correct answer and unit (1)

$$v = u + at$$

For the additional 3 seconds, the initial velocity is 28 m/s.

$$v = 28 + (4 \times 3)$$

$$= 28 + 12$$

- Correct formula (1)
- Correct substitution (1)
- Correct answer and unit (1)

Question 5 (9 marks)

a) Same direction:

(2 marks)

b) Opposite directions:

(2 marks)

c) Relative speed:

$$t = \frac{300}{150}$$

$$= 2 \text{ hours}$$

- Correct relative speed (1)
- Correct time formula and substitution (1)
- Correct answer (1)

d) Any two:

- Helps drivers estimate the speed of approaching vehicles.
- Helps drivers make safe overtaking decisions.
- Helps drivers estimate the time taken to meet other vehicles.
- Helps prevent accidents.
- Helps in safe road transport planning.

(2 marks)

Question 6 (9 marks)

a) Since the trains move in opposite directions:

$$v_r = 20 + 15$$

(2 marks)

b) Total length:

$$120 + 80 = 200 \text{ m}$$

(2 marks)

$$t = \frac{\text{Relative distance}}{\text{Relative speed}}$$

$$t = \frac{200}{35}$$

$$t = 5.71 \text{ s}$$

- Correct formula (1)
- Correct substitution (1)
- Correct answer (1)

d) Any two:

- Overtaking of vehicles.
- Two vehicles approaching each other.
- Movement of boats in rivers.
- Movement of trains.
- Comparing the speeds of athletes.
- Movement of passengers inside moving vehicles.

(2 marks)

Suggested learner project

Project title: Investigating the movement of vehicles around the school community

Learners should:

1. Identify a safe observation point under teacher supervision.
2. Observe vehicles moving along a road.
3. Identify vehicles moving in the same and opposite directions.
4. Discuss the dangers of careless overtaking.
5. Use hypothetical or teacher-provided speeds to calculate relative speed.
6. Prepare a short report explaining the importance of relative motion in road safety.
7. Present their findings to the class.

Expected project outcomes:

- Learners explain relative motion using practical examples.
- Learners calculate relative speeds correctly.
- Learners demonstrate awareness of road safety.
- Learners communicate mathematical findings clearly.

STRAND 2.0: MEASUREMENTS AND GEOMETRY

SUB-STRAND 2.3: COMMERCIAL ARITHMETIC (1D)

1.0 INTRODUCTION

Commercial arithmetic is the branch of mathematics that deals with calculations involving buying, selling, taxation, trade, banking and other business activities.

In Kenya, goods are bought and sold locally, imported from other countries and exported to international markets. The government collects various taxes and duties on some of these goods to raise revenue, regulate trade and support national development.

For example:

- A Kenyan trader importing electronics from China may be required to pay import duty, excise duty where applicable, and Value Added Tax (VAT).
- A company exporting certain minerals or agricultural products may be required to pay an export levy or duty where the law provides for it.
- A supermarket selling locally manufactured goods may charge VAT on taxable supplies.
- A person buying an imported motor vehicle may encounter customs valuation and several import taxes.

Understanding commercial arithmetic enables learners to calculate these charges, interpret trade information and make informed financial decisions.

2.0 IMPORTED AND EXPORTED GOODS IN KENYA

2.1 Meaning of imported goods

Definition: Imported goods are goods brought into a country from another country for use, consumption, processing or resale.

Kenya imports goods from countries such as China, India, Japan, the United Arab Emirates, the United States and other trading partners.

Examples of imported goods in Kenya

1. Motor vehicles and motor vehicle spare parts.
2. Computers, mobile phones and other electronic equipment.
3. Petroleum products and some fuel-related products.
4. Machinery and industrial equipment.
5. Medicines and medical equipment.
6. Textiles and clothing.
7. Fertilisers and agricultural chemicals.
8. Wheat and other food products.
9. Iron, steel and construction materials.
10. Plastics and manufactured household products.

2.2 Meaning of exported goods

Definition: Exported goods are goods produced, grown, mined or processed in a country and sold or transported to another country.

Kenya exports different agricultural, manufactured and mineral products to international markets.

Examples of goods exported from Kenya

1. Tea.
2. Coffee.
3. Flowers and other horticultural products.
4. Fresh vegetables.
5. Fruits such as avocados and mangoes.
6. Fish and fish products.
7. Cement and selected manufactured products.
8. Soda ash.

9. Titanium minerals and other mineral products.
10. Leather and leather products.

Figure 2: Examples of products produced in Kenya for export.

2.3 Differences between imports and exports

Imports	Exports
Goods are brought into Kenya.	Goods are sent from Kenya to other countries.
They enter the Kenyan market.	They are sold in foreign markets.
They may attract import duty.	Certain goods may attract export duty or levies.
They may attract excise duty where applicable.	Export taxes depend on the specific goods and applicable law.
They may attract import VAT.	Export supplies are generally treated differently under VAT rules, subject to statutory conditions.

2.4 Importance of imports to Kenya

Imports are important because they:

1. Provide machinery used in industries.
2. Supply medical equipment and medicines.
3. Provide raw materials for local manufacturing.
4. Increase the variety of goods available to consumers.
5. Provide technology and electronic equipment.
6. Support construction and infrastructure development.
7. Supply products that may not be produced in sufficient quantities locally.

2.5 Importance of exports to Kenya

Exports are important because they:

1. Earn foreign exchange.
2. Create employment opportunities.
3. Provide markets for locally produced goods.
4. Support agricultural and industrial development.
5. Improve Kenya's international trade relations.
6. Encourage investment in production.
7. Contribute to national economic growth.

2.6 Classroom activity: Identifying imported and locally produced goods

Materials: Product labels, packaging, newspapers, magazines, photographs and internet resources.

Procedure:

1. Learners work in small groups.
2. Collect or identify ten products found in Kenyan shops.
3. Examine their labels to identify their country of origin.
4. Classify them as locally produced or imported.
5. Identify examples of Kenyan goods that can be exported.
6. Present the findings to the class.

Sample table

Product	Country of origin	Classification

Product	Country of origin	Classification
Kenyan tea	Kenya	Locally produced
Mobile phone	China or another country	Imported
Kenyan coffee	Kenya	Locally produced
Imported tractor	Another country	Imported
Kenyan flowers	Kenya	Export product

Discussion questions

1. Why does Kenya import machinery?
2. How do exports benefit Kenyan farmers?
3. What challenges might Kenyan exporters experience?
4. Why is it important to identify the country of origin of a product?

3.0 EXPORT DUTY AND EXPORT LEVY

3.1 Meaning of export duty

Definition: Export duty is a tax imposed by the government on specified goods when they are exported from a country to another country.

In Kenya, export duty or export levy is not charged on every exported product. It applies to goods specified in the relevant legislation and customs tariff schedules.

For example, the First Schedule of Kenya's Miscellaneous Fees and Levies Act provides export levies for specified products such as molasses, iron ores, certain raw hides and skins, and specified mineral products. The Act also provides an exemption for goods exported to East African Community Partner States.

3.2 Meaning of export levy

An export levy is a government charge imposed on specified goods when they leave the country.

The levy may be calculated:

1. As a percentage of the customs value of the exported goods.
2. As a fixed amount per unit, such as per tonne.
3. Using whichever of two prescribed rates is higher, where the law provides for this.

Examples of export levy rates in Kenya

The following are examples drawn from the First Schedule of the Miscellaneous Fees and Levies Act, revised to 1 January 2026. They are included to illustrate how different rates are applied.

Exported product	Illustrative statutory rate
Molasses	20% of customs value
Iron ores and concentrates	USD 175 per tonne
Certain raw hides and skins	50% or USD 0.32 per kg, whichever is higher

Rates and product descriptions must be checked against the applicable tariff code and current legal schedule before a real export transaction. Some specific rates are subject to adjustment.

3.3 Ad valorem export duty

Definition: Ad valorem export duty is a tax calculated as a percentage of the customs value of exported goods.

Formula:

$$\boxed{\text{Export duty} = \frac{\text{Rate}}{100} \times \text{Customs value}}$$

Worked Example 1: Export duty on molasses

A Kenyan company exports molasses with a customs value of KSh 800,000. Assume the applicable export levy is 20%.

Calculate the export levy payable.

Solution

Given:

- Customs value = KSh 800,000
- Export levy rate = 20%

Formula:

$$\text{Export levy} = \frac{20}{100} \times 800{,}000$$

$$= 0.20 \times 800{,}000$$

$$= \text{KSh } 160{,}000$$

Answer: KSh 160,000.

Worked Example 2: Finding the value after export levy

A trader exports goods worth KSh 1,200,000. The goods attract an export levy of 20%.

Calculate:

a) The export levy.

b) The value remaining after deducting the export levy.

Solution

a) Export levy:

$$= \frac{20}{100} \times 1{,}200{,}000$$

$$= \text{KSh } 240{,}000$$

b) Remaining value:

$$= 1{,}200{,}000 - 240{,}000$$

$$= \text{KSh } 960{,}000$$

Answers:

- Export levy = KSh 240,000.
- Remaining value = KSh 960,000.

3.4 Specific export duty

Definition: Specific export duty is a fixed amount of tax charged per unit of quantity of exported goods.

The units may include:

- Kilogram (kg)
- Tonne
- Litre
- Number of items

Formula:

$$\boxed{\text{Export duty} = \text{Quantity} \times \text{Duty per unit}}$$

Worked Example 3: Export levy on iron ore

A Kenyan mining company exports 40 tonnes of iron ore. Assume the applicable statutory rate is USD 175 per tonne.

Calculate the export levy in US dollars.

Solution

Given:

- Quantity = 40 tonnes
- Rate = USD 175 per tonne

$$\text{Export levy} = 40 \times 175$$

$$= \text{USD } 7{,}000$$

Answer: USD 7,000.

Worked Example 4: Converting export levy into Kenya shillings

Suppose the export levy in Worked Example 3 is USD 7,000 and the exchange rate used for the exercise is KSh 130 per USD.

Calculate the equivalent amount in Kenya shillings.

Solution

$$7{,}000 \times 130$$

$$= \text{KSh } 910{,}000$$

Answer: KSh 910,000.

Note: The exchange rate is assumed for mathematical practice, not a quoted current market rate.

3.5 Comparing two export levy rates

Some goods may attract a percentage rate or a specific rate, with the law requiring the higher amount to be paid.

Worked Example 5

A consignment of raw hides has a weight of 2,000 kg and a customs value of KSh 1,000,000.

For this exercise, assume:

- Ad valorem rate = 50%.
- Specific rate = USD 0.32 per kg.
- Exchange rate = KSh 130 per USD.

Calculate the two possible levies and determine the higher amount.

Solution

Step 1: Calculate the ad valorem levy.

$$= \frac{50}{100} \times 1{,}000{,}000$$

$$= \text{KSh } 500{,}000$$

Step 2: Calculate the specific levy in dollars.

$$= 2{,}000 \times 0.32$$

$$= \text{USD } 640$$

Step 3: Convert into Kenya shillings.

$$= 640 \times 130$$

$$= \text{KSh } 83{,}200$$

Step 4: Compare the amounts.

Method	Levy
Ad valorem	KSh 500,000
Specific rate	KSh 83,200

The higher amount is KSh 500,000.

Answer: KSh 500,000.

3.6 Importance of export duties and levies

Export duties and levies may be used to:

1. Raise government revenue.
2. Encourage local processing of raw materials.
3. Discourage the uncontrolled exportation of certain resources.
4. Support local industries that use exported raw materials.
5. Regulate the export of selected products.
6. Promote value addition before goods are exported.

3.7 Practical activity: Calculating export levies

Learners work in groups and solve the following tasks:

1. A company exports molasses worth KSh 500,000 at an assumed export levy rate of 20%. Calculate the levy.
2. A trader exports 25 tonnes of iron ore at USD 175 per tonne. Calculate the levy in dollars.
3. Convert the levy in Question 2 into Kenya shillings using an assumed exchange rate of KSh 130 per USD.
4. Discuss why a government may impose a levy on the export of selected raw materials.

4.0 CUSTOMS VALUE OF IMPORTED GOODS

4.1 Meaning of customs value

Definition: Customs value is the value assigned to imported goods for purposes of assessing customs duties and other applicable taxes.

The customs value is generally based on the transaction value, which is the price actually paid or payable for the goods, adjusted according to customs valuation rules.

For many classroom calculations, the customs value is represented by the Cost, Insurance and Freight (CIF) value.

4.2 Meaning of CIF

CIF stands for:

- **C** – Cost
- **I** – Insurance
- **F** – Freight

Components of CIF

a) Cost of goods

This is the price paid or payable for the goods purchased from the foreign supplier.

Example: A Kenyan trader purchases computers from China for KSh 500,000.

b) Insurance

This is the cost of insuring the goods against loss or damage during transportation to Kenya.

Example: The trader pays KSh 20,000 to insure the computers.

c) Freight

This is the cost of transporting goods from the exporting country to the place of importation.

Example: The trader pays KSh 50,000 to transport the computers to Kenya.

4.3 Formula for customs value

For simplified school calculations:

$$\boxed{\text{Customs value} = \text{Cost} + \text{Insurance} + \text{Freight}}$$

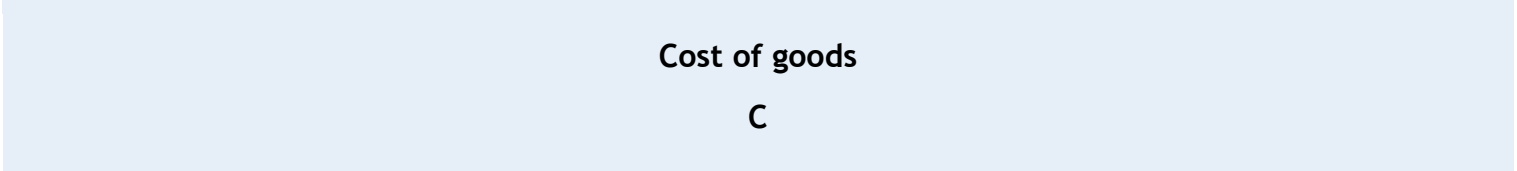
Or:

$$\boxed{CV = C + I + F}$$

Where:

- CV = Customs value
- C = Cost of goods
- I = Insurance
- F = Freight

Figure 3: Components of customs value



+

Insurance
I

+

Freight
F

=

Customs value (CIF)
C + I + F

4.4 Worked examples on customs value

Worked Example 6

A Kenyan importer purchases machinery from Japan at KSh 1,500,000. The insurance cost is KSh 80,000 while the freight cost is KSh 120,000.

Calculate the customs value.

Solution

Given:

- Cost = KSh 1,500,000
- Insurance = KSh 80,000
- Freight = KSh 120,000

Formula:

$$CV = C + I + F$$

Substitute:

$$CV = 1{,}500{,}000 + 80{,}000 + 120{,}000$$

$$CV = \text{KSh } 1{,}700{,}000$$

Answer: KSh 1,700,000.

Worked Example 7: Finding freight cost

The customs value of imported goods is KSh 950,000. The cost of the goods is KSh 800,000 and insurance is KSh 30,000.

Calculate the freight cost.

Solution

Formula:

$$CV = C + I + F$$

Rearrange:

$$F = CV - C - I$$

Substitute:

$$F = 950{,}000 - 800{,}000 - 30{,}000$$

$$F = \text{KSh } 120{,}000$$

Answer: KSh 120,000.

Worked Example 8: Customs value in foreign currency

A Kenyan business imports goods from India. The following details are given:

- Cost = USD 4,000
- Insurance = USD 200
- Freight = USD 500

Calculate the customs value in:

a) US dollars.

b) Kenya shillings, if the assumed exchange rate is KSh 130 per USD.

Solution

a) Customs value:

$$CV = 4{,}000 + 200 + 500$$

$$= \text{USD } 4{,}700$$

b) Convert into Kenya shillings:

$$4{,}700 \times 130$$

$$= \text{KSh } 611{,}000$$

Answers:

- Customs value = USD 4,700.
- Customs value = KSh 611,000.

4.5 Other factors considered in customs valuation

In actual customs valuation, the customs value may require adjustments under applicable valuation rules. These may include:

1. Packing costs.
2. Certain commissions and brokerage fees.
3. Royalties and licence fees where they qualify for inclusion.
4. Certain materials and services supplied by the buyer.
5. Transport and handling charges to the place of importation.
6. Insurance costs relating to transportation.

Certain post-importation costs may be excluded if they are separately identified and meet the applicable rules.

4.6 Importance of customs value

Customs value is important because it:

- Provides a basis for calculating import duty.
- Helps determine excise duty on applicable imported goods.
- Helps determine the taxable value for import VAT.
- Supports fair and consistent customs assessment.
- Helps prevent under-declaration of imported goods.
- Helps the government collect the correct revenue.

4.7 Practical activity: Calculating customs value

A Kenyan trader imports furniture from Malaysia with the following costs:

Item	Amount
Cost of furniture	KSh 600,000
Insurance	KSh 25,000
Freight	KSh 45,000

Learners should:

1. Identify the three components of customs value.
2. Calculate the customs value.
3. Explain why insurance and freight are included.
4. Discuss why a trader should keep invoices and shipping documents.

5.0 IMPORT DUTY (CUSTOMS DUTY)

5.1 Meaning of import duty

Definition: Import duty is a tax imposed by the government on certain goods brought into the country from other countries.

In Kenya, import duty is administered by the Kenya Revenue Authority (KRA) under the East African Community Customs Management Act and the East African Community Common External Tariff (EAC CET).

The applicable rate depends on the classification of the imported product, its tariff code, country of origin and any applicable preferential arrangement or exemption.

5.2 Reasons for imposing import duty

The government imposes import duty to:

1. Raise revenue for national development.
2. Protect local industries from unfair competition.
3. Encourage people to buy locally manufactured goods.
4. Regulate the importation of certain products.
5. Support industrial development.
6. Protect employment opportunities in local industries.
7. Manage the flow of goods into the country.

5.3 Import duty rates

Import duty rates are not the same for all imported goods.

The EAC Common External Tariff provides different rates according to product classification. The general tariff structure includes rates such as 0%, 10%, 25% and 35%, while certain sensitive products may attract higher rates. Exemptions and preferential rates may also apply. (www.treasury.go.ke)

For classroom calculations, the following assumed rates may be used:

Imported product (illustrative)	Assumed import duty rate
Selected raw materials	0%
Selected intermediate products	10%
Selected finished products	25%
Selected sensitive products	35% or higher

These are learning examples, not a tariff classification for the named product categories. Always use the applicable EAC tariff code to establish the actual rate.

5.4 Formula for calculating import duty

$$\text{Import duty} = \frac{\text{Import duty rate}}{100} \times \text{Customs value}$$

Or:

$$ID = \frac{r}{100} \times CV$$

Where:

- ID = Import duty
- r = Import duty rate
- CV = Customs value

5.5 Worked examples on import duty

Worked Example 9: Calculating import duty at 10%

A Kenyan company imports machinery with a customs value of KSh 800,000. Assume the applicable import duty rate is 10%.

Calculate the import duty payable.

Solution

Given:

$$CV = \text{KSh } 800{,}000$$

$$r = 10\%$$

Formula:

$$ID = \frac{10}{100} \times 800{,}000$$

$$= 0.10 \times 800{,}000$$

$$= \text{KSh } 80{,}000$$

Answer: KSh 80,000.

Worked Example 10: Calculating import duty at 25%

A trader imports furniture with a customs value of KSh 1,200,000. The applicable import duty rate is 25%.

Calculate the import duty.

Solution

$$ID = \frac{25}{100} \times 1{,}200{,}000$$

$$= 0.25 \times 1{,}200{,}000$$

$$= \text{KSh } 300{,}000$$

Answer: KSh 300,000.

Worked Example 11: Calculating import duty at 35%

A company imports selected goods with a customs value of KSh 500,000. Assume the goods attract an import duty rate of 35%.

Calculate the import duty.

Solution

$$ID = \frac{35}{100} \times 500{,}000$$

$$= 0.35 \times 500{,}000$$

$$= \text{KSh } 175{,}000$$

Answer: KSh 175,000.

Worked Example 12: Finding the customs value

An importer pays KSh 150,000 as import duty on goods attracting an import duty rate of 25%.

Calculate the customs value.

Solution

Formula:

$$ID = \frac{r}{100} \times CV$$

Rearrange:

$$CV = \frac{ID \times 100}{r}$$

Substitute:

$$CV = \frac{150{,}000 \times 100}{25}$$

$$CV = \text{KSh } 600{,}000$$

Answer: KSh 600,000.

Worked Example 13: Finding the import duty rate

An importer brings in goods with a customs value of KSh 400,000 and pays import duty of KSh 100,000.

Calculate the import duty rate.

Solution

$$r = \frac{\text{Import duty}}{\text{Customs value}} \times 100$$

$$= \frac{100{,}000}{400{,}000} \times 100$$

$$= 25\%$$

Answer: 25%.

5.6 Customs value plus import duty

After calculating import duty, it is useful to determine the value of the goods plus import duty.

$$\boxed{\text{Value after import duty} = CV + ID}$$

This amount forms part of the basis for calculating excise duty on imported excisable goods.

Worked Example 14

A trader imports goods with a customs value of KSh 700,000. The import duty rate is 25%.

Calculate:

a) Import duty.

b) Customs value plus import duty.

Solution

a) Import duty:

$$ID = \frac{25}{100} \times 700{,}000$$

$$= \text{KSh } 175{,}000$$

b) Total:

$$700{,}000 + 175{,}000$$

$$= \text{KSh } 875{,}000$$

Answers:

- Import duty = KSh 175,000.
- Customs value plus import duty = KSh 875,000.

6.0 EXCISE DUTY ON IMPORTED GOODS

6.1 Meaning of excise duty

Definition: Excise duty is a tax imposed on specified goods and services, including certain imported products.

In Kenya, excise duty is governed mainly by the Excise Duty Act, 2015, together with subsequent amendments and applicable schedules.

Not every imported product attracts excise duty. The product must fall within the categories specified by law.

6.2 Examples of goods that may attract excise duty

Depending on the applicable legislation and product classification, excise duty may apply to:

1. Alcoholic beverages.
2. Cigarettes and other tobacco products.
3. Certain motor vehicles.
4. Certain petroleum products.
5. Certain cosmetics and beauty products.
6. Certain electronic or digital products and services.
7. Other products specified in the Excise Duty Act.

Figure 4: Examples of product categories that may be subject to excise duty.

6.3 Types of excise duty rates

There are two main ways in which excise duty may be calculated.

A. Ad valorem excise duty

Definition: Ad valorem excise duty is calculated as a percentage of the prescribed taxable value of the goods.

Formula:

$$\boxed{\text{Excise duty} = \frac{\text{Excise rate}}{100} \times \text{Excise value}}$$

For simplified imported-goods calculations:

$$\boxed{\text{Excise value} = CV + ID}$$

Where:

- CV = Customs value
- ID = Import duty

Thus:

$$\boxed{ED = \frac{e}{100}(CV + ID)}$$

Where e is the assumed or applicable ad valorem excise rate.

The KRA explains that excise duty on imports is charged on the customs value plus applicable import duty.

B. Specific excise duty

Definition: Specific excise duty is a fixed amount of tax charged per unit of measure of an excisable product.

Examples of units include:

- Per litre.
- Per kilogram.
- Per cigarette.
- Per unit.

- Per millilitre.

Formula:

$$\boxed{\text{Excise duty} = \text{Quantity} \times \text{Rate per unit}}$$

The actual rate is determined by the applicable Excise Duty Act schedule.

6.4 Worked examples on ad valorem excise duty

Worked Example 15

A Kenyan trader imports goods with a customs value of KSh 500,000. The import duty rate is 25%, while the assumed excise duty rate is 10%.

Calculate:

a) Import duty.

b) Excise value.

c) Excise duty.

Solution

Step 1: Calculate import duty.

$$ID = \frac{25}{100} \times 500{,}000$$

$$ID = \text{KSh } 125{,}000$$

Step 2: Calculate excise value.

$$\text{Excise value} = CV + ID$$

$$= 500{,}000 + 125{,}000$$

$$= \text{KSh } 625{,}000$$

Step 3: Calculate excise duty.

$$ED = \frac{10}{100} \times 625{,}000$$

$$ED = \text{KSh } 62{,}500$$

Answers:

- Import duty = KSh 125,000.
- Excise value = KSh 625,000.
- Excise duty = KSh 62,500.

Worked Example 16: Excise duty at 20%

An imported product has a customs value of KSh 800,000. The applicable import duty rate is 10%, and the assumed excise duty rate is 20%.

Calculate the excise duty.

Solution

Step 1: Import duty.

$$ID = \frac{10}{100} \times 800{,}000$$

$$= \text{KSh } 80{,}000$$

Step 2: Excise value.

$$800{,}000 + 80{,}000$$

$$= \text{KSh } 880{,}000$$

Step 3: Excise duty.

$$ED = \frac{20}{100} \times 880{,}000$$

$$= \text{KSh } 176{,}000$$

Answer: KSh 176,000.

6.5 Worked examples on specific excise duty

Worked Example 17

An imported product attracts a specific excise duty of KSh 120 per litre. A trader imports 500 litres.

Calculate the excise duty.

Solution

$$ED = 500 \times 120$$

$$= \text{KSh } 60{,}000$$

Answer: KSh 60,000.

Worked Example 18

An importer brings in 2,400 units of a product. The specific excise duty is KSh 15 per unit.

Calculate:

a) Total excise duty.

b) The average excise duty per 100 units.

Solution

a) Total excise duty:

$$2{,}400 \times 15$$

$$= \text{KSh } 36{,}000$$

b) Duty per 100 units:

$$100 \times 15$$

$$= \text{KSh } 1{,}500$$

Answers:

- Total excise duty = KSh 36,000.
- Duty per 100 units = KSh 1,500.

6.6 Difference between ad valorem and specific excise duty

Ad valorem excise duty	Specific excise duty
Calculated as a percentage of value.	Calculated using a fixed amount per unit.
Depends on the taxable value.	Depends on the quantity.
Example: 20% of excise value.	Example: KSh 120 per litre.
Formula: Rate × value.	Formula: Rate per unit × quantity.

6.7 Importance of excise duty

Excise duty helps the government to:

1. Raise revenue.
2. Regulate the consumption of certain products.
3. Discourage harmful consumption where relevant.
4. Support public services.
5. Regulate selected industries.
6. Influence consumer choices through taxation.

6.8 Classroom activity: Calculating excise duty

A company imports a product with the following details:

- Customs value = KSh 600,000.
- Import duty rate = 25%.
- Assumed excise duty rate = 15%.

Learners should calculate:

1. Import duty.
2. Excise value.
3. Excise duty.
4. The total customs value, import duty and excise duty.

7.0 VALUE ADDED TAX (VAT) ON IMPORTED GOODS

7.1 Meaning of VAT

Definition: Value Added Tax (VAT) is a consumption tax imposed on taxable goods and services at different stages of the supply chain.

VAT may be charged on:

- Taxable goods sold within Kenya.
- Taxable services supplied within Kenya.
- Imported taxable goods.
- Imported taxable services.

The standard VAT rate in Kenya is 16%, while certain supplies are zero-rated or exempt under the VAT Act.

7.2 Why VAT is charged on imported goods

VAT is charged on imported taxable goods to:

1. Raise government revenue.
2. Ensure that imported taxable goods are treated consistently with locally supplied taxable goods.

3. Support the provision of public services.
4. Promote fairness in taxation.
5. Support the national budget.

7.3 VAT rates in Kenya

VAT category	Rate	Explanation
Standard-rated supplies	16%	General rate for taxable supplies not zero-rated.
Zero-rated supplies	0%	Supplies specifically listed in the VAT Act's Second Schedule.
Exempt supplies	Not subject to VAT	Supplies listed as exempt under the VAT Act.

Important: Zero-rated and exempt supplies are not the same. Zero-rated supplies are taxable at 0%, while exempt supplies are outside the charge to VAT under the applicable exemption provisions.

7.4 Taxable value for import VAT

For simplified school calculations involving taxable imported goods, the VAT base is taken as:

$$\text{VAT taxable value} = CV + ID + ED$$

Where:

- CV = Customs value.
- ID = Import duty.
- ED = Excise duty, where applicable.

The exact taxable value must follow the VAT Act and the applicable customs valuation provisions. The Treasury's tax expenditure report explains the inclusion of customs value, import duty and excise duty in the import VAT base. (www.treasury.go.ke)

7.5 Formula for calculating VAT

$$\text{VAT} = \frac{\text{VAT rate}}{100} \times \text{VAT taxable value}$$

For a standard-rated import:

$$\text{VAT} = 16\% \times (CV + ID + ED)$$

Where excise duty does not apply:

$$\text{VAT} = 16\% \times (CV + ID)$$

7.6 Worked examples on VAT

Worked Example 19: Calculating VAT when excise duty is not applicable

A trader imports machinery with the following details:

- Customs value = KSh 600,000.
- Import duty = KSh 120,000.
- Excise duty = Not applicable.
- VAT rate = 16%.

Calculate the VAT payable.

Solution

Step 1: Calculate VAT taxable value.

$$= 600{,}000 + 120{,}000$$

$$= \text{KSh } 720{,}000$$

Step 2: Calculate VAT.

$$VAT = \frac{16}{100} \times 720{,}000$$

$$= \text{KSh } 115{,}200$$

Answer: KSh 115,200.

Worked Example 20: Calculating VAT when excise duty applies

An importer brings in goods with the following details:

- Customs value = KSh 500,000.
- Import duty = KSh 125,000.
- Excise duty = KSh 62,500.
- VAT rate = 16%.

Calculate the VAT payable.

Solution

Step 1: Calculate the VAT taxable value.

$$= 500{,}000 + 125{,}000 + 62{,}500$$

$$= \text{KSh } 687{,}500$$

Step 2: Calculate VAT.

$$VAT = \frac{16}{100} \times 687{,}500$$

$$= \text{KSh } 110{,}000$$

Answer: KSh 110,000.

Worked Example 21: Finding the VAT taxable value

An importer pays VAT of KSh 96,000 at a rate of 16%. Calculate the VAT taxable value.

Solution

$$VAT = \frac{16}{100} \times \text{Taxable value}$$

Rearrange:

$$\text{Taxable value} = \frac{VAT \times 100}{16}$$

$$= \frac{96{,}000 \times 100}{16}$$

$$= \text{KSh } 600{,}000$$

Answer: KSh 600,000.

Worked Example 22: Finding the VAT rate

An importer pays VAT of KSh 128,000 on a taxable value of KSh 800,000.

Calculate the VAT rate.

Solution

$$\text{VAT rate} = \frac{\text{VAT}}{\text{Taxable value}} \times 100$$

$$= \frac{128{,}000}{800{,}000} \times 100$$

$$= 16\%$$

Answer: 16%.

7.7 Calculating the total taxes on imported goods

An importer may be required to pay more than one type of tax when goods enter Kenya.

For the school-level calculations in this sub-strand, consider:

1. Import duty.
2. Excise duty, where applicable.
3. VAT.

The total of these three taxes is:

$$\boxed{\text{Total taxes} = ID + ED + VAT}$$

The total cost including customs value is:

$$\boxed{\text{Total} = CV + ID + ED + VAT}$$

This simplified total does not include other applicable levies, regulatory charges, clearing fees, storage or transport costs.

Worked Example 23: Comprehensive calculation of import taxes

A Kenyan trader imports goods from China with the following details:

Item	Amount
Cost of goods	KSh 800,000
Insurance	KSh 40,000
Freight	KSh 60,000
Import duty rate	25%
Excise duty rate	10%
VAT rate	16%

Calculate:

a) Customs value.

b) Import duty.

c) Excise duty.

d) VAT.

e) Total taxes payable.

f) Total value including the three taxes.

Solution

Step 1: Calculate customs value.

$$CV = C + I + F$$

$$= 800{,}000 + 40{,}000 + 60{,}000$$

$$= \text{KSh } 900{,}000$$

Step 2: Calculate import duty.

$$ID = \frac{25}{100} \times 900{,}000$$

$$= \text{KSh } 225{,}000$$

Step 3: Calculate excise value.

$$= CV + ID$$

$$= 900{,}000 + 225{,}000$$

$$= \text{KSh } 1{,}125{,}000$$

Step 4: Calculate excise duty.

$$ED = \frac{10}{100} \times 1{,}125{,}000$$

$$= \text{KSh } 112{,}500$$

Step 5: Calculate VAT taxable value.

$$= CV + ID + ED$$

$$= 900{,}000 + 225{,}000 + 112{,}500$$

$$= \text{KSh } 1{,}237{,}500$$

Step 6: Calculate VAT.

$$VAT = \frac{16}{100} \times 1{,}237{,}500$$

$$= \text{KSh } 198{,}000$$

Step 7: Calculate total taxes.

$$= ID + ED + VAT$$

$$= 225{,}000 + 112{,}500 + 198{,}000$$

$$= \text{KSh } 535{,}500$$

Step 8: Calculate total value including taxes.

$$= 900{,}000 + 535{,}500$$

$$= \text{KSh } 1{,}435{,}500$$

Answer:

- Customs value = KSh 900,000.
- Import duty = KSh 225,000.
- Excise duty = KSh 112,500.
- VAT = KSh 198,000.
- Total taxes = KSh 535,500.
- Total including the three taxes = KSh 1,435,500.

The 25% import duty and 10% excise duty are assumed for this worked example. They are not asserted as the actual rates for every imported product.

7.8 Summary of the calculation process

Figure 5: Sequence for calculating taxes on an imported excisable product

1. Calculate customs value

$$CV = C + I + F$$

↓

2. Calculate import duty

$$ID = \frac{r}{100} \times CV$$

↓

3. Calculate excise duty

$$ED = \frac{e}{100} \times (CV + ID)$$

↓

4. Calculate VAT

$$VAT = 16\% \times (CV + ID + ED)$$

↓

5. Calculate total taxes

$$ID + ED + VAT$$

For a non-excisable product, omit the excise duty step. In actual customs declarations, also check applicable exemptions, preferential rates and other statutory fees and levies.

8.0 OTHER CHARGES THAT MAY APPLY TO IMPORTED GOODS

Besides import duty, excise duty and VAT, some imported goods may attract additional statutory fees and levies.

These charges are important when calculating the actual cost of importing goods into Kenya.

8.1 Import Declaration Fee (IDF)

Definition: Import Declaration Fee is a statutory fee charged on qualifying imported goods when they are entered for home use in Kenya.

Under the Miscellaneous Fees and Levies Act, the general statutory rate is 2.5% of customs value, subject to applicable exemptions and amendments. (www.ubutabera.com)

For a simplified calculation:

$$IDF = \frac{2.5}{100} \times CV$$

Worked Example 24

A trader imports goods with a customs value of KSh 400,000. Assuming the goods attract IDF at 2.5%, calculate the fee.

Solution

$$IDF = \frac{2.5}{100} \times 400{,}000$$

$$= \text{KSh } 10{,}000$$

Answer: KSh 10,000.

8.2 Railway Development Levy (RDL)

Definition: Railway Development Levy is a statutory levy imposed on qualifying imports to support railway development.

The applicable rate, exemptions and taxable base should be checked against the current Miscellaneous Fees and Levies Act and the relevant customs guidance.

For a classroom exercise, assume an RDL rate of 1.5% of customs value.

$$RDL = \frac{1.5}{100} \times CV$$

Worked Example 25

A company imports machinery with a customs value of KSh 2,000,000. Calculate the RDL using the assumed rate of 1.5%.

Solution

$$RDL = \frac{1.5}{100} \times 2{,}000{,}000$$

$$= \text{KSh } 30{,}000$$

Answer: KSh 30,000.

The rates in this section are clearly identified as classroom assumptions where necessary. Do not use them as a substitute for checking the current law and the actual customs entry.

8.3 Distinguishing the major import charges

Charge	Main purpose	Typical calculation basis
Import duty	Tax on imported goods	Customs value
Excise duty	Tax on specified excisable products	Customs value plus import duty, or specific quantity
VAT	Tax on imported taxable goods	Statutory VAT taxable value
IDF	Import declaration fee	Customs value, subject to law
RDL	Railway development levy	Customs value, subject to law

9.0 ROLE OF CUSTOMS DUTIES IN KENYA

Customs duties play an important role in the economic and social development of Kenya.

9.1 Raising government revenue

The government collects customs duties to finance public expenditure, including:

- Construction and maintenance of roads.
- Development of schools.
- Provision of healthcare services.
- Water supply projects.
- Security services.
- Public infrastructure.

9.2 Protecting local industries

Import duties may make selected imported products more expensive, helping local manufacturers compete with imported products.

For example, a duty on selected imported manufactured goods may support Kenyan producers of similar goods.

9.3 Encouraging local production

Customs duties may encourage businesses to:

- Manufacture products locally.
- Process agricultural products.
- Use locally available raw materials.
- Invest in local industries.

9.4 Regulating international trade

Customs duties help the government regulate the movement of goods across national borders.

They also help customs authorities monitor imports and exports.

9.5 Supporting economic development

Revenue collected through customs duties contributes to the government's ability to finance development projects.

9.6 Discouraging harmful products

Excise duty on certain products may influence consumer behaviour and help discourage the consumption of harmful products.

9.7 Promoting value addition

Export levies on selected raw materials may encourage local processing before exportation.

For example, encouraging local processing of hides and skins can create employment and increase the value of products sold internationally.

9.8 Challenges associated with customs duties

Despite their benefits, customs duties may also present challenges:

1. They may increase the cost of imported goods.
2. They may increase production costs for industries that rely on imported raw materials.
3. High duties may encourage smuggling and tax evasion.
4. Changes in duty rates may create uncertainty for businesses.
5. Complex customs procedures may increase the cost of international trade.

Practical digital learning activity

Task: Investigating import duty on selected products

Learners should:

1. Visit the KRA or InfoTrade Kenya website.
2. Select a product that may be imported into Kenya.
3. Identify its customs tariff classification where available.
4. Find the applicable import duty rate.
5. Identify whether excise duty applies.
6. Identify the applicable VAT treatment.
7. Record the information in a table.
8. Use a calculator or spreadsheet to estimate the taxes.

Suggested investigation table

Item	Details
Imported product	
Country of origin	
Customs value	
Import duty rate	
Import duty	
Excise duty rate, if applicable	
Excise duty	
VAT rate	
VAT payable	
Total taxes calculated	

SUMMARY OF IMPORTANT FORMULAE

Quantity	Formula
Customs value	$CV = C + I + F$
Export duty (percentage)	$\frac{r}{100} \times CV$
Export duty (specific)	Quantity × rate per unit
Import duty	$\frac{r}{100} \times CV$
Excise value	$CV + ID$
Ad valorem excise duty	$\frac{e}{100} \times (CV + ID)$
Specific excise duty	Quantity × rate per unit
VAT taxable value (simplified excisable import)	$CV + ID + ED$
VAT	$\frac{16}{100}$ × VAT taxable value
Total of the three main taxes	$ID + ED + VAT$
Total including customs value	$CV + ID + ED + VAT$
IDF (assumed general rate for exercises)	$\frac{2.5}{100} \times CV$
RDL (assumed classroom rate)	$\frac{1.5}{100} \times CV$

Where:

- CV = Customs value.
- C = Cost of goods.
- I = Insurance.
- F = Freight.
- ID = Import duty.
- ED = Excise duty.
- r = Applicable duty rate.
- e = Applicable excise rate.

REVISION EXERCISES

SECTION A: 30 MARKS

SECTION A: MULTIPLE-CHOICE QUESTIONS

Instructions: Answer all questions. Choose the correct answer from alternatives A, B, C and D. Each question carries 1 mark.

1. What is meant by importation?

A. Sending goods from Kenya to another country.

B. Bringing goods into Kenya from another country.

C. Manufacturing goods within Kenya.

D. Transporting goods between Kenyan counties.

2. Which of the following is an example of an agricultural product exported from Kenya?

A. Wheat imported from another country.

B. Mobile phones.

C. Tea.

D. Motor vehicle spare parts.

3. Which institution is responsible for administering customs duties in Kenya?

A. Kenya Revenue Authority.

B. Kenya National Examinations Council.

C. Teachers Service Commission.

D. Kenya National Highways Authority.

4. What does CIF stand for?

A. Cost, Insurance and Freight.

B. Customs, Import and Finance.

C. Cost, Import and Fees.

D. Customs, Insurance and Finance.

5. Which of the following is included in the simplified calculation of customs value?

A. Import duty only.

B. Excise duty only.

C. Cost, insurance and freight.

D. VAT and income tax.

6. A trader imports goods costing KSh 400,000. Insurance is KSh 20,000 and freight is KSh 30,000. Calculate the customs value.

A. KSh 400,000.

B. KSh 420,000.

C. KSh 450,000.

D. KSh 500,000.

7. A company imports machinery with a customs value of KSh 600,000. If the import duty rate is 10%, calculate the import duty.

A. KSh 6,000.

B. KSh 30,000.

C. KSh 60,000.

D. KSh 600,000.

8. Goods have a customs value of KSh 800,000. The import duty rate is 25%. Calculate the import duty.

A. KSh 100,000.

B. KSh 160,000.

C. KSh 200,000.

D. KSh 250,000.

9. A trader pays import duty of KSh 75,000 on goods attracting a duty rate of 15%. Calculate the customs value.

A. KSh 300,000.

B. KSh 400,000.

C. KSh 500,000.

D. KSh 600,000.

10. What is the meaning of excise duty?

A. A tax imposed on specified goods and services.

B. A tax imposed only on exported agricultural goods.

C. A payment made to transport goods.

D. A tax charged only on salaries.

11. Which of the following is an example of a product category that may attract excise duty?

A. Fresh unprocessed vegetables.

B. Certain alcoholic beverages.

C. All school textbooks.

D. All locally grown maize.

12. Which Act mainly governs excise duty in Kenya?

A. Education Act.

B. Employment Act.

C. Excise Duty Act, 2015.

D. Basic Education Act.

13. A trader imports goods with a customs value of KSh 500,000. The import duty rate is 20%. Calculate the import duty.

A. KSh 50,000.

B. KSh 100,000.

C. KSh 150,000.

D. KSh 200,000.

14. An imported product has a customs value of KSh 400,000 and import duty of KSh 80,000. Calculate the excise value.

A. KSh 320,000.

B. KSh 400,000.

C. KSh 480,000.

D. KSh 500,000.

15. An imported product has an excise value of KSh 600,000. If the assumed excise duty rate is 10%, calculate the excise duty.

A. KSh 6,000.

B. KSh 30,000.

C. KSh 60,000.

D. KSh 100,000.

16. A product attracts specific excise duty of KSh 50 per litre. Calculate the duty on 200 litres.

A. KSh 2,500.

B. KSh 5,000.

C. KSh 10,000.

D. KSh 20,000.

17. What is the standard VAT rate in Kenya for general taxable supplies?

A. 5%.

B. 10%.

C. 16%.

D. 25%.

18. Which of the following best describes VAT?

A. A tax charged on taxable goods and services.

B. A payment made only by exporters.

C. A fee paid to transport goods.

D. A tax charged only on agricultural land.

19. Imported goods have a customs value of KSh 500,000 and import duty of KSh 100,000. Excise duty is not applicable. Calculate the VAT taxable value using the simplified formula.

A. KSh 400,000.

B. KSh 500,000.

C. KSh 600,000.

D. KSh 700,000.

20. Calculate VAT at 16% on a taxable value of KSh 600,000.

A. KSh 60,000.

B. KSh 80,000.

C. KSh 96,000.

D. KSh 120,000.

21. An imported product has a customs value of KSh 400,000 and import duty of KSh 100,000. The excise duty is KSh 50,000. Calculate the VAT taxable value using the simplified formula.

A. KSh 400,000.

B. KSh 450,000.

C. KSh 500,000.

D. KSh 550,000.

22. Calculate VAT at 16% on a taxable value of KSh 750,000.

A. KSh 75,000.

B. KSh 100,000.

C. KSh 120,000.

D. KSh 150,000.

23. What is ad valorem duty?

A. A fixed amount charged per kilogram.

B. A tax calculated as a percentage of value.

C. A tax charged per kilometre.

D. A tax charged per hour.

24. A company exports goods worth KSh 500,000. If the assumed export levy rate is 20%, calculate the levy.

A. KSh 50,000.

B. KSh 75,000.

C. KSh 100,000.

D. KSh 200,000.

25. An exporter sends 30 tonnes of iron ore at an assumed specific export levy of USD 175 per tonne. Calculate the levy.

A. USD 1,750.

B. USD 3,500.

C. USD 5,250.

D. USD 7,000.

26. A trader imports goods with a customs value of KSh 1,000,000. Using an assumed IDF rate of 2.5%, calculate the IDF.

A. KSh 2,500.

B. KSh 15,000.

C. KSh 25,000.

D. KSh 50,000.

27. A trader imports goods with a customs value of KSh 800,000. Using an assumed RDL rate of 1.5%, calculate the RDL.

A. KSh 8,000.

B. KSh 12,000.

C. KSh 15,000.

D. KSh 18,000.

28. Which of the following is an important role of customs duties in Kenya?

A. Preventing all international trade.

B. Raising government revenue.

C. Eliminating local manufacturing.

D. Preventing the production of agricultural goods.

29. Which of the following is a correct statement about zero-rated and exempt supplies?

A. Both are always taxed at 16%.

B. Zero-rated supplies are taxable at 0%, while exempt supplies are not taxable supplies.

C. Exempt supplies are always taxed at 10%.

D. Zero-rated supplies are not recognised by the VAT Act.

30. Why should an importer use the correct customs tariff classification?

A. To avoid paying for transportation.

B. To establish the correct applicable customs duty and other charges.

C. To avoid preparing import documents.

D. To determine the selling price of locally manufactured goods only.

SECTION B: STRUCTURED QUESTIONS

Instructions: Answer all questions. Show all your working clearly. Use Kenya shillings unless otherwise stated.

QUESTION 1: IMPORTED AND EXPORTED GOODS (7 MARKS)

a) Define the following terms:

i. Importation. (1 mark)

ii. Exportation. (1 mark)

b) State three examples of goods exported from Kenya. (3 marks)

c) State two reasons why Kenya imports goods from other countries. (2 marks)

QUESTION 2: CUSTOMS VALUE (8 MARKS)

A Kenyan business imports office equipment from Japan. The following information is provided:

Item	Amount
Cost of equipment	KSh 750,000
Insurance	KSh 35,000
Freight	KSh 65,000

a) Explain the meaning of customs value. (1 mark)

b) State the meaning of CIF. (1 mark)

c) Calculate the customs value of the equipment. (3 marks)

d) If the customs value is KSh 850,000 and the import duty rate is 10%, calculate the import duty. (3 marks)

QUESTION 3: IMPORT DUTY AND EXCISE DUTY (8 MARKS)

A trader imports selected excisable goods with a customs value of KSh 600,000.

The applicable rates for this mathematical exercise are:

- Import duty = 25%.
- Excise duty = 10%.

Calculate:

a) The import duty. (2 marks)

b) The excise value. (2 marks)

c) The excise duty payable. (2 marks)

d) The total customs value, import duty and excise duty. (2 marks)

QUESTION 4: VALUE ADDED TAX (8 MARKS)

A company imports taxable goods with the following details:

- Customs value = KSh 900,000.
- Import duty = KSh 180,000.
- Excise duty = KSh 72,000.
- VAT rate = 16%.

Calculate:

a) The VAT taxable value. (3 marks)

b) The VAT payable. (3 marks)

c) The total of import duty, excise duty and VAT. (2 marks)

QUESTION 5: EXPORT DUTY AND LEVY (9 MARKS)

A Kenyan exporter exports a consignment of molasses with a customs value of KSh 1,500,000. Assume an export levy rate of 20%.

a) Define export levy. (1 mark)

b) Calculate the export levy payable. (3 marks)

c) Calculate the value remaining after deducting the export levy. (2 marks)

d) An exporter sends 40 tonnes of a product that attracts an assumed specific levy of USD 175 per tonne. Calculate the levy in US dollars. (2 marks)

e) State one importance of export levies to the government. (1 mark)

QUESTION 6: COMPREHENSIVE COMMERCIAL ARITHMETIC (10 MARKS)

A Kenyan importer purchases goods from another country. The following information is provided:

Item	Amount
Cost of goods	KSh 1,000,000
Insurance	KSh 50,000
Freight	KSh 100,000
Import duty rate	25%
Excise duty rate	10%
VAT rate	16%

Calculate:

a) The customs value. (2 marks)

b) The import duty. (2 marks)

c) The excise duty. (2 marks)

d) The VAT payable. (2 marks)

e) The total taxes payable, excluding IDF and RDL. (2 marks)

TOTAL: 50 MARKS

13.0 ANSWER KEY AND MARKING SCHEME

Section A: Multiple-choice answers

Question	Answer	Question	Answer
1	B	16	C
2	C	17	C
3	A	18	A
4	A	19	C
5	C	20	C
6	C	21	D
7	C	22	C
8	C	23	B

Question	Answer	Question	Answer
9	B	24	C
10	A	25	C
11	B	26	C
12	C	27	B
13	B	28	B
14	C	29	B
15	C	30	B

Section B: Suggested marking scheme

Question 1 (7 marks)

i. Importation is bringing goods into Kenya from another country. (1)

ii. Exportation is sending goods from Kenya to another country. (1)

b) Any three:

- Tea.
- Coffee.
- Flowers.
- Avocados.
- Fresh vegetables.
- Fish products.
- Soda ash.
- Selected mineral products.

(3 marks)

c) Any two:

- To obtain machinery and equipment.
- To obtain raw materials.
- To obtain medicines and medical equipment.
- To access goods not produced locally.
- To increase the variety of products available.

(2 marks)

Question 2 (8 marks)

a) Customs value is the value of imported goods determined for purposes of assessing customs duties and taxes. (1)

b) CIF means Cost, Insurance and Freight. (1)

$$CV = C + I + F$$

$$= 750{,}000 + 35{,}000 + 65{,}000$$

$$= \text{KSh } 850{,}000$$

- Correct formula (1)
- Correct substitution (1)
- Correct answer and unit (1)

$$ID = \frac{10}{100} \times 850{,}000$$

$$= \text{KSh } 85{,}000$$

- Correct formula (1)
- Correct substitution (1)
- Correct answer (1)

Question 3 (8 marks)

$$ID = \frac{25}{100} \times 600{,}000$$

$$= \text{KSh } 150{,}000$$

(2 marks)

$$\text{Excise value} = CV + ID$$

$$= 600{,}000 + 150{,}000$$

$$= \text{KSh } 750{,}000$$

(2 marks)

$$ED = \frac{10}{100} \times 750{,}000$$

$$= \text{KSh } 75{,}000$$

(2 marks)

$$600{,}000 + 150{,}000 + 75{,}000$$

$$= \text{KSh } 825{,}000$$

(2 marks)

Question 4 (8 marks)

$$\text{VAT taxable value} = CV + ID + ED$$

$$= 900{,}000 + 180{,}000 + 72{,}000$$

$$= \text{KSh } 1{,}152{,}000$$

(3 marks)

$$VAT = \frac{16}{100} \times 1{,}152{,}000$$

$$= \text{KSh } 184{,}320$$

(3 marks)

$$180{,}000 + 72{,}000 + 184{,}320$$

$$= \text{KSh } 436{,}320$$

(2 marks)

Question 5 (9 marks)

a) Export levy is a government charge imposed on specified goods exported from the country. (1)

$$\text{Export levy} = \frac{20}{100} \times 1{,}500{,}000$$

$$= \text{KSh } 300{,}000$$

- Correct formula (1)
- Correct substitution (1)
- Correct answer (1)

$$1{,}500{,}000 - 300{,}000$$

$$= \text{KSh } 1{,}200{,}000$$

(2 marks)

$$40 \times 175$$

$$= \text{USD } 7{,}000$$

(2 marks)

e) Any one:

- Raising government revenue.
- Regulating the export of selected products.
- Encouraging local processing.
- Supporting national development.

(1 mark)

Question 6 (10 marks)

a) Customs value:

$$CV = 1{,}000{,}000 + 50{,}000 + 100{,}000$$

$$= \text{KSh } 1{,}150{,}000$$

(2 marks)

b) Import duty:

$$ID = \frac{25}{100} \times 1{,}150{,}000$$

$$= \text{KSh } 287{,}500$$

(2 marks)

c) Excise value:

$$1{,}150{,}000 + 287{,}500$$

$$= \text{KSh } 1{,}437{,}500$$

Excise duty:

$$ED = \frac{10}{100} \times 1{,}437{,}500$$

$$= \text{KSh } 143{,}750$$

(2 marks)

d) VAT taxable value:

$$1{,}150{,}000 + 287{,}500 + 143{,}750$$

$$= \text{KSh } 1{,}581{,}250$$

VAT:

$$VAT = \frac{16}{100} \times 1{,}581{,}250$$

$$= \text{KSh } 253{,}000$$

(2 marks)

e) Total taxes:

$$287{,}500 + 143{,}750 + 253{,}000$$

$$= \text{KSh } 684{,}250$$

(2 marks)

Suggested learner project

Project title: Investigating the cost of importing goods into Kenya

Learners should work in groups to:

1. Select one product that is imported into Kenya.
2. Identify its country of origin.
3. Obtain a hypothetical or researched purchase price.
4. Identify the freight and insurance costs.
5. Calculate the customs value.
6. Research the applicable import duty rate.
7. Establish whether excise duty applies.
8. Calculate VAT using the correct taxable value.
9. Prepare a report showing the calculations.
10. Explain how the government benefits from customs revenue.

Expected project outcomes:

- Learners apply commercial arithmetic to real-life trade.
- Learners use mathematical skills to solve taxation problems.
- Learners develop digital literacy and research skills.
- Learners appreciate the importance of government revenue.

STRAND 2.0: MEASUREMENTS AND GEOMETRY

SUB-STRAND 2.4: ROTATION

1. Introduction to rotation

Rotation is one of the transformations studied in geometry. It involves turning a shape around a fixed point through a specified angle and in a particular direction.

Rotation is commonly observed in everyday activities and objects. For example:

- The hands of a clock rotate around the centre of the clock.
- A bicycle wheel rotates around its axle.
- A fan rotates around its central shaft.
- A windmill rotates as its blades move around a fixed centre.

- A door rotates about its hinges when opened or closed.
- A potter's wheel rotates around its central axis.

In mathematics, rotation changes the position and orientation of a plane figure but preserves its size and shape.

Learning outcomes

By the end of this sub-strand, the learner should be able to:

1. Determine the properties of rotation in different situations.
2. Rotate an object on a plane given the centre and angle of rotation.
3. Determine the centre and angle of rotation given an object and its image.
4. Determine the order of rotational symmetry of plane figures.
5. Appreciate the application of rotation in real-life situations.

Key inquiry questions

1. How is rotation applied in real-life situations where objects rotate about a fixed point in a particular direction?
2. Why is rotation useful in day-to-day life activities?

Prerequisite knowledge

The learner should be able to:

- Identify plane figures such as triangles, squares, rectangles and circles.
- Plot points on a Cartesian plane.
- Identify coordinates of points.
- Measure angles using a protractor.
- Draw line segments and perpendicular lines.
- Recognise clockwise and anticlockwise directions.

2.4.1 Meaning of rotation

Definition of rotation

Rotation is a transformation in which a point or plane figure is turned about a fixed point called the *centre of rotation* through a given angle in a specified direction.

A rotation is described using three important elements:

1. **Centre of rotation:** The fixed point around which a figure turns.
2. **Angle of rotation:** The angle through which the figure turns.
3. **Direction of rotation:** The direction in which the figure turns, either clockwise or anticlockwise.

For example, when a point is rotated through $90°$ anticlockwise about the origin, its position changes according to the rotation rule.

Important terms used in rotation

Term	Meaning
Object	The original figure before rotation
Image	The new figure after rotation
Centre of rotation	The fixed point about which the figure turns
Angle of rotation	The amount of turning, measured in degrees
Clockwise rotation	Turning in the direction of the hands of a clock

Term	Meaning
Anticlockwise rotation	Turning in the direction opposite to the hands of a clock
Corresponding points	Points on the object and image that match after rotation
Invariant point	A point that remains in the same position during a transformation

The original figure is commonly labelled ABC, while its image is labelled $A'B'C'$.

The prime symbol $(')$ indicates that a point belongs to the image.

For example:

- A is a point on the original object.
- A' is the corresponding point on the rotated image.
- B corresponds to B'.
- C corresponds to C'.

2.4.2 Direction of rotation

There are two main directions of rotation.

A. Clockwise rotation

A clockwise rotation is a turn in the same direction as the movement of the hands of an ordinary clock.

Examples include:

- Turning a key clockwise to lock a door.
- The movement of clock hands.
- Turning a water tap clockwise to close it.

B. Anticlockwise rotation

An anticlockwise rotation is a turn in the direction opposite to the movement of the hands of a clock.

Examples include:

- Turning a key anticlockwise to unlock a door.
- Turning a tap anticlockwise to open it.
- Turning a bicycle pedal in the opposite direction.

Important facts about direction

- Clockwise rotation is sometimes represented by a negative angle.
- Anticlockwise rotation is represented by a positive angle.
- A complete clockwise turn is $-360°$.
- A complete anticlockwise turn is $+360°$.

For example:

Rotation	Direction
$90°$	Anticlockwise
$-90°$	Clockwise
$180°$	Either direction gives the same final position

Rotation	Direction
$270°$ anticlockwise	Equivalent to $90°$ clockwise
$360°$	One complete turn

Note: In this sub-strand, unless otherwise stated, positive angles represent anticlockwise rotations and negative angles represent clockwise rotations.

C. Measuring angles of rotation

Angles of rotation are measured in degrees.

A complete revolution is $360°$.

Fraction of a complete turn	Angle
One-quarter turn	$90°$
Half turn	$180°$
Three-quarter turn	$270°$
One complete turn	$360°$
Two complete turns	$720°$

Worked Example 1

A bicycle wheel makes half a revolution. Determine the angle through which it rotates.

Solution

One complete revolution is $360°$.

Half a revolution is:

$$\frac{1}{2} \times 360° = 180°$$

Answer: $180°$**.**

Worked Example 2

A fan blade turns through three-quarters of a complete revolution. Find the angle of rotation.

Solution

$$\frac{3}{4} \times 360°$$

$$= 270°$$

Answer: $270°$**.**

2.4.3 Properties of rotation

Rotation has several important properties. These properties help us identify and construct rotated images accurately.

1. Rotation preserves the size of a figure

The object and its image have the same dimensions.

For example, if a square has sides of 4 cm before rotation, its image will also have sides of 4 cm.

$$AB = A'B'$$

2. Rotation preserves the shape of a figure

The image has the same shape as the original object.

A triangle remains a triangle, a rectangle remains a rectangle and a circle remains a circle.

3. Rotation preserves lengths

The distance between any two corresponding points remains unchanged.

For example:

$$AB = A'B'$$

$$BC = B'C'$$

$$AC = A'C'$$

4. Rotation preserves angles

Corresponding angles in the object and image are equal.

For example:

$$\angle ABC = \angle A'B'C'$$

5. The centre of rotation remains fixed

The centre of rotation does not move during the transformation.

If O is the centre of rotation, then:

$$O' = O$$

6. Every point moves through the same angle

All points of the object rotate through the same angle and in the same direction.

For example, if a triangle is rotated through $90°$ anticlockwise, each of its vertices turns through $90°$ anticlockwise.

7. Corresponding points remain at equal distances from the centre

The distance from the centre of rotation to an original point is equal to the distance from the centre to its image.

$$OA = OA'$$

$$OB = OB'$$

$$OC = OC'$$

8. Rotation preserves orientation

A rotation does not reverse the order of the vertices of a figure. If the original vertices are followed in a particular order, their images retain that order.

Summary of properties

Property	Explanation
Shape	Remains unchanged
Size	Remains unchanged

Property	Explanation
Lengths	Corresponding lengths are equal
Angles	Corresponding angles are equal
Centre	Remains fixed
Direction	All points turn in the same direction
Angle	All points turn through the same angle
Distance	Each point and its image are equally distant from the centre

Main point: Rotation is an example of a rigid transformation because it changes the position of a figure without changing its shape or size.

2.4.4 Rotation about a fixed point

A fixed point is a point that does not move during rotation. It is called the centre of rotation.

When a figure is rotated about a fixed point:

- The centre remains unchanged.
- Each vertex moves along a circular path.
- The distance of each vertex from the centre remains constant.
- The figure turns through the specified angle.

Understanding rotation using a triangle

Consider triangle ABC rotated about centre O through $90°$ anticlockwise.

The corresponding points are:

- A moves to A'.
- B moves to B'.
- C moves to C'.

The following relationships hold:

$$OA = OA'$$

$$OB = OB'$$

$$OC = OC'$$

Also:

$$\angle AOA' = \angle BOB' = \angle COC' = 90°$$

Worked Example 3

A point P is 5 cm from a fixed point O. It is rotated through $90°$ anticlockwise about O. Find the distance of its image P' from O.

Solution

Rotation preserves the distance from the centre.

Therefore:

$$OP = OP'$$

$$OP' = 5 \text{ cm}$$

Answer: 5 cm.

2.4.5 Rotation on a Cartesian plane

A Cartesian plane is a flat surface formed by two perpendicular number lines called axes.

The two axes are:

1. **The x-axis:** The horizontal number line.
2. **The y-axis:** The vertical number line.

The axes meet at a point called the origin, represented by $O(0,0)$.

Rotation on a Cartesian plane involves turning a point or plane figure around a specified centre through a given angle.

A. Coordinates of a point

A point on the Cartesian plane is represented by an ordered pair:

$$P(x, y)$$

Where:

- x represents the horizontal coordinate.
- y represents the vertical coordinate.

For example, point $P(3,2)$ is located 3 units to the right and 2 units upwards from the origin.

Point $Q(-2,3)$ is located 2 units to the left and 3 units upwards from the origin.

B. Rotation of a point about the origin

The origin is the point $(0,0)$. When a point is rotated about the origin, its coordinates change according to the angle and direction of rotation.

The following rules are essential.

Rotation about origin	Coordinate rule
$90°$ anticlockwise	$(x, y) \rightarrow (-y, x)$
$180°$ anticlockwise	$(x, y) \rightarrow (-x, -y)$
$270°$ anticlockwise	$(x, y) \rightarrow (y, -x)$
$360°$ anticlockwise	$(x, y) \rightarrow (x, y)$
$90°$ clockwise	$(x, y) \rightarrow (y, -x)$
$180°$ clockwise	$(x, y) \rightarrow (-x, -y)$
$270°$ clockwise	$(x, y) \rightarrow (-y, x)$

Remember: A $90°$ clockwise rotation gives the same result as a $270°$ anticlockwise rotation. Similarly, a $270°$ clockwise rotation gives the same result as a $90°$ anticlockwise rotation.

C. Rotation through $90°$ anticlockwise

The rule is:

$$(x, y) \rightarrow (-y, x)$$

This means:

- The original y-coordinate becomes the new x-coordinate with an opposite sign.
- The original x-coordinate becomes the new y-coordinate.

Worked Example 4

Rotate point $A(3,2)$ through $90°$ anticlockwise about the origin.

Solution

The rule is:

$$(x, y) \rightarrow (-y, x)$$

Substitute $x = 3$ and $y = 2$.

$$A(3,2) \rightarrow A'(-2,3)$$

Answer

$A'(-2,3)$.

D. Rotation through $180°$ about the origin

The rule is:

$$(x, y) \rightarrow (-x, -y)$$

Both coordinates change their signs.

Worked Example 5

Rotate point $B(4,3)$ through $180°$ anticlockwise about the origin.

Solution

$$(x, y) \rightarrow (-x, -y)$$

$$B(4,3) \rightarrow B'(-4, -3)$$

Answer: $B'(-4, -3)$.

E. Rotation through $270°$ anticlockwise

The rule is:

$$(x, y) \rightarrow (y, -x)$$

Worked Example 6

Rotate point $C(2,5)$ through $270°$ anticlockwise about the origin.

Solution

$$(x, y) \rightarrow (y, -x)$$

$$C(2,5) \rightarrow C'(5, -2)$$

Answer: $C'(5, -2)$.

F. Rotation through $90°$ clockwise

The rule is:

$$(x, y) \rightarrow (y, -x)$$

Worked Example 7

Rotate point $D(3,4)$ through $90°$ clockwise about the origin.

Solution

$$D(3,4) \rightarrow D'(4,-3)$$

Answer: $D'(4,-3)$.

G. Rotation through $270°$ clockwise

The rule is:

$$(x,y) \rightarrow (-y,x)$$

Worked Example 8

Rotate point $E(5,2)$ through $270°$ clockwise about the origin.

Solution

$$E(5,2) \rightarrow E'(-2,5)$$

Answer: $E'(-2,5)$.

2.4.6 Graphical representation of rotation

Rotation can be represented by plotting the original figure and its image on the same Cartesian plane.

Example 9: Rotation of a triangle through $90°$ anticlockwise

Triangle ABC has vertices:

$$A(1,1), \quad B(4,1), \quad C(2,3)$$

Rotate the triangle through $90°$ anticlockwise about the origin.

Step 1: State the rotation rule.

$$(x,y) \rightarrow (-y,x)$$

Step 2: Calculate the image coordinates.

For point $A(1,1)$:

$$A(1,1) \rightarrow A'(-1,1)$$

For point $B(4,1)$:

$$B(4,1) \rightarrow B'(-1,4)$$

For point $C(2,3)$:

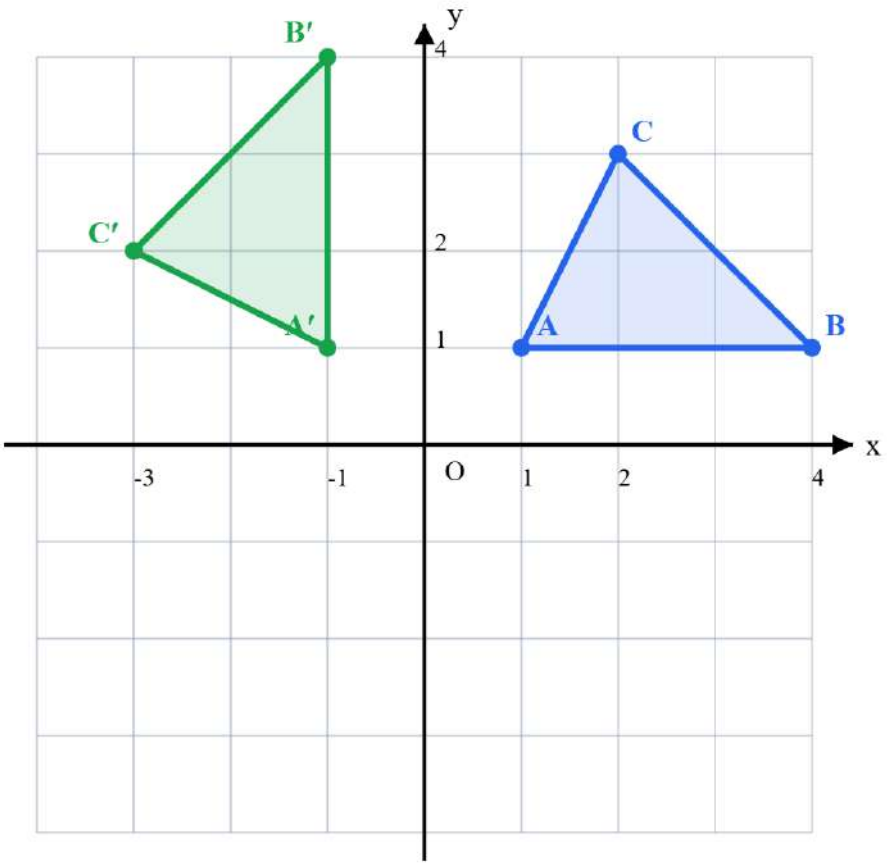
$$C(2,3) \rightarrow C'(-3,2)$$

Step 3: Record the coordinates.

Original point	Original coordinates	Image point	Image coordinates
A	(1, 1)	A′	(-1, 1)
B	(4, 1)	B′	(-1, 4)
C	(2, 3)	C′	(-3, 2)

Step 4: Plot the original triangle and its image.

Graph 1: Triangle ABC rotated through 90° anticlockwise



Original triangle ABC

Image A'B'C'

Each vertex is turned 90° anticlockwise about the origin. The graph uses the coordinates calculated above.

Conclusion: The image of triangle ABC is triangle $A'B'C'$. The shape and size remain unchanged, but its position changes.

Example 10: Rotation through $180°$

A quadrilateral has vertices:

$$P(1,2),\ Q(4,2),\ R(4,4),\ S(1,4)$$

Rotate it through $180°$ about the origin.

Solution

The rule is:

$$(x, y) \rightarrow (-x, -y)$$

Therefore:

$$P(1,2) \rightarrow P'(-1,-2)$$

$$Q(4,2) \rightarrow Q'(-4,-2)$$

$$R(4,4) \rightarrow R'(-4,-4)$$

$$S(1,4) \rightarrow S'(-1,-4)$$

The image vertices are:

$$P'(-1,-2),\ Q'(-4,-2),\ R'(-4,-4),\ S'(-1,-4)$$

Exercise: Rotation on a Cartesian plane

1. Rotate $A(2,3)$ through $90°$ anticlockwise about the origin.
2. Rotate $B(5,2)$ through $180°$ about the origin.
3. Rotate $C(3,4)$ through $270°$ anticlockwise about the origin.
4. Rotate $D(4,1)$ through $90°$ clockwise about the origin.
5. Triangle XYZ has vertices $X(1,2)$, $Y(3,2)$ and $Z(2,4)$. Find the coordinates of its image after a $180°$ rotation about the origin.

2.4.7 Rotation about a centre other than the origin

Rotation does not have to take place about the origin. A figure can be rotated about any point on the Cartesian plane.

For example, a triangle can be rotated about $O(2,1)$, $C(3,4)$ or any other specified point.

When the centre is not the origin, the rotation must be performed relative to that centre.

Method of rotating a point about a different centre

Suppose a point $P(x,y)$ is rotated about centre $C(a,b)$.

The general method is:

1. Subtract the centre coordinates from the point coordinates.
2. Apply the appropriate rotation rule about the origin.
3. Add the centre coordinates to the resulting coordinates.

For a $90°$ anticlockwise rotation:

$$\boxed{(x,y) \rightarrow (a-(y-b),\ b+(x-a))}$$

This can also be written as:

$$\boxed{(x,y) \rightarrow (a+b-y,\ b+x-a)}$$

Useful formulae for a centre $C(a,b)$

Rotation	Image coordinates
$90°$ anticlockwise	$(a+b-y,\ b+x-a)$
$180°$	$(2a-x,\ 2b-y)$
$270°$ anticlockwise	$(a+y-b,\ b+a-x)$
$90°$ clockwise	$(a+y-b,\ b+a-x)$

Worked Example 11: Rotation about $C(2,1)$

Point $P(4,2)$ is rotated through $90°$ anticlockwise about centre $C(2,1)$. Find the image coordinates.

Step 1: Identify the coordinates.

$$P(4,2), \quad C(2,1)$$

Step 2: Translate the point relative to the centre.

$$(4-2,\ 2-1) = (2,1)$$

Step 3: Apply the $90°$ anticlockwise rule.

$$(x, y) \to (-y, x)$$

$$(2,1) \to (-1,2)$$

Step 4: Add the centre coordinates.

$$(-1+2,\ 2+1)$$

$$= (1,3)$$

Answer

$P'(1,3)$.

Worked Example 12: Rotation about a centre through $180°$

Point $A(5,4)$ is rotated through $180°$ about centre $C(2,1)$. Find its image.

Solution

For a $180°$ rotation:

$$(x, y) \to (2a - x,\ 2b - y)$$

Where:

$$a = 2, \quad b = 1$$

Therefore:

$$x' = 2(2) - 5$$

$$x' = -1$$

And:

$$y' = 2(1) - 4$$

$$y' = -2$$

Answer:

$$A'(-1, -2)$$

Worked Example 13: Rotation of a triangle about a non-origin centre

Triangle ABC has vertices:

$$A(3,2), \quad B(5,2), \quad C(4,4)$$

Rotate it through $90°$ anticlockwise about $O(1,1)$.

Solution

Use the rule:

$$(x, y) \rightarrow (a + b - y,\ b + x - a)$$

Here $a = 1$ and $b = 1$.

For $A(3,2)$:

$$A' = (1 + 1 - 2,\ 1 + 3 - 1)$$

$$A' = (0,3)$$

For $B(5,2)$:

$$B' = (1 + 1 - 2,\ 1 + 5 - 1)$$

$$B' = (0,5)$$

For $C(4,4)$:

$$C' = (1 + 1 - 4,\ 1 + 4 - 1)$$

$$C' = (-2,4)$$

Final image coordinates:

$$A'(0,3), \quad B'(0,5), \quad C'(-2,4)$$

2.4.8 Determining the centre of rotation

Sometimes a learner is given an original figure and its image, but the centre of rotation is not provided.

The learner is required to determine the centre.

Definition

The **centre of rotation** is the fixed point that is equally distant from a point on the original figure and its corresponding image.

If point A is rotated to A' about centre O, then:

$$OA = OA'$$

The centre of rotation lies on the perpendicular bisector of the line segment joining A and A'.

Construction method for finding the centre

To find the centre of rotation:

1. Identify a point on the original figure and its corresponding image point.
2. Join the two corresponding points using a straight line.
3. Construct the perpendicular bisector of the line segment.
4. Repeat the process using another pair of corresponding points.
5. The point where the perpendicular bisectors intersect is the centre of rotation.

Worked Example 14

A point A and its image A' are given. Describe how the centre of rotation can be determined.

Solution

- Join A to A'.
- Find the midpoint of AA'.
- Draw a perpendicular line through the midpoint.
- Select another pair of corresponding points, B and B'.

- Draw the perpendicular bisector of BB'.
- The intersection of the two perpendicular bisectors gives the centre of rotation.

Important: Two pairs of corresponding points are generally required for an accurate construction. If the angle is $180°$, the centre is the midpoint of each corresponding point pair.

Finding the centre using coordinates

For a $180°$ rotation, the centre is the midpoint of the line segment joining a point and its image.

The midpoint formula is:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Worked Example 15

Point $A(2,4)$ is rotated through $180°$ to $A'(-4,2)$. Find the centre of rotation.

Solution

The centre is the midpoint of AA'.

$$C = \left(\frac{2 + (-4)}{2}, \frac{4 + 2}{2}\right)$$

$$C = \left(\frac{-2}{2}, \frac{6}{2}\right)$$

$$\boxed{C(-1,3)}$$

Answer: The centre of rotation is $(-1,3)$.

2.4.9 Determining the angle and direction of rotation

The angle of rotation is the angle through which a figure turns about its centre.

It is measured between the line joining the centre to an original point and the line joining the centre to its corresponding image.

For example, if A is rotated to A' about centre O, the angle of rotation is:

$$\angle AOA'$$

Steps for determining the angle of rotation

1. Identify the centre of rotation.
2. Select a point on the original figure.
3. Identify its corresponding image point.
4. Join the original point to the centre.
5. Join the image point to the centre.
6. Measure the angle between the two lines using a protractor.
7. Identify whether the movement is clockwise or anticlockwise.

Worked Example 16

A point P is rotated about centre O to position P'. The angle between OP and OP' is $120°$, measured anticlockwise. State the angle and direction of rotation.

Solution

The measured angle is $120°$.

The direction is anticlockwise.

Answer: $120°$ **anticlockwise.**

Worked Example 17

A point moves from $A(2,1)$ to $A'(1,-2)$ under rotation about the origin. Determine the angle and direction.

Solution

The original point is:

$$A(2,1)$$

The image is:

$$A'(1,-2)$$

The rotation rule:

$$(x,y) \rightarrow (y,-x)$$

gives:

$$(2,1) \rightarrow (1,-2)$$

This is a $90°$ clockwise rotation.

Answer: $90°$ **clockwise.**

Worked Example 18: Identifying the rotation from coordinates

Point $B(-3,4)$ is mapped to $B'(-4,-3)$ about the origin. Determine the angle and direction.

Solution

The rule for a $90°$ anticlockwise rotation is:

$$(x,y) \rightarrow (-y,x)$$

Substituting:

$$(-3,4) \rightarrow (-4,-3)$$

Therefore, the transformation is a $90°$ anticlockwise rotation.

Answer: $90°$ **anticlockwise.**

2.4.10 Rotational symmetry of plane figures

Meaning of symmetry

Symmetry refers to a situation where a figure has a balanced arrangement of parts that correspond to one another.

There are different types of symmetry, including:

- Line symmetry.
- Rotational symmetry.

Definition of rotational symmetry

Rotational symmetry is the property of a plane figure that allows it to fit exactly onto its original outline after being rotated through an angle greater than $0°$ and less than or equal to $360°$.

A figure has rotational symmetry if it matches its original position during a complete turn.

For example:

- A square has rotational symmetry.
- An equilateral triangle has rotational symmetry.
- A regular hexagon has rotational symmetry.
- A non-square rectangle has rotational symmetry.
- A circle has rotational symmetry through any angle.

A. Order of rotational symmetry

Definition

The **order of rotational symmetry** is the number of times a plane figure fits exactly onto its original outline during one complete revolution of $360°$.

It is determined by counting the number of matching positions, including the starting position but excluding the final position at $360°$, which is the same as the starting position.

Formula

$$\text{Order of rotational symmetry} = \frac{360°}{\text{Smallest angle of rotation}}$$

The smallest angle of rotation is the least positive angle through which the figure turns and still fits exactly onto its original outline.

B. Rotational symmetry of common plane figures

Plane figure	Order of rotational symmetry	Smallest angle
Equilateral triangle	3	$120°$
Square	4	$90°$
Rectangle	2	$180°$
Rhombus	2	$180°$
Parallelogram	2	$180°$
Regular pentagon	5	$72°$
Regular hexagon	6	$60°$
Regular octagon	8	$45°$
Circle	Infinite	Any angle
Non-square rectangle	2	$180°$
Ordinary scalene triangle	1	$360°$ only

Note: A figure with order 1 has no non-trivial rotational symmetry. It only returns to its original position after one complete turn of $360°$.

C. Rotational symmetry of a square

A square has four equal sides and four right angles.

It fits onto its original outline at:

- $90°$.
- $180°$.
- $270°$.
- $360°$, the completed turn.

Therefore:

$$\text{Order} = \frac{360°}{90°}$$

$$\boxed{\text{Order} = 4}$$

D. Rotational symmetry of an equilateral triangle

An equilateral triangle has three equal sides and three equal angles of $60°$.

It fits onto its original outline after:

- $120°$.
- $240°$.
- $360°$.

Therefore:

$$\text{Order} = \frac{360°}{120°}$$

$$\boxed{\text{Order} = 3}$$

E. Rotational symmetry of a rectangle

A rectangle has two pairs of equal opposite sides and four right angles.

It fits onto its original outline after a half turn of $180°$.

Therefore:

$$\text{Order} = \frac{360°}{180°}$$

$$\boxed{\text{Order} = 2}$$

F. Rotational symmetry of a regular hexagon

A regular hexagon has six equal sides and six equal interior angles.

The smallest angle of rotation is:

$$\frac{360°}{6} = 60°$$

Therefore:

$$\boxed{\text{Order} = 6}$$

G. Rotational symmetry of a circle

A circle has infinite rotational symmetry because it fits onto its original outline after rotation through any angle about its centre.

For example:

- $30°$.

- $45°$.
- $90°$.
- $180°$.
- $270°$.

All these rotations leave the circular outline unchanged.

Therefore, a circle has infinite order of rotational symmetry.

Worked Example 19

A regular polygon has an order of rotational symmetry of 8. Find its smallest angle of rotation.

Solution

$$\text{Smallest angle} = \frac{360°}{\text{Order}}$$

$$= \frac{360°}{8}$$

$$\boxed{45°}$$

Answer: $45°$.

Worked Example 20

A plane figure fits onto its original outline after every $60°$ rotation. Determine its order of rotational symmetry.

Solution

$$\text{Order} = \frac{360°}{60°}$$

$$\boxed{6}$$

Answer: Order 6.

Worked Example 21

A learner traces a figure and finds that it fits onto its original outline 5 times in one complete turn. Determine:

a) Its order of rotational symmetry.

b) Its smallest angle of rotation.

Solution

a) The number of matching positions is 5.

$$\text{Order} = 5$$

$$\text{Smallest angle} = \frac{360°}{5}$$

$$\boxed{72°}$$

Answer: Order = 5; smallest angle = $72°$.

H. How to determine the order of rotational symmetry experimentally

The learner can use paper cut-outs to investigate rotational symmetry.

Materials:

- Plain paper.
- Pencil.
- Ruler.
- Scissors.
- Tracing paper.
- Protractor.

Procedure:

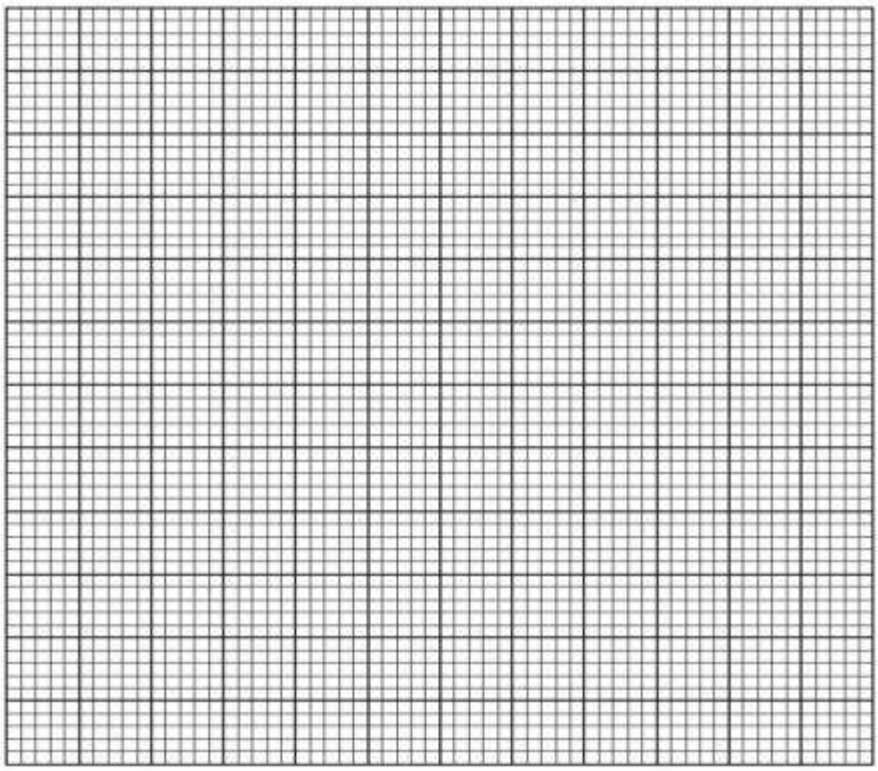
1. Draw a square on a piece of paper.
2. Cut out the square.
3. Place it over an identical outline.
4. Keep one corner or the centre fixed, as appropriate.
5. Rotate the square through equal angles.
6. Count the number of times it fits exactly onto the outline during one complete turn.
7. Record the number of matching positions.
8. Repeat the activity using a rectangle, equilateral triangle and regular hexagon.

Observation table

Figure	Matching positions	Order
Square	4	4
Rectangle	2	2
Equilateral triangle	3	3
Regular hexagon	6	6

PRACTICE QUESTION

A square ABCD has vertices A (-5,-5), B (-4, -5), C (-4,-4), and D (-5,-4) (a) On the grid below, draw the square ABCD (1 mark)



a. On the same axes, draw A'B'C'D', the image of the square ABCD under a transformation described as enlargement, centre (4, 3) and scale factor -3. (2 marks)

b. The square A"B"C"D" with the vertices A (1, 5)", B (4, 5)", C (4, 2)" and D (1, 2)" is the image of A'B'C'D' under a rotation. On the same axes, draw the square A"B"C"D" hence describe the rotation. (4 marks)

c. On the same axes, draw the square A'"B'"C'"D'", the image of A"B"C"D" under reflection in the line y = 0. (1 mark)

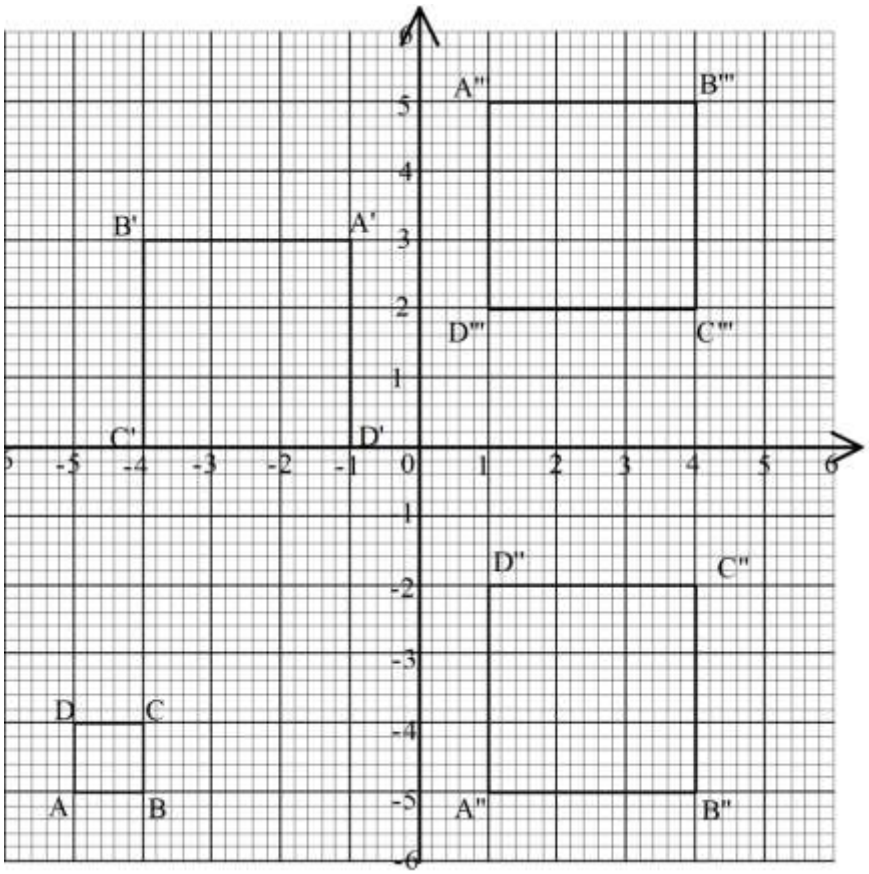
d. State a pair of squares that are:

i. Directly congruent (1 mark)

ii. Oppositely congruent (1 mark)

Marking Scheme Answer

a. Enlargement



b.

Rotation

Centre (o, -1)

Angle = 180°

c. Reflection in the line y = 0

d.i. A'B'C'D' and A'B'C'D'

ii. A"B"C"D" and A''B''C''D''

2.4.11 Difference between line symmetry and rotational symmetry

These two forms of symmetry are related but different.

Line symmetry	Rotational symmetry
A figure is divided into two matching halves by a line.	A figure fits onto itself after rotation.
The line is called a line of symmetry.	The turning takes place around a centre.
It involves reflection.	It involves rotation.
A figure may have one or more lines of symmetry.	A figure has an order of rotational symmetry.
Example: an isosceles triangle has one line of symmetry.	Example: a square has rotational symmetry of order 4.

Important: A figure may have line symmetry, rotational symmetry, both or neither.

For example, a general parallelogram has rotational symmetry of order 2 but does not have line symmetry.

2.4.12 Real-life applications of rotation in Kenya

Rotation is used in many activities, machines and structures within the Kenyan community.

1. Clocks and watches
   - The minute and hour hands rotate around the centre of the clock.
   - The minute hand completes one revolution in 60 minutes.
   - The hour hand completes one revolution in 12 hours.

2. Bicycle and motor vehicle wheels
   - Wheels rotate around their axles.
   - Rotation enables bicycles, motorcycles, matatus and other vehicles to move.
   - The rotation of wheels helps transport people and goods.

3. Electric fans
   - Fan blades rotate around a central shaft.
   - Rotation creates airflow, helping to cool rooms.
   - The same principle is used in industrial ventilation systems.

4. Windmills and wind turbines
   - Wind causes the blades to rotate.
   - The movement can be used to pump water or generate electricity.
   - Wind turbines convert wind energy into electrical energy.

5. Agricultural machinery

Examples include:

   - Rotating blades in grass cutters.
   - Rotating shafts in tractors.
   - Rotating wheels in irrigation equipment.
   - Rotating drums in threshing machines.

6. Water pumps
   - Some water pumps use rotating impellers.
   - The rotation helps move water from one location to another.
   - This is useful in irrigation and domestic water supply.

7. Doors and gates
   - A door rotates around its hinges.
   - The hinges provide the fixed axis of rotation.
   - Rotation allows the door to open and close.

8. Sports and recreation

Rotation occurs in:

   - A football spinning after being kicked.
   - A basketball spinning on a player's finger.
   - A discus rotating during a throw.

- A bicycle wheel during cycling.
- A merry-go-round turning around its central point.

9. Industry and manufacturing

- Rotating machines are used in factories.
- Lathes rotate materials during shaping.
- Drilling machines rotate drill bits.
- Grinding machines rotate grinding wheels.

10. Traditional and modern technology

Rotation is used in:

- Potter's wheels.
- Grinding stones.
- Water mills.
- Grain milling machines.
- Electric motors.

Importance of understanding rotation

Understanding rotation helps learners to:

1. Understand how rotating machines operate.
2. Interpret diagrams and technical drawings.
3. Design mechanical devices.
4. Understand the movement of wheels and gears.
5. Develop problem-solving skills.
6. Apply geometry in engineering and technology.
7. Understand rotational symmetry in design and decoration.

2.4.13 Rotation using digital technology

Digital devices provide useful tools for investigating transformations.

A learner can use:

- GeoGebra.
- Dynamic geometry software.
- Computer drawing applications.
- Interactive mathematics simulations.

Activity: Rotating a triangle using digital software

Instructions:

1. Open a dynamic geometry application such as GeoGebra.
2. Draw a triangle on the coordinate grid.
3. Mark a point to act as the centre of rotation.
4. Select the rotation transformation.
5. Enter an angle of $90°$.
6. Select anticlockwise as the direction.
7. Observe the new image.
8. Repeat with $180°$ and $270°$.
9. Compare the original figure with the rotated images.

10. Record the coordinates of the vertices.

What learners should observe

- The shape does not change.
- The size does not change.
- The vertices move around the centre.
- Each point turns through the same angle.
- The distance from the centre to each point remains constant.

2.4.14 Summary of important formulae and rules

Concept	Formula or rule
Complete revolution	$360°$
Half revolution	$180°$
Quarter revolution	$90°$
Three-quarter revolution	$270°$
Order of rotational symmetry	$\frac{360°}{\text{Smallest angle}}$
Smallest angle	$\frac{360°}{\text{Order}}$
$90°$ anticlockwise about origin	$(x, y) \rightarrow (-y, x)$
$180°$ about origin	$(x, y) \rightarrow (-x, -y)$
$270°$ anticlockwise about origin	$(x, y) \rightarrow (y, -x)$
$90°$ clockwise about origin	$(x, y) \rightarrow (y, -x)$
$360°$ about origin	$(x, y) \rightarrow (x, y)$
Centre of $180°$ rotation	Midpoint of corresponding points

2.4.15 Competency-Based learning activities

Activity 1: Investigating rotation in the community

Task: Identify five objects in the school or home environment that rotate around a fixed point.

For each object, record:

- Name of the object.
- Centre or axis of rotation.
- Direction of rotation.
- Purpose of rotation.
- Whether the object makes a complete revolution.

Activity 2: Constructing rotated figures

Task: Draw a triangle on graph paper and rotate it through:

- $90°$ anticlockwise.
- $180°$ clockwise.
- $270°$ anticlockwise.

Use the same centre of rotation and compare the resulting images.

Activity 3: Investigating rotational symmetry

Task: Draw and cut out the following shapes:

- Square.
- Rectangle.
- Equilateral triangle.
- Regular pentagon.
- Regular hexagon.

Rotate each figure about its centre and record its order of rotational symmetry.

Activity 4: Designing a rotational pattern

Task: Use a simple triangular or square shape to create a decorative pattern by rotating the shape around a fixed point.

The learner should explain:

- The centre of rotation.
- The angle of rotation.
- The number of rotations.
- The pattern produced.

Competencies developed

- Critical thinking and problem-solving.
- Creativity and imagination.
- Communication and collaboration.
- Digital literacy.
- Learning to learn.

Values promoted

- Cooperation.
- Responsibility.
- Respect for other learners' ideas.
- Accuracy.
- Perseverance.

Pertinent and contemporary issues

- Application of mathematics in technology.
- Safe use of rotating machines.
- Energy conservation.
- Innovation and design.
- Environmental awareness through wind energy.

REVISION EXERCISES

Section A: Multiple-choice questions (30 marks)

Instructions: Answer all questions. Choose the correct answer from the alternatives A, B, C and D. Each question carries 1 mark.

1. Which of the following best defines rotation?

A. Sliding a figure from one position to another.

B. Turning a figure about a fixed point through a given angle.

C. Enlarging a figure by a given scale factor.

D. Reflecting a figure across a straight line.

2. The fixed point around which a figure rotates is called the:

A. Image point.

B. Origin.

C. Centre of rotation.

D. Vertex.

3. Which direction is opposite to the movement of the hands of a clock?

A. Clockwise.

B. Horizontal.

C. Vertical.

D. Anticlockwise.

4. A complete revolution is equivalent to:

A. $90°$.

B. $180°$.

C. $270°$.

D. $360°$.

5. A bicycle wheel makes half a revolution. Through what angle has it rotated?

A. $45°$.

B. $90°$.

C. $180°$.

D. $270°$.

6. Which of the following is a property of rotation?

A. It changes the size of a figure.

B. It changes the shape of a figure.

C. It preserves the lengths of corresponding sides.

D. It always changes the number of vertices.

7. A point $P(3,2)$ is rotated through $90°$ anticlockwise about the origin. Its image is:

A. $(2,-3)$.

B. $(-3,-2)$.

C. $(-2,3)$.

D. $(3,-2)$.

8. A point $A(4,5)$ is rotated through $180°$ about the origin. Its image is:

A. $(5,4)$.

B. $(-5,4)$.

C. $(-4,-5)$.

D. $(4,-5)$.

9. The coordinate rule for a $90°$ clockwise rotation about the origin is:

A. $(x,y) \to (-y,x)$.

B. $(x,y) \to (y,-x)$.

C. $(x,y) \to (-x,-y)$.

D. $(x,y) \to (x,y)$.

10. Point $B(2,6)$ is rotated through $270°$ anticlockwise about the origin. Find its image.

A. $(6,-2)$.

B. $(-6,2)$.

C. $(-2,-6)$.

D. $(2,-6)$.

11. Which point remains fixed when a figure is rotated about the origin?

A. $(2,3)$.

B. $(0,0)$.

C. $(3,4)$.

D. $(-2,1)$.

12. Point $P(5,2)$ is rotated through $90°$ clockwise about the origin. Its image is:

A. $(-2,5)$.

B. $(-5,-2)$.

C. $(2,-5)$.

D. $(5,-2)$.

13. A square is rotated through $90°$ about its centre. What happens to its size?

A. It doubles.

B. It becomes smaller.

C. It remains unchanged.

D. It becomes larger.

14. The smallest angle of rotation of a regular hexagon is:

A. $30°$.

B. $45°$.

C. $60°$.

D. $90°$.

15. The order of rotational symmetry of a square is:

A. 1.

B. 2.

C. 4.

D. 8.

16. A rectangle that is not a square has an order of rotational symmetry of:

A. 1.

B. 3.

C. 2.

D. 4.

17. An equilateral triangle has an order of rotational symmetry of:

A. 1.

B. 2.

C. 3.

D. 4.

18. A regular pentagon has a smallest angle of rotation of:

A. $72°$.

B. $90°$.

C. $120°$.

D. $180°$.

19. A plane figure has an order of rotational symmetry of 6. Its smallest angle of rotation is:

A. $30°$.

B. $45°$.

C. $60°$.

D. $90°$.

20. A figure fits onto its original outline 8 times during one complete turn. Its order of rotational symmetry is:

A. 4. B. 6.

C. 8. D. 10.

21. Which of the following plane figures has infinite rotational symmetry?

A. Square.

B. Rectangle.

C. Equilateral triangle.

D. Circle.

22. The centre of a $180°$ rotation lies at the:

A. End point of the original figure.

B. Midpoint of the line joining corresponding points.

C. Highest point of the image.

D. Lowest point of the image.

23. The centre of rotation can be determined by constructing:

A. Parallel lines only.

B. Perpendicular bisectors of corresponding point segments.

C. Horizontal lines only.

D. Vertical lines only.

24. A point $A(2,4)$ is rotated through $180°$ about the origin. Find its image.

A. $(4,2)$.

B. $(-2,4)$.

C. $(-2,-4)$.

D. $(2,-4)$.

25. Point $P(4,3)$ is rotated through $90°$ anticlockwise about the origin. Its image is:

A. $(3,-4)$.

B. $(-4,-3)$.

C. $(-3,4)$.

D. $(4,-3)$.

26. Which of the following is an example of rotation in agriculture?

A. Spreading manure.

B. Planting seeds.

C. Rotating blades in a grass-cutting machine.

D. Measuring the length of a farm.

27. The hands of a clock rotate around:

A. The edge of the clock.

B. The centre of the clock.

C. The numbers on the clock.

D. The minute hand.

28. A regular polygon has a rotational symmetry order of 10. Find its smallest angle of rotation.

A. $10°$.

B. $36°$.

C. $45°$.

D. $90°$.

29. Which of the following statements about rotation is correct?

A. It changes the shape of a figure.

B. It preserves the size and shape of a figure.

C. It always changes the number of sides.

D. It changes the lengths of corresponding sides.

30. A point $P(3,5)$ is rotated through $90°$ anticlockwise about centre $C(1,1)$. Find its image.

A. $(-3,3)$.

B. $(-3,5)$.

C. $(-1,3)$.

D. $(3,-3)$.

Section B: Structured questions (50 marks)

Instructions: Answer all questions. Show all working clearly. Use graph paper where necessary.

Question 1: Basic concepts and properties of rotation (8 marks)

a) Define the term rotation. (2 marks)

b) State four properties of rotation. (4 marks)

c) A wheel makes three-quarters of a complete revolution.

Calculate the angle through which the wheel rotates. (2 marks)

Question 2: Rotation about the origin (8 marks)

Triangle ABC has vertices:

$$A(2,1), \quad B(4,1), \quad C(3,3)$$

a) Rotate triangle ABC through $90°$ anticlockwise about the origin. Determine the coordinates of A', B' and C'. (3 marks)

b) Rotate the original triangle through $180°$ about the origin. Determine the coordinates of the new image. (3 marks)

c) State two properties that are preserved in both rotations. (2 marks)

Question 3: Rotation about a different centre (8 marks)

Point $P(5,4)$ is rotated through $90°$ anticlockwise about centre $C(2,1)$.

a) State the coordinates of the centre of rotation. (1 mark)

b) Calculate the coordinates of the image P'. (5 marks)

c) Explain why the distance between C and P is equal to the distance between C and P'. (2 marks)

Question 4: Centre and angle of rotation (8 marks)

a) Point $A(3,5)$ is rotated through $180°$ to point $A'(-1,-3)$.

Determine the centre of rotation. (3 marks)

b) Point $B(2,1)$ is mapped to $B'(1,-2)$ by a rotation about the origin.

Determine the angle and direction of rotation. (3 marks)

c) State the construction used to find the centre of rotation when an object and its image are given. (2 marks)

Question 5: Rotational symmetry (9 marks)

a) Define rotational symmetry. (2 marks)

b) Complete the table below. (4 marks)

Plane figure	Order of rotational symmetry
Square	_____
Equilateral triangle	_____
Regular hexagon	_____
Non-square rectangle	_____

c) A regular polygon has an order of rotational symmetry of 12. Calculate its smallest angle of rotation. (2 marks)

d) State one plane figure that has infinite rotational symmetry. (1 mark)

Question 6: Application of rotation and graphical construction (9 marks)

A learner draws triangle XYZ on a Cartesian plane with vertices:

$$X(1,1), \quad Y(4,1), \quad Z(2,3)$$

The triangle is rotated through $90°$ anticlockwise about the origin.

a) Calculate the coordinates of X', Y' and Z'. (3 marks)

b) On graph paper, plot triangle XYZ and its image $X'Y'Z'$ on the same Cartesian plane. Join the vertices in the correct order. (4 marks)

c) State the centre and angle of rotation. (2 marks)

4.0 ANSWER KEY AND MARKING GUIDE

Section A: Multiple-choice answers

Question	Answer	Question	Answer	Question	Answer
1	B	11	B	21	D
2	C	12	C	22	B
3	D	13	C	23	B
4	D	14	C	24	C
5	C	15	C	25	C
6	C	16	C	26	C
7	C	17	C	27	B
8	C	18	A	28	B
9	B	19	C	29	B
10	A	20	C	30	A

Note on Question 30: The centre is $C(1,1)$. Relative to this centre, P is $(4,3)$. A $90°$ anticlockwise rotation gives $(-3,4)$; adding the centre gives $(-2,5)$. Therefore, none of the listed alternatives is correct. The question should be corrected as follows:

Corrected Question 30: Point $P(3,5)$ is rotated through $90°$ anticlockwise about centre $C(1,1)$. Find its image.

A. $(-3,3)$.

B. $(-3,5)$.

C. $(-1,3)$.

D. $(3,-3)$.

The correct image is $(-3,3)$, so the correct answer is **A**.

Section B: Suggested marking scheme

Question 1 (8 marks)

a) Rotation is a transformation in which a figure is turned about a fixed point through a given angle in a specified direction. (2 marks)

b) Any four:

- Preserves size. (1)
- Preserves shape. (1)
- Preserves lengths. (1)
- Preserves angles. (1)
- The centre remains fixed. (1)
- All points rotate through the same angle. (1)

Award a maximum of 4 marks.

$$\frac{3}{4} \times 360° = 270°$$

- Correct method (1)
- Correct answer $270°$ (1)

Question 2 (8 marks)

a) $90°$ anticlockwise:

$$(x, y) \rightarrow (-y, x)$$

$$A(2,1) \rightarrow A'(-1,2)$$

$$B(4,1) \rightarrow B'(-1,4)$$

$$C(3,3) \rightarrow C'(-3,3)$$

Award 1 mark for each correct image coordinate. (3 marks)

b) $180°$:

$$(x, y) \rightarrow (-x, -y)$$

$$A'(-2,-1)$$

$$B'(-4,-1)$$

$$C'(-3,-3)$$

Award 1 mark for each correct image coordinate. (3 marks)

c) Any two:

- Shape is preserved.
- Size is preserved.
- Corresponding lengths are equal.
- Corresponding angles are equal.

(2 marks)

Question 3 (8 marks)

$$C(2,1)$$

(1 mark)

b) Translate relative to the centre:

$$(5-2,\ 4-1)=(3,3)$$

Rotate $90°$ anticlockwise:

$$(3,3)\rightarrow(-3,3)$$

Add the centre coordinates:

$$(-3+2,\ 3+1)=(-1,4)$$

$$\boxed{P'(-1,4)}$$

Suggested allocation:

- Correct relative coordinates (2)
- Correct rotation rule and application (2)
- Correct final image coordinates (1)

c) Rotation preserves distance from the centre; therefore:

$$CP=CP'$$

(2 marks)

Question 4 (8 marks)

a) The centre is the midpoint of AA'.

$$C=\left(\frac{3+(-1)}{2},\frac{5+(-3)}{2}\right)$$

$$C=(1,1)$$

(3 marks)

$$(2,1)\rightarrow(1,-2)$$

This follows the rule:

$$(x,y)\rightarrow(y,-x)$$

Therefore, the rotation is $90°$ clockwise. (3 marks)

- Join corresponding points. (1)
- Construct perpendicular bisectors of two corresponding point segments; their intersection gives the centre. (1)

Question 5 (9 marks)

a) Rotational symmetry is the property of a plane figure that enables it to fit exactly onto its original outline after rotation through an angle greater than $0°$ and less than or equal to $360°$. (2 marks)

Plane figure	Order
Square	4
Equilateral triangle	3
Regular hexagon	6
Non-square rectangle	2

(4 marks)

$$\text{Smallest angle} = \frac{360°}{12}$$

$$= 30°$$

(2 marks)

d) Circle. (1 mark)

Question 6 (9 marks)

a) The rotation rule is:

$$(x, y) \rightarrow (-y, x)$$

For $X(1,1)$:

$$X'(-1,1)$$

For $Y(4,1)$:

$$Y'(-1,4)$$

For $Z(2,3)$:

$$Z'(-3,2)$$

Award 1 mark for each correct coordinate. (3 marks)

b) Graphical construction:

- Correct axes and suitable scale. (1)
- Correct plotting of original triangle. (1)
- Correct plotting of image. (1)
- Correct joining and labelling of corresponding vertices. (1)

(4 marks)

- Centre: Origin $(0,0)$. (1)
- Angle and direction: $90°$ anticlockwise. (1)

STRAND 2.0: MEASUREMENTS AND GEOMETRY

SUB-STRAND 2.5: SCALE DRAWING

The Scale

Scale is a ratio that represents the relationship between the dimensions of a model and the corresponding dimensions on the actual figure or object.

The figure below shows the relative positions of Mombasa, Nairobi and Nakuru. The distance between Mombasa and Nairobi on a straight line is 450 km and that between Nairobi and Nakuru is 142 km.

(i). Measure in centimetres the distance between:

(a) Mombasa and Nairobi.

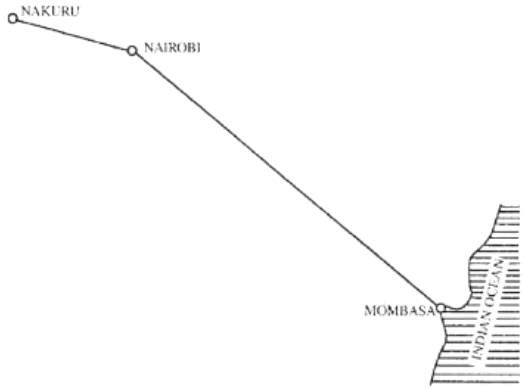
(b) Nairobi and Nakuru.

(ii). How many kilometres does 1 cm represent between:

(a) Mombasa and Nairobi?

(b) Nairobi and Nakuru?

You should notice that I cm on the map represents 50 km on the ground. Since 50 km is equal to 5 000 000 cm, this statement can be written in ratio form as 1: 5 000 000.



As a representative fraction (R.F.), 1: 5000 000 is written as $\frac{1}{5,000,000}$

- The ratio of the distance on a map to the actual distance on the ground is called ***the scale of the map***.

Example;

(1). The scale of a map is given in a statement as '1 cm represents 4 km.' Convert this ratio form.

Solution

1 cm represents 4 x 100 000 cm

Therefore, the ratio is 1: 400 000 and the R.F. is $\frac{1}{400,000}$

(2). The scale of a map is given as 1: 250 000. Write this as a statement.

Solution

1: 250 000 means 1 cm on the map represents 250 000 cm on the ground. km, i.e., 1 cm represents 2.5 km.

Therefore, 1 cm represents $\frac{250,000}{100,000}$

EXCERCISE

(1). On a map, 1 cm represents 4 kilometres. Re-write as a ratio.

(2). What distance on the ground is represented by 3.7 cm on the map if 1 cm represents 4 kilometres?

(3). Two towns A and B are 42.8 km apart on the ground. What is this distance on the map given that 1 cm represents 1.5 kilometres?

(4). A map is drawn to a scale of 1:50 000. Write this scale as a statement connecting map distance to ground distance.

(5). What is the actual distance if the distance on the map is 12.7 cm given that 1 cm represents 2.5 kilometres?

(6). On a map, 1 cm represents 5 kilometres. A railway line measures 8.3 km. What is its length on the map?

(7). The scale of a map is given in a statement as '1 cm represents 4.5 km.' Convert this to a representative fraction (R.F.).

(8). The scale of a map is given as 1: 350 000. Write this as a statement.

(9). A map is drawn to a scale of 1:500 000. What is the actual distance if the distance on the map is 14.7 cm?

(10). What is the distance on the map of Two towns P and Q which are 62.8 km apart on the ground given that 1 cm represents 2.5 kilometres?

Scale Diagrams

Scale Diagrams.

- A scale drawing is an enlargement of an object.
- An enlargement changes the sizes of an object by multiplying each of the lengths by a scale factor to make it larger or smaller.
- It is usually stated as a ratio.

Note:

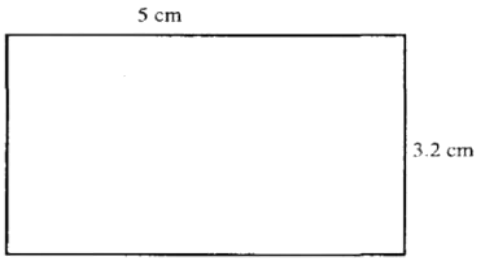
- One should be careful in choosing the right scale, so that the drawing fits on the paper without much detail being lost.

Example.

1. The length of a classroom is 10 metres and its width 6.4 metres. By scale drawing, represent this on a figure.

Solution

First look for a scale. Consider a scale of 1 cm to represent 2 m. Hence, the classroom will be 10/2 = 5



cm by 6.4/2 = 3.2 cm.

Assignment

(1). The length of a classroom is 10 metres and its width 6.4 metres. By scale drawing, represent this on a figure.

(2). A plot of land in form of a rectangle has dimensions 120 m by 180 m. Draw this on a paper.

(3). A rectangular field measures 40 m by 100 m. The length of the field on the map is 5 cm. What is the area of the field on the map?

(4). A rectangular field measures 40 m by 100 m. The length of the field on the map is 5 cm. Write the scale of the map as a representative fraction and hence the width of the field on the map?

(5). A plot of land in form of a rectangle measures 100 m by 80 m. Draw this on a paper.

(6). Two villages M and N are connected by a straight road 750 m long on a level ground. A third village L is 450 m from M and 650 m from N. Using a suitable scale, draw the diagram and find the shortest distance of L from the road.

(7). The length of a rectangular field on the map is 4 cm, If the rectangular field measures 50 m by 120 m write the scale of the map as a representative fraction and hence the width of the field on the map?

(8). Represent the dimensions of a plot of land in form of a rectangle measuring 150 m by 70 m. on a paper.

(9). Two Towns S and T are connected by a straight road 650 m long on a level ground. A third village R is 350 m from S and 550 m from T. Using a suitable scale, draw the diagram and find the shortest distance of R from the road.

(10). Using a suitable scale, draw the diagram of a rectangular plot of land measuring 6 km by 3 km.

Compass Bearing

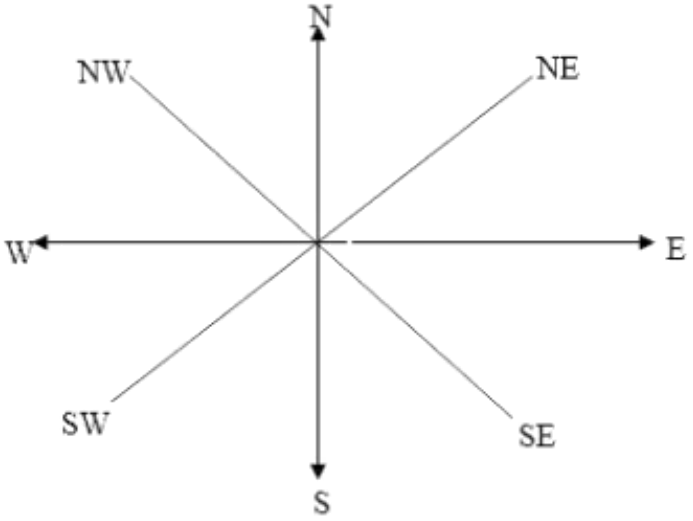
Bearing and Distance.

- **Bearing** is the angle measured in a clockwise or counterclockwise direction from the north direction or South direction to a given direction.

- **Distance** is the length between the two places. Bearing and distance can be used to locate the position of a given point from another reference point.

Points of the compass

- The figure below shows the eight points of the compass.
- The four main points of the compass are North (N), South (S), East (E) and West (W). The other four points shown are secondary and include the North East (NE), South East (SE), South West (SW) and North West (NW). Each angle formed at the centre of the compass of the eight directions is 45°. The angle between N and E is 90°.



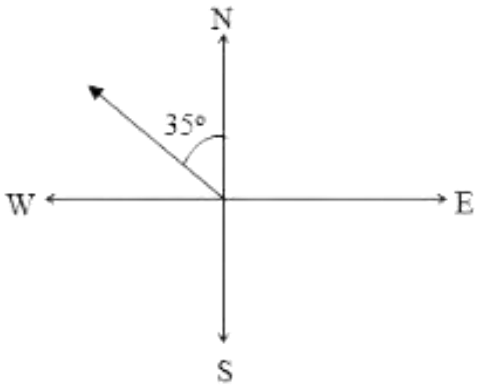
Compass Bearing

- It is measured in either clockwise direction or anti-clockwise direction from North or South and the angle is an acute angle. E.g. N 45° W, S 60° E

Example;

Draw a sketch to show the bearing marking the angle N35°W clearly.

Solution



N35°W means from N measure 35° toward the W or 35°W of N.

Assignment

(1). Three boys Isaac, Alex and Ken are standing in different parts of a football field. Isaac is 100 metres north of Alex and Ken is 120 metres east of Alex. Find the Compass bearing of Ken from Isaac.

(2). Kilo school is 12 kilometres from Sokomoko on a bearing of N 50°W. Tiba dispensary is 10 kilometres from Kilo on a bearing of S 60°E. Find the compass bearing of Sokomoko from Tiba.

(3). Survey posts R, Q and P are situated such that they form a triangle. If Q is on a bearing of S 60° W and 12 kilometres away and R is on a bearing of S 30°E and 8 kilometres away from P, find the compass bearing of Q from R.

(4). Kisumu and Nanyuki are situated in such a way that Nanyuki is on a bearing of N 75° E from Nakuru and Kisumu on a bearing of N 80° W from Nakuru. If Kisumu is 190 km and Nanyuki is 160 km from Nakuru,

Find:

a. The compass bearing of Kisumu from Nanyuki.

b. The distance of Kisumu from Nanyuki.

(5). From a meteorological weather station P on a plateau, a hill Q is 5 km on a bearing N 78° E and a railway station, R, is 1.5 km away on a bearing S20°W. Use scale drawing to find the compass bearing of Q from the railway station.

(6). A town P is 200 km West of Q. Town R is at a distance of 80 km on a bearing of N490E from P. Town S is due East of R and due North of Q. Determine the compass bearing of S from P.

(7). A route for safari rally has five sections AB, BC, CD, DE and EA. B is S 20° W km on a bearing N 50° E from A.C is 500km from B. The bearing of B from C is N 60° W. D is 400km on a bearing S 50° W from C. E is 250km on a bearing N 25° E from D. Using the scale 1cm for 50km draw the diagram representing the route for the rally. From the diagram determine:

a. The distance in km of A from E

b. The compass bearing of E from A.

(8). Manyatta village is 74 km North West of Nyangata village. Chamwe village is 42 km west of Nyangate. By using an appropriate scale drawing, find the compass bearing of Chamwe from Manyatta.

(9). Four towns R, T K and G are such that T is 84 km directly to the north of R, and K is on a distance of N 65° W from R at a distance of 60 km. G is on a bearing of N 20° W from K and a distance of 30 km. Using a scale of 1cm to represent 10km, make an accurate scale drawing to show the relative positions of the towns. Find the distance and the compass bearing of G from T.

(10). Using the scale: 1 cm represents 10 km, construct a diagram showing the positions of B, C, Q and D. Determines the distance between B and C and the compass bearing of D from B.

True Bearing

Bearing and Distance.

True Bearing.

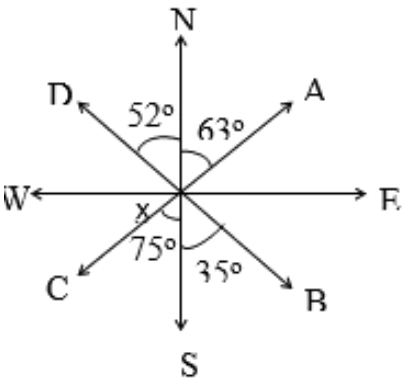
- It is also known as ***Three-figure bearings***.
- True bearing is given as the number of degrees from north, measured in a clockwise direction.
- Three-digit are always used for given angles but for angles less than 100° extra zero(s) is always added in front of the digits e.g. 008° , 060°, 070° up to 099°.

- The true bearings of due north is given as 000°. Due South East as 135° and due North West as 315°, etc. In figure 22.7, the true bearing of Q from P is 138°.

Example.

Find the three-figure bearings of A, B, C, and D from X.

Solution



(1). The arrow N shows the direction N, NXA = 63°. the bearing of A from X is 063°

(2). NXB = 180 - 35 = 145°. The bearing of B from X is 145°

(3). NXC clockwise = 180 + 75 = 255°. The bearing of C from X is 255° 4. NXD clockwise = 360 - 52 = 308°. The bearing of D from X is 308°.

Assignment

(1). A coastguard at a port observes two steamships approaching the harbour. The first ship P appears on a bearing 100° and the second ship Q on a bearing 020°. If the guard estimates the distances of the ships to be 120 km and 80 km respectively, find:

(a) the distance between ships P and Q

(b) the bearing of Q from P.

(2). A prison guard on a watchtower sees a bridge 120 m away on a bearing of 230° and a bus stop 80 m away on a bearing of 090°. Use scale drawing to find:

(a) The bearing of the bridge from the bus stop.

(b) The distance between the bus stop and the bridge

(3). From a point P, the bearing of a house is 060°. From a point Q 100m due east of P, the bearing is 330°. Draw a labelled sketch to show the positions of P, Q and the house.

(4). Three boys Isaac, Alex and Ken are standing in different parts of a football field. Isaac is 100 metres north of Alex and Ken is 120 metres east of Alex. Find the True bearing of Ken from Isaac.

(5). Kilo school is 12 kilometres from Sokomoko on a bearing of 320°. Tiba dispensary is 10 kilometres from Kilo on a bearing of 120°. Find the True bearing of Sokomoko from Tiba.

(6). Survey posts R, Q and P are situated such that they form a triangle. If Q is on a bearing of 210° and 12 kilometres away and R is on a bearing of 150° and 8 kilometres away from P, find the true bearing of Q from R.

(7). Kisumu and Nanyuki are situated in such a way that Nanyuki is on a bearing of 075° from Nakuru and Kisumu on a bearing of 280° from Nakuru. If Kisumu is 190 km and Nanyuki is 160 km from Nakuru, find:

(a) The bearing of Kisumu from Nanyuki.

(b) The distance of Kisumu from Nanyuki.

(8). Town A is on a bearing 050° from town C. Town B is on a bearing 020° from C. If B is 500 km from C and A is 500 km from B, find by scale drawing:

(a) The distance of A from C.

(b) The bearing of B form A.

(9). A route for safari rally has five sections AB, BC, CD, DE and EA. B is 200 km on a bearing 050° from A.C is 500km from B. The bearing of B from C is 300° . D is 400km on a bearing 230° from C. E is 250km on a bearing 025° from D. Using the scale 1cm for 50km draw the diagram representing the route for the rally. From the diagram determine

a. The distance in km of A from E

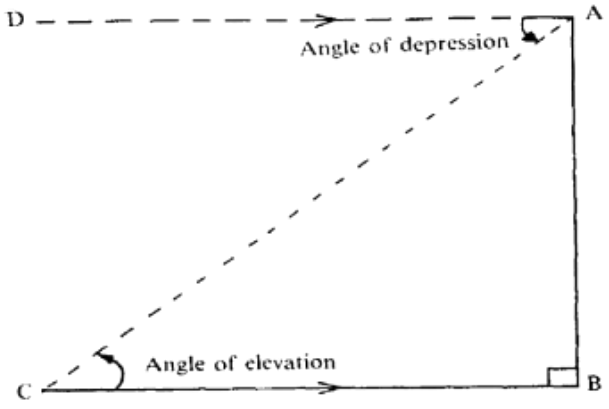
b. The bearing of E from A

(10). A town P is 200 km West of Q. Town R is at a distance of 80 km on a bearing of 049° from P. Town S is due East of R and due North of Q. Determine the bearing of S from P.

Angle of Elevation and Depression

Angle of Elevation and the Angle of Depression.

- The angle of elevation is the angle between the horizontal line of sight and the line of sight up to an object.
- The angle of depression is the angle between the horizontal line of sight and the line of sight down to an object.
- The angle of elevation is equal to the angle of depression.



- Angles of depression and elevation can be measured by use of an instrument called clinometer.
- A simple clinometer can be made from a cardboard in the shape of a protractor.

N/B:

- The angle of elevation increases as the observer moves towards the object and decreases as the observer moves away from the object.

Example.

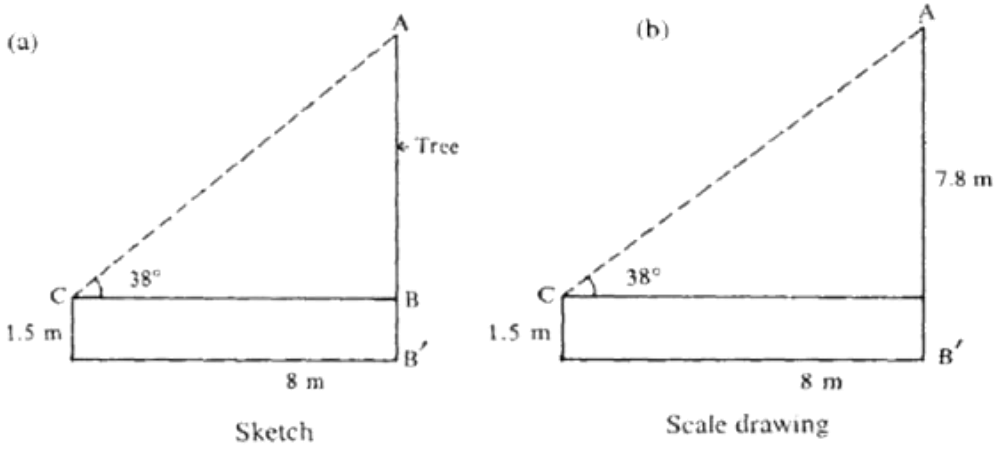
A boy 1.5 m tall and 8 m from a tree finds that the angle of elevation to the top of the tree is 38°. Find the height of the tree by scale drawing.

Solution

The figure below shows the sketch and scale drawing.

Using a scale of 1 cm to 2 m. The measurement of AB' on the scale is 3.9 cm.

(1). cm represents 2 m, therefore 3.9 cm represents (3.9 x 2) m = 7.8 m.



Assignment

(1). A boy 1.5 m tall and 8 m from a tree finds that the angle of elevation to the top of the tree is 38°. Find the height of the tree by scale drawing.

(2). The angle of elevation of the top of a flagpole from a point A, 14 metres away, is 36°. Use scale drawing to find the height of the flagpole.

(3). The angle of depression from the top of a cliff to a stationary boat is 48°. Find the horizontal distance by accurate drawing if the height of the cliff is 80 metres. Measure the angle of elevation of the top of the cliff from the boat. What to you notice?

(4). A building tower casts a shadow 33 metres away. The angle of elevation of the tower from the tip of the shadow is 21°. Find the height of the tower by scale drawing.

(5). The angle of elevation of a church tower from a point A, 50 metres away from the foot of the church, is 24°. Find the distance between A and B if the angle of elevation of the tower from B is 20°.

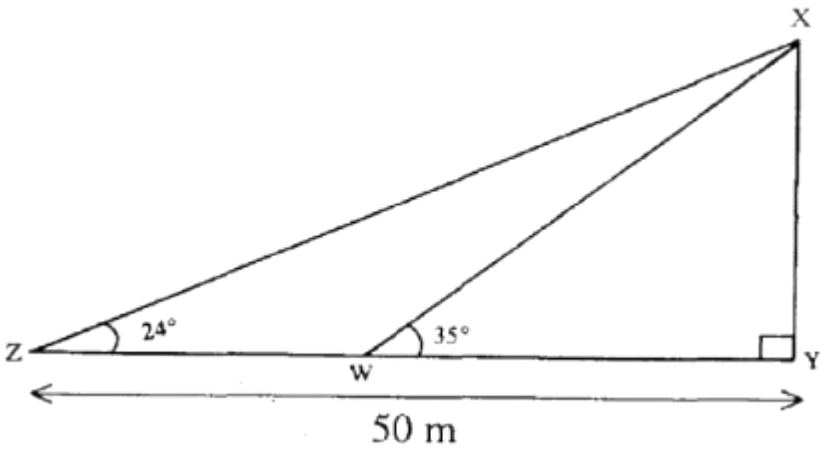
(6). From a viewing tower 15 metres above the ground, the angle of depression of an object on the ground is 30° and the angle of elevation of an aircraft vertically above the object is 42°. By choosing a suitable scale, find the height of the aircraft above the object.

(7). A soldier standing on top of a cliff 100 m high notices two enemy boats in line, whose angles of depression are 10 and 23°. Find, by scale drawing. the distance between the boats.

(8). The angles of elevation of an aircraft from two villages A and B 1 km apart are 67° and 53° respectively. Find the height of the aircraft in metres by scale drawing.

(9). The angle of elevation of a stationary hot air balloon 50 m above the ground from a man on the ground is 17°. The balloon moves vertically upwards so that the angle of elevation from the man is 30°. Find, by scale drawing, the distance the balloon moves. How far above the ground is the balloon?

(10). In figure below, Z is 50 m away from Y. Use scale drawing to find the distance from W to Y, given that the angle of elevation of X from Z and W are 24° and 35° respectively.



Survey A

Simple Survey Techniques.

- Surveying an area of land involves taking field measurements of the area so that a map of the area can be drawn to scale. Pieces of land are usually surveyed in order to fix boundaries (land adjudication) of land for different owners, for town planning, road construction, water supplies, mineral development, etc.
- There are two simple methods used in surveying.

1. Triangulation.
2. Survey of an area by use of Compass Bearing and Distances.

(a). Triangulation.

- This is a method in which the area to be surveyed is divided into convenient geometrical figures, or is covered by a suitable geometrical figure.

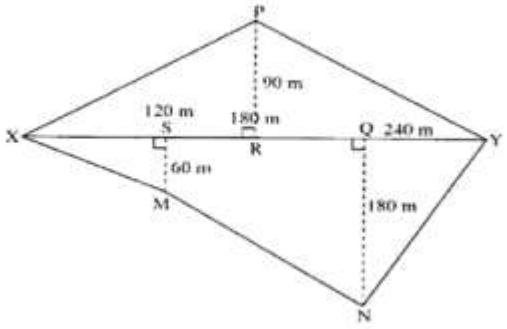
(b). Survey of an area by use of Compass Bearing and Distances.

- In this survey method, only bearings and distances from a chosen point are considered.

In this chapter, we will narrow down into Triangulation into details.

Example.

(1). A survey of a small island was drawn as shown below. Record the measurements shown in the figure in a tabular form in a field book.



Solution

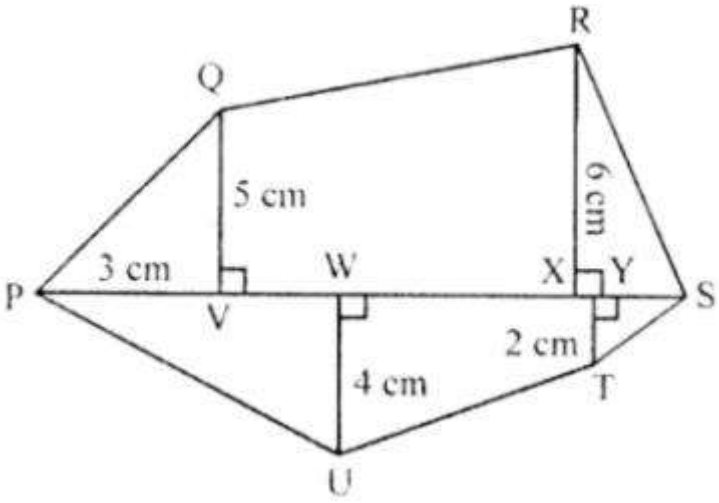
The line XY is called base lines of the survey. The lines perpendicular to the base lines and joining the points M, N, and P at the edge S, R and Q are called offsets.

The measurements are recorded in a field book in a tabular form as follows:

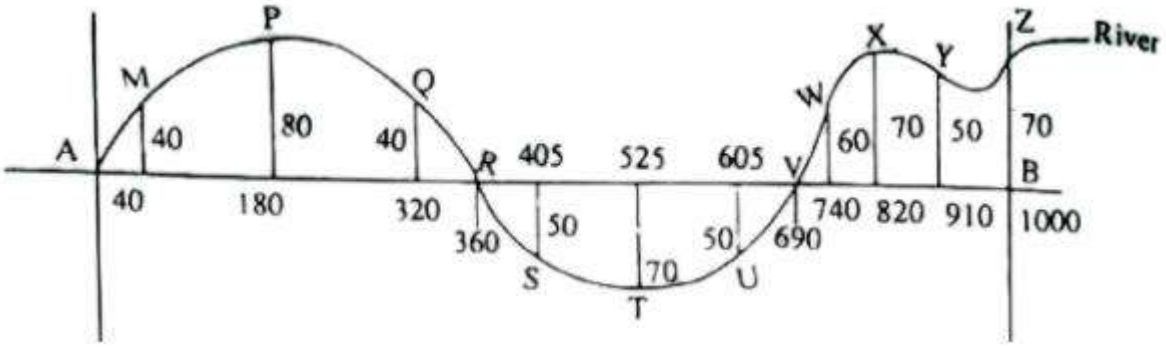
	Y	
	240	**180 to N**
To R90	**180**	
	120	**60 to M**
	X	

Assignment

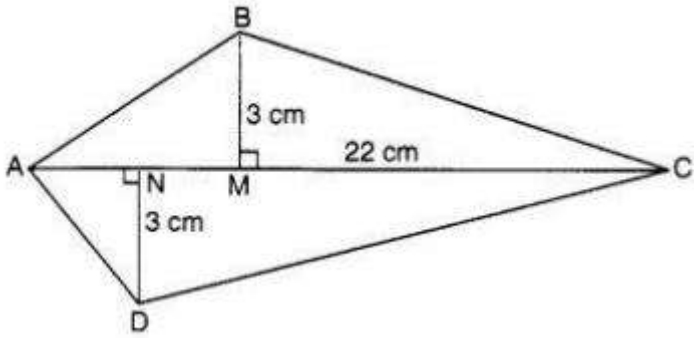
(1). A survey of a small island was made by using a triangle PQRSTU as shown in the sketch of figure below. Record the measurements shown in the figure in a tabular form in a field book.



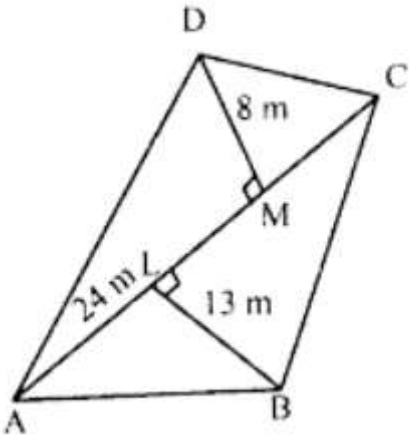
(2). A river can also be surveyed and its map made using base line AB as shown in the sketch of figure below. Record the measurements from the sketch in a field book.



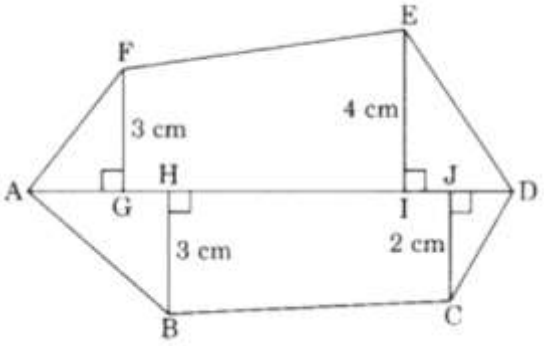
(3). A plot of land is draw as shown below. Record the measurements from the plot in a field book given that AN = NM = 2 cm. Use a scale of 1cm rep 25 m.



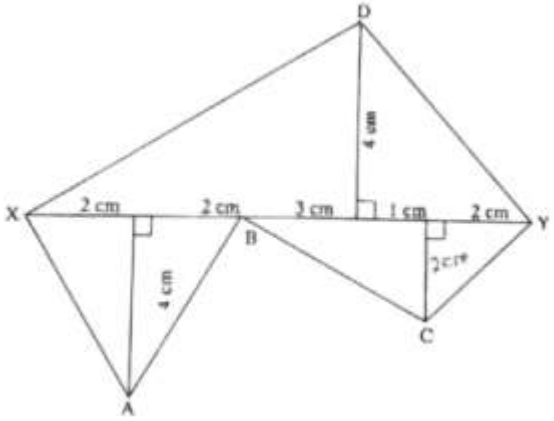
(4). A piece of land is drawn with a base line AC as shown below. Record the measurements from the plot in a field book given that LM = 12 m MC = 20 m.



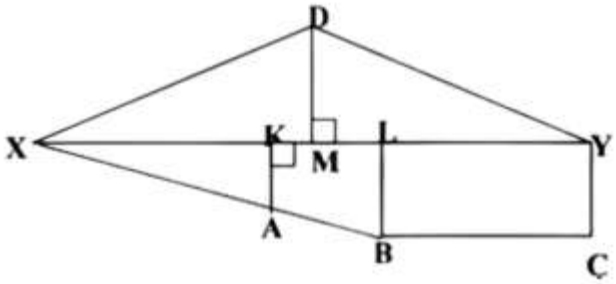
(5). A survey of a small island was drawn as shown below. Record the measurements shown in the figure in a tabular form in a field book given that baseline AD = 10 cm and AG = JD = 2.5 cm. Use a scale of 1cm rep 45 m.



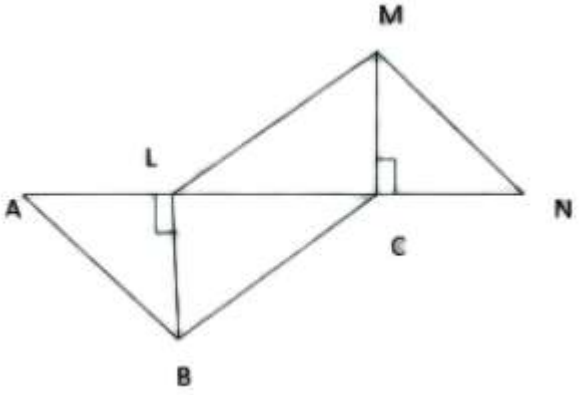
(6). Record the measurements shown in the figure in a tabular form in a field book for a survey of a small piece of land. Use a scale of 1cm rep 60m.



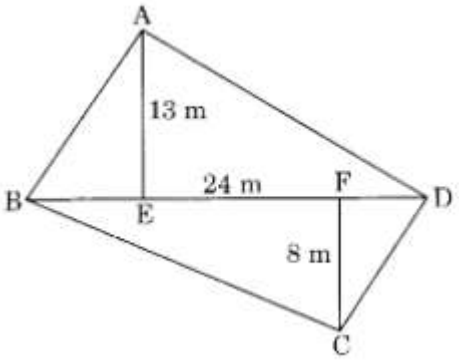
(7). The figure below represents a surveyor's sketch of a plot of land. Record the measurements from the sketch in a field book given that XY = 50m, XK = 20m, XM = 25m, XL = 35m, KA = 40m, MD = 38m and LB = YC = 60m.



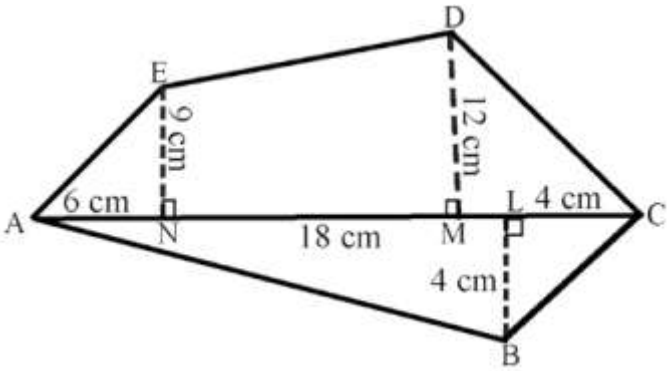
(8). A surveyor's sketch of a plot of land is as shown below. Record the measurements from the sketch in a field book given that AN = 10 cm, AL = 2 cm, LC = 2.5 cm, CN = 3.5 cm, LB = 4 cm, CM = 3.8 cm. use a scale of 1 cm rep 50 m.



(9). The figure below represents a surveyor's plot of land. Record the measurements from the plot in a field book.



(10). Using a scale of 1cm rep 20 m, record the measurements shown in the figure below in a field book for a survey of a small piece of land.



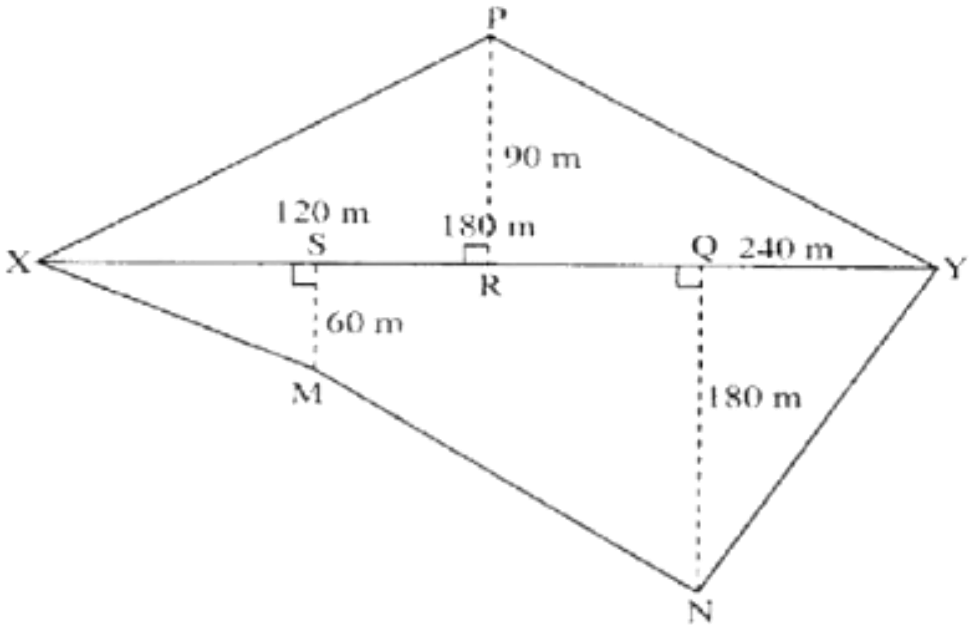
Survey

- Areas of pieces of land which have irregular shapes can be obtained by subdividing them into convenient geometrical shapes, e.g., triangles, rectangles or trapezia. This is done by the use of baseline and offsets of the area required.

Example.

(1). Find the area in hectares of a coffee field whose measurements are entered in a field book as follows. (Take XY = 360 m as the base line)

	Y	
	240	180 to N
To R90	180	
	120	60 to M
	X	



Solution

Area of:

- *Triangle XPR is* $^1/_2$ *x 180 x 90 m² = 8 100 m²*
- *Triangle PRY is* $^1/_2$ *x 180 x 90 m² = 8 100 m²*
- *Triangle XSM is* $^1/_2$ *x 120 x 60 m² = 3 600 m²*
- *Triangle QNY is* $^1/_2$ *x 120 x 180 m² = 10 800 m²*
- *Trapezium SQNM =* $^1/_2$ *(QN+SM) x SQ m²*
- $^1/_2$ *(180+60) x 120 m² = 14 400 m²*
- *Total area = 45 000 m²*
- *Therefore, the area of the field is 4.5 ha.*

Assignment

(1). Find the area in hectares of a coffee field whose measurements are entered in a field book as follows. (Take XY = 400 m as the base line).

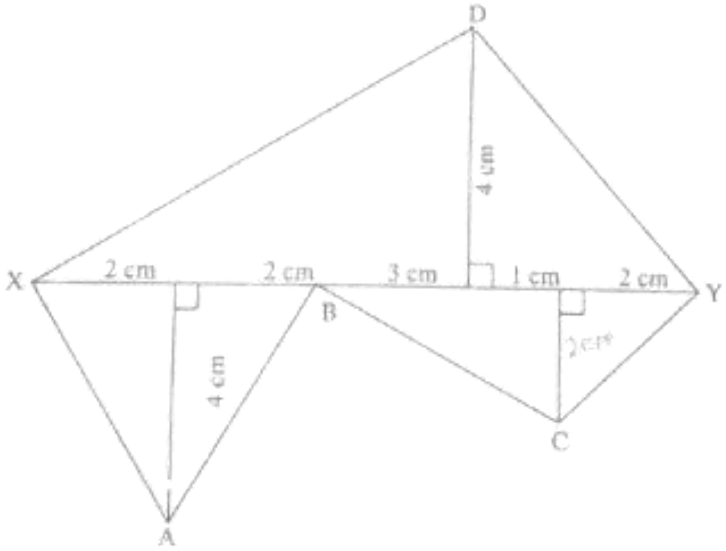
	Y	
	360	80 to Q
To R80	280	
To S160	200	
	80	200 to P
	X	

(2). A farm of land of measurements are shown in field book below. XY = 500 m. Find the area of the farm in hectares.

	Y	
To B100	450	
	300	20 to C
	120	70 to D
To A40	60	
	X	

(3). Use a scale of 1 cm to 40 m to draw the map of the coffee field.

(a). Find the area in hectares of a maize farm XABCYD in figure below which is drawn to a scale of 1 cm to 50 m.



(b). Taking XY as the base line and that the survey is from X to Y, enter the actual measurements of the farm in a field book.

(4). Measurements of a maize field using a base line XY were recorded as shown below. (Measurements are in metres)

Use a suitable scale to draw the map of the maize field hence find the area of the field in hectares.

		240	
	60	190	
		180	75 TO Q
		150	50 TO P
TO S	100	120	
		100	100 TO N
	30	50	
		20	20 TO M

(5). Find the area in hectares of farms whose measurements are shown in field book as in the tables below. AB = 600 m.

	B	
	550	120 to L
To K150	450	
	250	90 to M
To J60	40	
	A	

(6). A surveyor recorded the measurements of a small field in a field book using base lines AB = 75 cm,

	B		C		A
R9	55	W5	80	Z17	70
Q7	42	V6	70	Y5	50
P15	30	U7	60	X6	25
A	A	S10	20		C
			B		

as shown below, draw the map of the field.

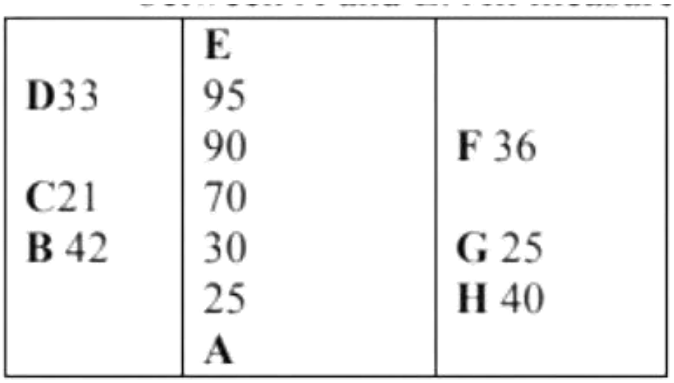
(7). Use a suitable scale to draw the map of the tea field hence find the area of the field in hectares. If a surveyor recorded the measurements of the tea field in a field book using base lines BC = 100 cm, as shown below.

	B		C		A
R9	55	W5	80	Z17	70
Q7	42	V6	70	Y5	50
P15	30	U7	60	X6	25
A	A	S10	20		C
			B		

(8). The table below gives a field book showing the results of a survey of a section of a piece of land between A and E. All measurements are in metres.

(a) Draw a sketch of the land.

(b) Calculate the area of this piece of land.



(9). A field was surveyed and its measurements recorded in a field book as shown below.

(a) Using a scale of 1cm to represent 10m, draw a map of the field.

(b) Calculate the area of the field in hectares.

	F	
	100	
E 40	80	
	60	D 50
C 40	40	
	20	B 30
	A	

(10). A tea farm in Kakamega forest was surveyed and the results were recorded in the surveyors note book as shown below. The measurements are in meters. Using a scale of 1: 25, draw the map of the plot and hence calculate the area of the plot in Hectares.

	250	**Y**
	240	D70
C80	170	
	70	B60
A60	50	
X	**0**	

SUMMARY TO SCALE DRAWING

1.0 INTRODUCTION TO SCALE DRAWING

Scale drawing is an important area of mathematics used in map reading, architecture, engineering, surveying, construction, town planning and many other fields.

In everyday life, people use drawings and maps to represent large areas or objects on smaller surfaces. For example:

- A map of Kenya represents the whole country on a small sheet of paper.
- An architect prepares a scaled plan of a house before construction.
- A surveyor uses scale drawings to represent land boundaries and distances.
- Engineers use scale drawings when designing roads, bridges and buildings.

- A school map can show the positions of classrooms, offices, laboratories, playgrounds and other facilities.

Scale drawing makes it possible to represent real distances and dimensions accurately on paper or a computer screen.

2.0 MEANING OF SCALE DRAWING

2.1 Definition of scale drawing

A **scale drawing** is a drawing of an object, place or area in which all dimensions are reduced or enlarged in the same proportion according to a specified scale.

The scale shows the relationship between a measurement on the drawing and the corresponding actual measurement.

For example, a drawing of a classroom may be made using a scale of 1 cm represents 2 metres. This means that every 1 cm measured on the drawing represents 2 m in the actual classroom.

Important terms used in scale drawing

Term	Meaning
Scale	The relationship between drawing measurements and actual measurements
Scale drawing	A drawing made using a specified scale
Actual distance	The real distance between two points
Drawing distance	The distance measured on a map or drawing
Linear scale	A scale showing the relationship between lengths
Statement scale	A scale expressed in words
Ratio scale	A scale expressed as a ratio or representative fraction
Representative fraction (RF)	The ratio of drawing distance to actual distance in the same units
Enlargement	Increasing the dimensions of an object proportionally
Reduction	Decreasing the dimensions of an object proportionally
Map scale	The scale used to represent actual geographical distances on a map

2.2 Why scale drawing is important

Scale drawing is useful because:

1. It represents large areas on small sheets of paper.
2. It allows accurate measurement of distances.
3. It helps architects plan buildings before construction.
4. It helps engineers design roads, bridges and other structures.
5. It enables surveyors to prepare land plans.
6. It helps learners understand maps and geographical locations.
7. It assists in estimating materials required for construction.
8. It makes it easier to compare dimensions of different objects.
9. It is useful in designing furniture, machines and equipment.

10. It supports proper planning and use of available land.

1. Town X is 13.5km from town Y on a bearing of 028°. A matatu leaves y at 7:35 a.m towards a bearing of 080°. The matatu is at point Z due south of X at 8:55a.m.

   a. Calculate the average speed of the matatu from Y to Z

   b. If the matatu continues on the same bearing, calculate the distance it covers from Z when it is East of X.

2. Three towns X, Y and Z are such that Y is 500km on a bearing of 315° from X. Z is on a bearing of 230° from X. given that the distance between Y and Z is 800km.

   a. using a scale of 1cm to represent 100km, draw a scale diagram to show the position of the Towns

   b. Find the bearing of;

      i. X from Z

      ii. Z from Y

   c. Use the scale drawing to find the distance from X to Z

3. Two aeroplanes S and R leave an airport at the same time. S flies on the bearing of 240° at 750Km/h while R flies due East at 600Km/hr.

   a.

      i. Calculate the distance of each aeroplane after 30minutes

      ii. Using a scale of 1cm to represent 50km make an accurate scale drawing to show the positions of the aeroplanes after 30minutes

   b.

      i. Use the scale drawing to find the distance between the two aeroplanes after 30minutes

      ii. If each aeroplane landed after 30minutes and S received a signal to join R in 45minutes. Find its speed.

   c. Determine the bearing of:

      i. S from R

      ii. R from S

4. The table below gives a field book showing the results of a survey of a section of a piece of l measurements are in metres.

	E	
D33	95	
	90	F 36
C21	70	
B 42	30	G 25
	25	H 40
	A	

a. Draw a sketch of the land.

b. Calculate the area of this piece of land.

5. Three towns A B and C are situated such that town A is 40km from B on a bearing of 280°. C is 60km from B on a bearing of 130°. Another town D is only 10km from C on a bearing of 210°.

a. Drawing accurately and using a scale of 1cm to 10km find the distance from A to C and the bearing of A from C.

b.

i. Distance of B from D

ii. Distance of A from D

iii. Bearing of A from D

iv. Bearing of C from D

6. A train left Naivasha for Nakuru at 1000hours. It traveled at an average speed of 45km/h and reached Gilgil after 40minutes. It then covered the remaining 50km in 1½ hours. A second train left Nakuru for Naivasha at 1015 hours and arrived at Gilgil at the same time as the first train arrived at Nakuru.

a. Using a scale of 1cm to represent 10minutes in the time axis and 1cm to represent 10km on the distance axis, draw on the same axes the graphs to show the movement of the two trains.

b. use your graph to find;

i. the distance between Naivasha and Nakuru

ii. the time at which the train met

c. calculate the average speed, in km/h of the second train

7. On a certain map, a road 20km long is represented by a line 4cm long. Calculate the area of a rectangular plot represented by dimensions 2.4cm by 1.5cm on this map – leaving your answer in hectares.

8. A port B is on a bearings of 080° from a port A and at a distance of 95km. A submarine is stationed at a port D, which is on a bearing of 200° from A, and a distance of 124km from B. A ship leaves B and moves directly southwards to an island P, which is on a bearing of 140° from A. The submarine at D on realizing that the ship was heading for the island P, decides to head straight for the island to intercept the ship.

   a. Using a scale of 1cm to represent 10km draw a diagram to show the positions of A,B,D, and P

   b. Hence;
   Determine;

      i. the distance from A to D

      ii. the bearing of the submarine from the ship when the ship was setting off from B

      iii. the bearing of the island P from D

      iv. the distance the submarine had to cover to reach the island P

9. Use a scale of 1cm represents 50km in these questions. Five towns A, B, C, D and E are situated such that A is 200 km from B on a bearing of 050° from E. C is 300 km from B on a bearing of 150° from B. D is 350km on a bearing of 240° from C. E is 200km from D and the bearing of D from E is 100°

   a. Draw the diagram representing the positions of the towns

   b. From the diagram, determine;

      i. The distance in km of A from E

      ii. The bearing of D from B

10. Four towns P, Q, R & S are such that P is 280 km North of R, S is 190 km from R on a bearing of 310° and Q is 240 km from P on a bearing of 105°.

   a. Using scale of 1 cm rep. 50 km, locate the four towns.

   b. Find;

      i. distance SQ.

      ii. Bearing of S from Q.

      iii. The shortest distance between P and side QR.

11. Four ships are at sea such that a streamliner S is 150km on a bearing of 025° from a cargo ship C. A trawler T is 300km on a bearing of 145° from the cargo ship and a yacht Y is due West of C and on a bearing of 300° from T.

    a. Using a scale of 1cm= 50km, draw on accurate scale drawing showing the positions of S, C, T and Y

    b. By measurement from your scale drawing determine:

        i. The distance and bearing of Y from S

        ii. The distance ST

        iii. The distance YT

12. A tea farm in Kakamega forest was surveyed and the results were recorded in the surveyors note book as shown below. The measurements are in meters.

	250	Y
	240	D70
C80	170	
	70	B60
A60	50	
X	0	

Using a scale of 1:25, draw the map of the plot and hence calculate the area of the plot in Hectares.

13. The information below shows the entries in a surveyor's field book after a survey of a farm. XY = 280m is the baseline. All measurements are in metres.

	280	Y
B 105	230	110E
	190	
	160	45E
A 100	90	
	40	95G
X	O	

    a. Use a scale of 1cm represents 20m to draw the map of the farm

    b. Estimate the area of the farm in hectares

    c. If the point Y lies due north of X, find correct to 1 decimal place, the :

        i. Bearing of E from X

        ii. Distance of E from X

14. nts of a flower garden were recorded in a surveyor's field book as shown.

	250	Y
C80	240 170 70	D 70 B 60
X	0	

Draw a sketch of the field and find its area. (Measurements are in m)

15. A map has a scale 1:40,000:

a. Calculate the distance between two points on the ground if the corresponding distance shown on the map is 3.25cm

b. Calculate the area in the map of woodland which occupies 36ha on the ground

16. Three scouts John, Peter and Samwel stand on three adjacent peaks of equal altitude on mountain range. The distance between John and Peter is 800metres and the bearing of Peter from John is 020°. The distance between John and Samwel is 1500metres, and the bearing of Samwel from John is 320°.

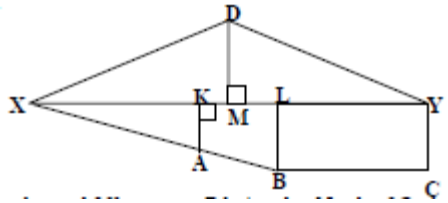
a. Calculate the bearing of John from Peter

b. Calculate:-

i. the distance

ii. the bearing of Samwel from Peter

17. The figure below represents a surveyor's sketch of a plot of land. Calculate the area of the plot in square metres given that XY = 50m, XK = 20m, XM = 25m, XL = 35m, KA = 40m, MD = 38m and LB = YC = 60m.



18. Two boats P and Q are located 30km apart; P being due North of Q. An observer at P spots a ship whose bearing he finds as S 56°E from Q, the bearing of the same ship is 038°. Calculate the distance of the ship from Q to 2 decimal places

19. A map is drawn to scale of 1:100,000. What area in km², is represented by a rectangle measuring 4.5cm by 5.4 cm

20. Town B is 180 km on a bearing of 050° from town A. Another town C is on a bearing of 110° from town A and on a bearing of 150° from town B. A fourth town D is 240 km on a bearing of 320° from A. Without using a scale drawing, calculate to the nearest kilometer.

a. The distance AC

b. The distance CD

Answers

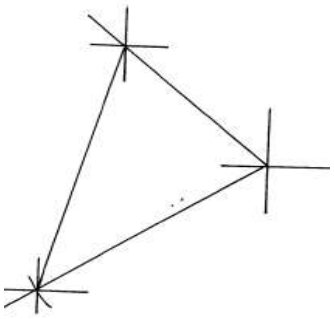
1.

a. $^{YZ}/_{\text{Sin } 28^o} = {}^{13.5}/_{\text{sin } 100^o}$
Duration of travel = 8:55am - 7:35am
$= {}^{4}/_{3}$
Speed = $6.436 \div {}^{4}/_{3}$
= 4.827 km/hr

b. $^{13.5}/_{\text{Sin } 10^o} = {}^{(6.436 + ZQ)}/_{\text{Sin } 118^o}$
Sin 118o
$6.436 + ZQ = 13.5 \times \sin 118^o = 68.659$
$ZQ = 68.659 - 6.436$
= 62.223

2.

a.



b.

i. 049 ± 1

ii. 190 ± 1

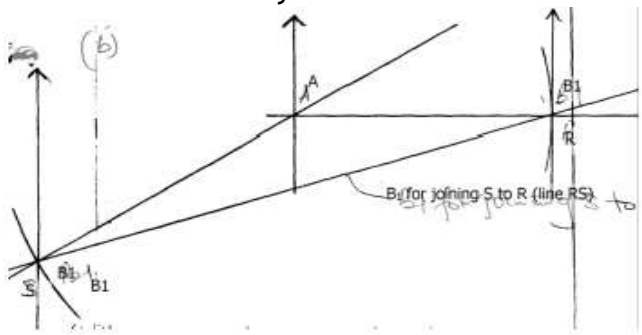
c. 6.7 ± 0.1
670 ± 10

3.

a.

i. Distance covered by s
= (750 x ½)km = 375 km

Distance covered by R



b.

i. Distance between the two aeroplanes
= 12.5 x 50 = 625 ± 5 km

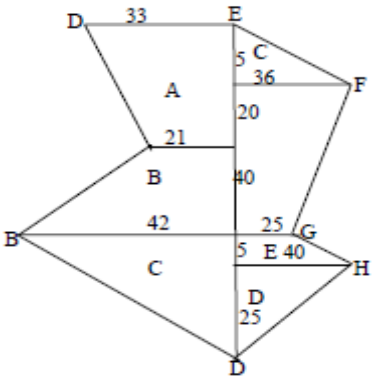
ii. Speed = ($^{625}/_{45}$ x 60) km/hr
$833^{1}/_{3}$ km/h

c.

i. Bearing of S from R = 225°

ii. The bearing of R from S = 72°

4.



Area A: ½ x 25 (33 + 21) = 675
Area B: ½ x 40 (21 x 42) = 1260
Area C: ½ x 30 x 42 = 630
Area D: ½ x 25 x 40 = 500
Area E: ½ x 5 (40 + 25) = 162.5
Area F: ½ X 60 (25 + 36) = 1830
Area G: ½ x 5 x 36 = 90 ∫
= 5,147.5m^2

5.

a. A to C = 96 ± 1 km
Bearing = 300°

b.

i. 62 ± 1km

ii. 97 ± 1 km

iii. 304°

iv. 030°

6.

a. Graph

b.

i. 80 km

ii. 11.06a.m

c. Average speed of the 2nd train
Time taken = $80 \div 1^{11}/_{12}$
$= 80 \times {}^{12}/_{23}$
= 41.74km/h

7. L.S.F = $^{4}/_{2000000} = {}^{1}/_{500000}$
A.S.F = $^{1}/_{5 \times 10^{5}} = {}^{1}/_{2.5 \times 10^{11}}$
Area of rectangle = $(2.4 \times 1.5)\ cm^2$
$= 3.6cm^2$
Actual area
$= \frac{3.6 \times 2.5 \times 10^{11}}{100 \times 10000}$ ha
$= 9 \times 10^{5}$
= 900,000ha

8.

a. Δ ABD correctly constructed
Δ ABP

b.

i. AD = 4.5 + 0.1cm
Distance A + D
= 4.5 X 10 = 45km

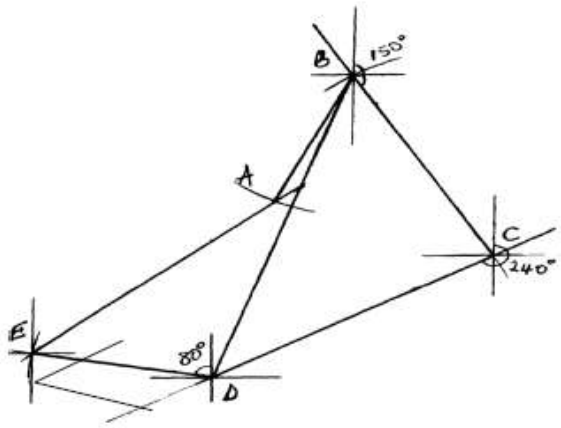
ii. Bearing of (i) from B
= 241 + 1

iii. Bearing P from D
= 123 = 2

iv. Dp = 12.9 + 0.2 am
Distance D + P = 12.9 X 10
= 129 km

9.

a.



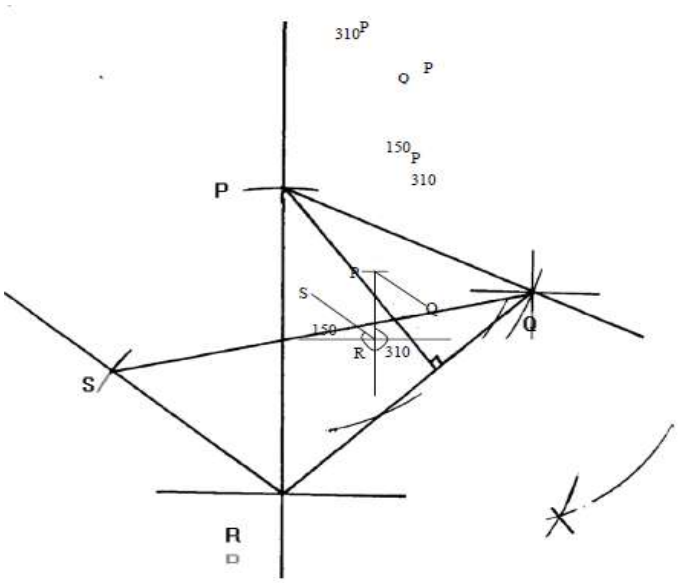
b.

i. 6.8 + 0.1cm
Distance Ae = 340 + 5 km

ii. 180 + 18 = 198 + 2

10.

a.



b.

i. SP = 7.8 x 50 = 390 km ± 5 km

ii. S & Q = 255° ± 1°

iii. 4 x 50 = 200 km + 5 km

11.

a. Scale = 50km
Drawing accurately<NCE = 25°
<NCT = 145°
<NTY = 90°
Lines drawn //

b. By measurement:

i. Distance SY = 6.9 x 50 = 345 ± 5km
Bearing Y For S = 360° - 114 = 246±1°

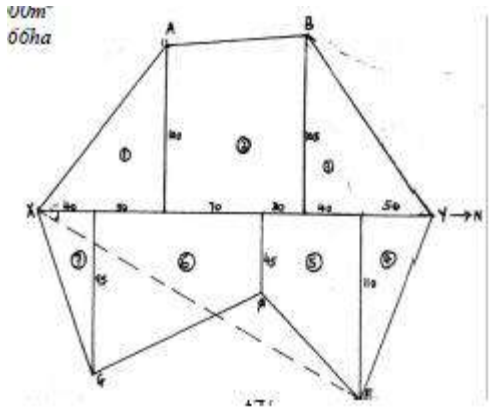
ii. distance ST = 7.9 x 50 = 39.5 ± 5km

iii. distance YT = 9.8 x 50 = 490 ± 5km

12. Area of A = ½ x 50 x 60 = 1500m^2
B = ½ x 70 x 60 = 2100m^2
C = ½ (60 +80) x 120=11050m^2
D = ½ x 80 x 80 = 3200m^2
F = ½ x 10 x 70 = 350m^2
Total area = 26600m^2
Ha = $^{26600}/_{10000}$ = 2.66ha

13.

a.



b.

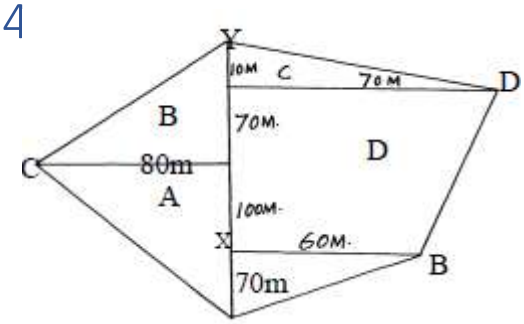
Total area = area (1) + (2) + (3) + (4) +(5) + (6) + (7)
Area (1)= ½ x 90 x 100 = 4500m^2
(2) = $\frac{(100 + 105)}{2}$10 = 10250m^2
(3) = ½ x 90 x 105 = 4725m^2
(4) = ½ x 50 x 110 = 2750m^2
(5) = ½ x (110 + 45)70 = 5425m^2
(6) = $\frac{(45 + 95)}{}$ 120 = 8400m^2
(7) = ½ x 40 x 95 = 1900m^2
Total area = 37,950m^2
In hectares = $^{37950}/_{10000}$ ha = 3.795ha

c.

i. bearing of E from x is 0.25 ± 1°

ii. Distance Ex = (12.8 x 0.1 x 20m) = 256 ± 2m

14



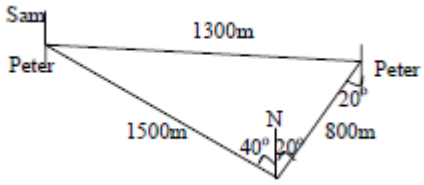
Area A = ½ x 170 x 80 = 6800
B = ½ x 80 x 80 = 3200
C = ½ x 10 x 70 = 350
D = ½ x 170 x 130 = 11050
E = ½ x 70 x 60 = 2100
Total = 23,500 m2

15.

a. $^{L.s.f}/_{40,000} = 1$
$^{1}/_{40,000} = {}^{3.25}/_{x}$
x = 130,000cm

b. A.s.f = $(^{1}/_{40000})^2$
$(^{1}/_{40000})^2 = {}^{x}/_{36000000}$
x = 0.0225cm^2

16.



a. bearing = 180 + 20 = 200°

b. a^2 = 1500 +
$a^2 = b^2 + c^2 - 2bc \cos A$
$a^2 = 1500^2 + 800^2 - 2 \times 1500 \times 800 \cos 60$
$= 2250000 + 640000 - 1200000$
$= 1690000$
a = 1300m

c. $^{1300}/_{\text{Sin } 60} = {}^{1500}/_{\text{sin c}}$

1300 sin c = 1500 sin 60

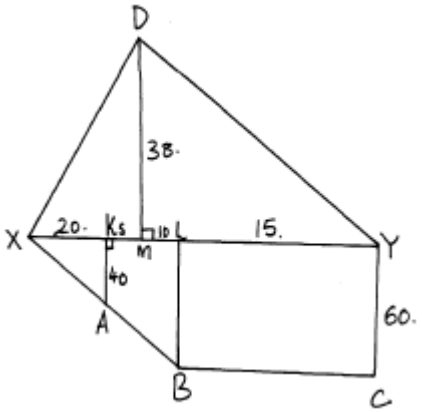
Sin c = $\frac{1500 \sin 60}{1300}$

= 0.9993

c = 87.79°

c = 87.80°

17.



A of Δ XYD = ½ x 50 x 38 = 950m^2

A of XBCY = ½ (50 + 15) 60

= ½ x 65 x 60

= 1950m^2

Total A = (950 + 1950)m^2

= 2900m^2

18. B1 for 86°

$^{30}/_{\text{Sin } 86^\circ} = {}^{QS}/_{\text{Sin } 56^\circ}$

QS = $\frac{30 \sin 56^\circ}{\text{Sin } 86^\circ}$

= 24.93km

19. 1cm for 100000cm

1cm^2 = (100000cm)^2

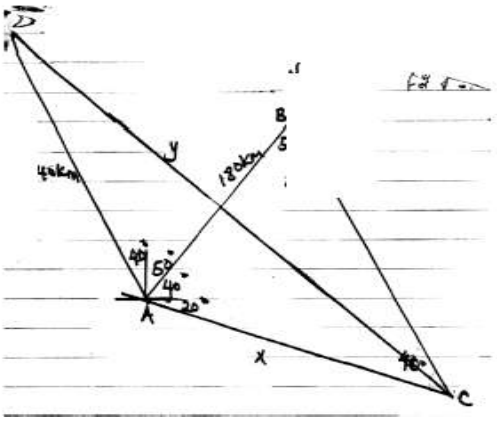
Area = 5.4 x 4.5 x 100000 cm^2

= $\frac{5.4 \times 4.5 \times 100000 \times 100000 \text{Km}^2}{100000 \times 100000}$

= 24.3km^2

20.

a.



For ✓ sketch
For ✓ exp. of x
For ✓ ans.
For ✓ Sub.
✓ cos 150
For taking sq. root.
For exp. of BC

Use sine rule in ΔABC

$^{X}/_{\text{Sin } 80^{o}} = {}^{180}/_{\text{Sin } 40^{o}} \Rightarrow x = {}^{180 \sin 80o}/_{\sin 40^{o}}$

= 275.8

Hence AC = 276 km

b. Use the cosine rule in Δ AD when < DAC = 150^{o}

$y^2 = 240^2 + 276^2 - 2 \times 240 \times 276 \cos 150^{o}$

= 576000 + 76180 – 132 480 (-cos 30^{o})

= 133776 + 114731 = 248507

y = √248507

= 498.5

Hence CD = 499 km

c. Using sine rule in ΔABC we have

$\frac{BC}{\text{Sin } 60^{o}} = \frac{180}{\sin 40^{o}}$

d. BC =

$\frac{180 \sin 60}{\text{Sin } 40}$

= 242.5

= 243 km

3.0 LINEAR SCALE

3.1 Definition of linear scale

A **linear scale** is a scale that expresses the relationship between lengths on a drawing or map and their corresponding actual lengths.

It is commonly used to represent distances on maps, building plans and engineering drawings.

For example:

1 cm represents 5 km

This means that a distance of 1 cm on the map represents an actual distance of 5 km on the ground.

There are three common ways of expressing a linear scale:

1. Statement form.
2. Ratio form.
3. Linear or graphical scale bar.

3.2 Statement form of a scale

Definition

A **statement scale** is a scale expressed in words, showing the relationship between a unit of measurement on a drawing and the corresponding actual distance.

Examples:

- 1 cm represents 1 km.
- 1 cm represents 5 km.
- 2 cm represents 10 km.
- 1 inch represents 2 miles.

In Kenya, map distances are commonly expressed in centimetres, while actual ground distances may be expressed in metres or kilometres.

Example 1

A map of a region uses a scale of:

$$1 \text{ cm represents } 4 \text{ km}$$

Interpret this statement.

Solution

The statement means that every 1 cm measured on the map represents 4 km on the ground.

Therefore:

- 2 cm represents 8 km.
- 3 cm represents 12 km.
- 5 cm represents 20 km.
- 10 cm represents 40 km.

Example 2

A school compound is represented using a scale of 1 cm represents 3 metres.

What actual distance does 7 cm represent?

Solution

Given:

$$1 \text{ cm} = 3 \text{ m}$$

Therefore:

$$7 \text{ cm} = 7 \times 3 \text{ m}$$

$$= 21 \text{ m}$$

Answer: 21 metres.

3.3 Ratio form of a scale

Definition

A **ratio scale** is a scale expressed as a ratio between a distance on a drawing and the corresponding actual distance, with both distances expressed in the same units.

It is also known as a representative fraction (RF).

The ratio scale is written as:

$$\boxed{1:n}$$

Where:

- 1 represents one unit on the drawing.
- n represents the corresponding number of the same units on the ground.

For example:

$$1:100\,000$$

This means that:

- 1 cm on the map represents 100,000 cm on the ground.
- 1 mm on the map represents 100,000 mm on the ground.
- 1 metre on the drawing represents 100,000 metres on the ground.

Important rule

The two quantities in a ratio scale must be expressed in the same units before the ratio is simplified.

For example, it is incorrect to write:

$$1\text{ cm}:5\text{ km} = 1:5$$

The units must first be converted to the same unit.

4.0 CONVERSION OF LINEAR SCALES

4.1 Converting statement scale to ratio scale

To convert a statement scale into ratio form:

1. Write the statement as a ratio.
2. Convert both measurements into the same unit.
3. Simplify the ratio.
4. Express the scale in the form $1:n$, where possible.

Worked Example 3

Convert the following scale into ratio form:

$$1\text{ cm represents }2\text{ km}$$

Step 1: Write the scale as a ratio.

$$1\text{ cm}:2\text{ km}$$

Step 2: Convert kilometres into centimetres.

We know that:

$$1 \text{ km} = 1\,000 \text{ m}$$

and:

$$1 \text{ m} = 100 \text{ cm}$$

Therefore:

$$1 \text{ km} = 100\,000 \text{ cm}$$

Thus:

$$2 \text{ km} = 200\,000 \text{ cm}$$

Step 3: Write the ratio using the same units.

$$1 \text{ cm}: 200\,000 \text{ cm}$$

Step 4: Simplify.

$$\boxed{1:200\,000}$$

Answer: $1:200\,000$.

Worked Example 4

Convert 1 cm represents 500 metres into ratio form.

Solution

Given:

$$1 \text{ cm}: 500 \text{ m}$$

Convert 500 m into centimetres:

$$500 \times 100 = 50\,000 \text{ cm}$$

Therefore:

$$1:50\,000$$

Answer: $1:50\,000$.

Worked Example 5

Convert 2 cm represents 8 km into ratio form.

Solution

Given:

$$2 \text{ cm}: 8 \text{ km}$$

Convert 8 km into centimetres:

$$8 \times 100\,000 = 800\,000 \text{ cm}$$

Therefore:

$$2:800\,000$$

Divide both terms by 2:

$$\boxed{1:400\,000}$$

Answer: $1:400\,000$.

Worked Example 6

Convert 5 cm represents 250 metres into ratio form.

Solution

Convert 250 metres into centimetres:

$$250 \times 100 = 25\,000 \text{ cm}$$

The ratio is:

$$5:25\,000$$

Divide both terms by 5:

$$\boxed{1:5\,000}$$

Answer: $1:5\,000$.

4.2 Converting ratio scale to statement scale

To convert a ratio scale into statement form:

1. Read the ratio carefully.
2. Identify the drawing measurement.
3. Identify the corresponding actual measurement.
4. Convert the actual measurement into a suitable unit.
5. Express the relationship in words.

Worked Example 7

Convert the scale $1:100\,000$ into statement form using kilometres.

Solution

The ratio means:

$$1 \text{ cm}: 100\,000 \text{ cm}$$

Convert 100,000 cm into kilometres.

$$100\,000 \div 100 = 1\,000 \text{ m}$$

$$1\,000 \div 1\,000 = 1 \text{ km}$$

Therefore:

$$\boxed{1 \text{ cm represents } 1 \text{ km}}$$

Worked Example 8

Convert the scale $1:250\,000$ into statement form.

Solution

$$1 \text{ cm}: 250\,000 \text{ cm}$$

Convert into kilometres:

$$250\,000 \div 100 = 2\,500 \text{ m}$$

$$2\,500 \div 1\,000 = 2.5 \text{ km}$$

Answer:

$$\boxed{1 \text{ cm represents } 2.5 \text{ km}}$$

Worked Example 9

Convert the scale $1:50\,000$ into statement form using metres.

Solution

$$1 \text{ cm}: 50\,000 \text{ cm}$$

Convert to metres:

$$50\,000 \div 100 = 500 \text{ m}$$

Answer:

$$\boxed{1 \text{ cm represents } 500 \text{ m}}$$

4.3 Summary of unit conversions used in scale drawing

Unit conversion	Equivalent
1 kilometre	1,000 metres
1 kilometre	100,000 centimetres
1 metre	100 centimetres
1 centimetre	10 millimetres
1 metre	1,000 millimetres
1 kilometre	1,000,000 millimetres

Remember these three steps when converting a scale:

1. Write the drawing distance and actual distance as a ratio.
2. Convert both distances to the same unit.
3. Simplify the ratio or express it in words.

5.0 USING LINEAR SCALES TO CALCULATE DISTANCES

A scale can be used to calculate:

1. The actual ground distance from a map distance.
2. The map distance from an actual ground distance.
3. The scale of a map when both distances are known.

5.1 Calculating actual distance

Formula

$$\boxed{\text{Actual distance} = \text{Map distance} \times \text{Scale factor}}$$

When using a ratio scale $1:n$:

$$\boxed{\text{Actual distance} = \text{Map distance} \times n}$$

The map distance and actual distance must be expressed in the same units before multiplication.

Worked Example 10

A map of Nakuru County has a scale of $1:100\,000$. The distance between Nakuru town and a nearby location on the map is 6 cm. Calculate the actual distance.

Solution

Given:

- Scale = $1:100\,000$
- Map distance = 6 cm

$$\text{Actual distance} = 6 \times 100\,000$$

$$= 600\,000 \text{ cm}$$

Convert centimetres to kilometres:

$$600\,000 \div 100 = 6\,000 \text{ m}$$

$$6\,000 \div 1\,000 = 6 \text{ km}$$

Answer

The actual distance is 6 km.

Worked Example 11

A map has a scale of $1:250\,000$. The distance between two towns is 8 cm on the map. Calculate the actual distance in kilometres.

Solution

$$\text{Actual distance} = 8 \times 250\,000$$

$$= 2\,000\,000 \text{ cm}$$

Convert to kilometres:

$$2\,000\,000 \div 100\,000 = 20 \text{ km}$$

Answer: 20 km.

Worked Example 12

A map has a scale of 1 cm represents 4 km. The distance between two trading centres is 5.5 cm. Find the actual distance.

Solution

$$1 \text{ cm} = 4 \text{ km}$$

$$5.5 \text{ cm} = 5.5 \times 4$$

$$= 22 \text{ km}$$

Answer: 22 km.

5.2 Calculating map distance

Sometimes the actual distance is known, but the map distance is required.

Formula

$$\boxed{\text{Map distance} = \frac{\text{Actual distance}}{\text{Scale factor}}}$$

For a ratio scale $1:n$:

$$\boxed{\text{Map distance} = \frac{\text{Actual distance}}{n}}$$

Ensure that the actual distance is first converted into the same unit as the map distance.

Worked Example 13

The actual distance between two towns is 15 km. A map uses a scale of $1:300\,000$. Calculate the distance between the towns on the map in centimetres.

Solution

Convert 15 km into centimetres:

$$15 \times 100\,000 = 1\,500\,000 \text{ cm}$$

Using the scale:

$$\text{Map distance} = \frac{1\,500\,000}{300\,000}$$

$$= 5 \text{ cm}$$

Answer: 5 cm.

Worked Example 14

The distance between two buildings in a school compound is 24 metres. The school map uses a scale of 1 cm represents 4 metres. Calculate the distance on the map.

Solution

$$\text{Map distance} = \frac{24}{4}$$

$$= 6 \text{ cm}$$

Answer: 6 cm.

5.3 Determining the scale of a map

When the map distance and actual distance are known, the scale can be calculated.

Formula

$$\boxed{\text{Scale} = \text{Map distance:Actual distance}}$$

Both distances must be converted into the same units.

Worked Example 15

A distance of 5 cm on a map represents 10 km on the ground. Find the scale in ratio form.

Solution

Convert 10 km into centimetres:

$$10 \times 100\,000 = 1\,000\,000 \text{ cm}$$

Write the ratio:

$$5 : 1\,000\,000$$

Divide both terms by 5:

$$\boxed{1 : 200\,000}$$

Answer: $1 : 200\,000$.

Worked Example 16

A school compound is 80 metres long. It is represented by a drawing that is 16 cm long. Find the scale in ratio form.

Solution

Convert 80 metres into centimetres:

$$80 \times 100 = 8\,000 \text{ cm}$$

Write the ratio:

$$16 : 8\,000$$

Divide both terms by 16:

$$\boxed{1 : 500}$$

Answer: $1 : 500$.

6.0 GRAPHICAL OR LINEAR SCALE BAR

6.1 Definition

A **graphical scale** is a scale represented by a graduated line or bar divided into sections, each representing a known actual distance.

It is also called a linear scale bar.

It is commonly found on maps and atlases.

Features of a graphical scale

- It is drawn as a straight line.
- It is divided into equal sections.
- Each section represents a known actual distance.
- It has clearly labelled units.
- It can be used to measure distances on a map.

Worked Example 17

A graphical scale is divided into five equal sections. Each section represents 2 km. What actual distance is represented by four sections?

Solution

$$4 \times 2 = 8 \text{ km}$$

Answer: 8 km.

Advantages of a graphical scale

1. It is easy to interpret visually.
2. It allows direct measurement of distances.
3. It remains useful when a map is enlarged or reduced proportionally along with its scale bar.
4. It is convenient for map reading.
5. It helps compare distances between different locations.

Limitation of a graphical scale

A graphical scale may be difficult to read accurately if the printed map or screen image is blurred, distorted or resized without preserving its proportions.

7.0 BEARINGS

Bearings are used in navigation, surveying, map reading, aviation, marine transport and construction.

They help us describe the direction of one point or place from another.

For example, a learner may be asked to describe the direction of a hospital from a school, or the direction of a market from a police station.

7.1 Definition of bearing

A **bearing** is the angle measured clockwise from the north direction at a given point to the direction of another point.

Bearings are measured in degrees from $000°$ to $360°$.

A bearing is always written using three digits.

Examples:

- $045°$
- $090°$
- $120°$
- $180°$
- $270°$
- $315°$

Important: Bearings are measured clockwise from north, not anticlockwise.

7.2 The four main compass directions

The four main compass directions are:

1. North (N).
2. East (E).
3. South (S).
4. West (W).

The directions are separated by angles of $90°$.

Direction	Bearing
North	$000°$ or $360°$
East	$090°$
South	$180°$
West	$270°$

7.3 The eight main compass directions

Between the four main directions are four intermediate directions:

- North-East (NE).
- South-East (SE).
- South-West (SW).
- North-West (NW).

Direction	Bearing
North	000°
North-East	045°
East	090°
South-East	135°
South	180°
South-West	225°
West	270°
North-West	315°

7.4 Important rules when measuring bearings

1. Always begin measuring from the north line.
2. Measure clockwise.
3. Use a protractor accurately.
4. Write bearings using three digits.
5. Identify the starting point before measuring.
6. Draw a north line at the starting point.
7. Do not measure from the east, south or west unless the question specifically requires a different angle.

Example of three-digit notation

Angle	Correct bearing notation
5°	005°
30°	030°
75°	075°
90°	090°
120°	120°
180°	180°
270°	270°

8.0 DETERMINING THE BEARING OF ONE POINT FROM ANOTHER

To determine the bearing of point B from point A:

1. Mark point A as the starting point.
2. Draw a vertical north line through A.
3. Join A to B using a straight line.
4. Measure the angle clockwise from north at A to line AB.
5. Write the answer using three digits.

Graph 1: Bearing of B from A

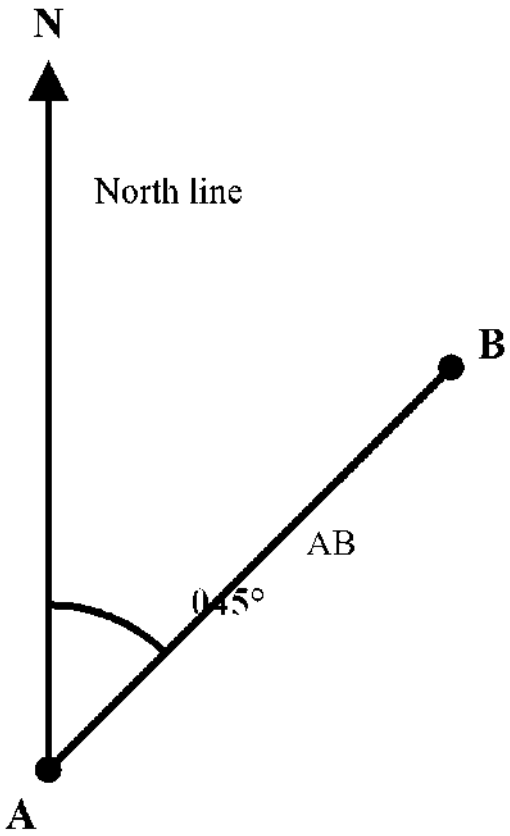


Illustration: the clockwise angle from north at A to AB is $045°$.

Worked Example 18

A point B is directly east of point A. Determine the bearing of B from A.

Solution

East is $90°$ clockwise from north.

Therefore:

$$\boxed{090°}$$

Worked Example 19

A point C is south-west of point A. Determine the bearing of C from A.

Solution

South-west is halfway between south and west.

$$180° + 45° = 225°$$

Answer: $225°$**.**

Worked Example 20

A point D is north-west of point A. Find the bearing of D from A.

Solution

North-west is halfway between west and north when measured clockwise from north.

$$360° - 45° = 315°$$

Answer: $315°$**.**

Worked Example 21

The bearing of a hospital from a school is $065°$. State the direction of the hospital from the school.

Solution

The bearing $065°$ is between north and east.

Therefore, the hospital is in the north-eastern direction of the school.

Answer: North-East, at a bearing of $065°$**.**

9.0 DRAWING AND MEASURING BEARINGS USING A PROTRACTOR

A protractor is an instrument used to measure and draw angles.

To draw a bearing of $120°$:

1. Mark the starting point A.
2. Draw a vertical north line through A.
3. Place the centre of the protractor at A.
4. Align the $0°$ line with north.
5. Measure $120°$ clockwise.
6. Mark the angle.
7. Draw a ray from A through the mark.
8. Label the new point B.

Worked Example 22

Draw the bearing of point P from point O as $070°$.

Construction

- Draw a north line at O.
- Measure $70°$ clockwise from north.
- Draw a ray in that direction.
- Mark point P on the ray.

The bearing of P from O is:

$$\boxed{070°}$$

Worked Example 23

A point Q is on a bearing of $240°$ from point P. Determine the general compass direction of Q from P.

Solution

The bearing $240°$ lies between:

$$180° \text{ (South)}$$

and:

$$270° \text{ (West)}$$

Therefore, Q is south-west of P.

Answer: South-West.

10.0 BACK BEARINGS

10.1 Definition

A **back bearing** is the bearing of one point from another when the direction is reversed.

For example:

- The bearing of B from A is the forward bearing.
- The bearing of A from B is the back bearing.

Rules for calculating back bearings

If the forward bearing is less than $180°$, add $180°$.

If the forward bearing is greater than $180°$, subtract $180°$.

$$\text{Back bearing} = \begin{cases} \text{Forward bearing} + 180°, & \text{if below } 180° \\ \text{Forward bearing} - 180°, & \text{if above } 180° \end{cases}$$

Worked Example 24

The bearing of B from A is $065°$. Find the bearing of A from B.

Solution

Since $065°$ is less than $180°$:

$$065° + 180° = 245°$$

Answer: $245°$**.**

Worked Example 25

The bearing of a market from a school is $235°$. Find the bearing of the school from the market.

Solution

Since $235°$ is greater than $180°$:

$$235° - 180° = 055°$$

Answer: $055°$**.**

Worked Example 26

The bearing of a hospital from a health centre is $180°$. Find the back bearing.

Solution

$$180° + 180° = 360°$$

A bearing of $360°$ is equivalent to $000°$.

Answer: $000°$**.**

Summary table of forward and back bearings

Forward bearing	Back bearing
$030°$	$210°$
$065°$	$245°$
$090°$	$270°$
$120°$	$300°$
$150°$	$330°$
$180°$	$000°$
$225°$	$045°$
$270°$	$090°$
$315°$	$135°$

11.0 SCALE DRAWING USING BEARINGS

Scale drawing and bearings can be combined to represent the relative positions of places accurately.

For example, a map can show:

- The position of a hospital from a school.
- The direction of a market from a police station.
- The distance between two trading centres.
- The location of a water tank from a classroom.
- The direction of a road from a particular junction.

Steps for making a scale drawing using bearings

1. Identify the starting point.
2. Select an appropriate scale.
3. Draw a north line at the starting point.
4. Measure the required bearing clockwise from north.
5. Convert the actual distance into drawing distance using the scale.
6. Measure the calculated drawing distance using a ruler.
7. Draw the line in the correct direction.
8. Mark and label the destination point.
9. Check that the scale and bearing are correctly represented.

Worked Example 27

A health centre is 6 km from a school on a bearing of $060°$. Draw a scale representation of the health centre from the school using a scale of 1 cm represents 2 km.

Solution

Step 1: Identify the scale.

$$1 \text{ cm} = 2 \text{ km}$$

Step 2: Calculate the drawing distance.

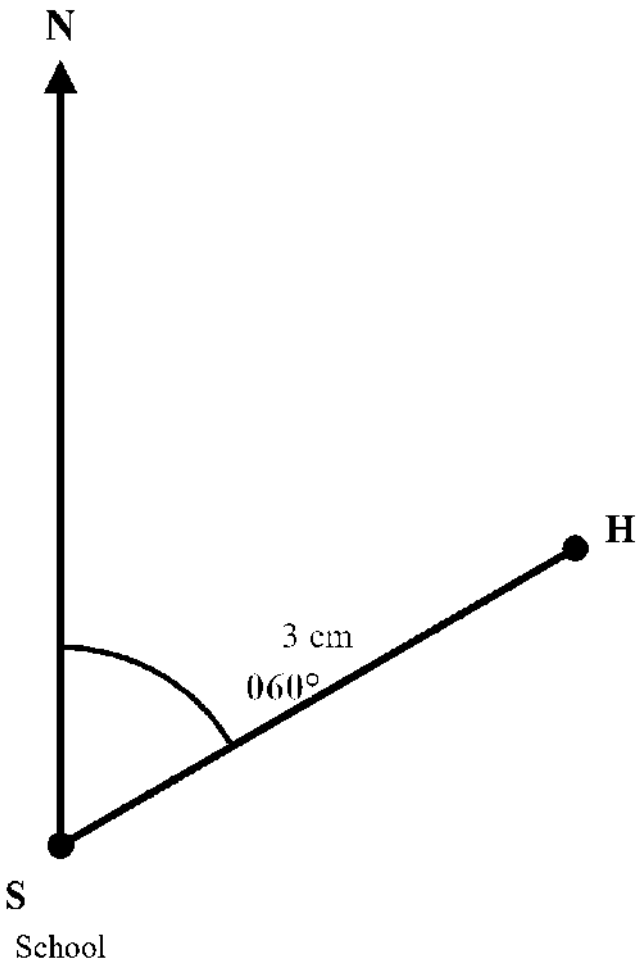
$$\text{Drawing distance} = \frac{6}{2}$$

$$= 3 \text{ cm}$$

Step 3: Construct the drawing.

- Mark the school as point S.
- Draw a north line through S.
- Measure $060°$ clockwise from north.
- Draw a ray in the measured direction.
- Measure 3 cm along the ray.
- Mark the health centre as H.

Graph 2: School and health centre



The health centre is 3 cm from the school on a bearing of $060°$. The drawing is schematic.

Answer: The health centre should be plotted 3 cm from the school at a bearing of $060°$.

12.0 MAKING SCALE DRAWINGS OF OBJECTS AND PLACES

A scale drawing may be used to represent an object, a building, a piece of land or an entire school compound.

12.1 Scale drawings of rectangular objects

Worked Example 28

A classroom is 12 metres long and 8 metres wide. Draw a scale plan using a scale of 1 cm represents 2 metres.

Solution

Length:

$$\frac{12}{2} = 6 \text{ cm}$$

Width:

$$\frac{8}{2} = 4 \text{ cm}$$

Therefore, draw a rectangle measuring:

$$\boxed{6 \text{ cm} \times 4 \text{ cm}}$$

12.2 Scale drawings of square objects

Worked Example 29

A square playground has an actual side length of 40 metres. Draw the playground using a scale of 1 cm represents 10 metres.

Solution

$$\text{Drawing side} = \frac{40}{10}$$

$$= 4 \text{ cm}$$

Draw a square with each side measuring 4 cm.

12.3 Scale drawings of irregular shapes

Irregular areas can also be represented using scale drawings.

Examples include:

- Farms.
- School compounds.
- Market areas.
- Roads.
- Rivers.
- Forest boundaries.

To draw an irregular area:

1. Identify the important boundary points.
2. Measure the distances between the points.
3. Select an appropriate scale.
4. Convert the actual distances into drawing distances.
5. Plot the points.
6. Join the points in the correct order.
7. Label the important features.

12.4 Enlargement and reduction in scale drawing

Scale drawings may involve reducing or enlarging the original dimensions.

A. Reduction

Reduction occurs when an object is represented by a smaller drawing while preserving its proportions.

Examples:

- Drawing a house plan on paper.
- Representing a large farm on a map.
- Drawing a road network on a county map.

B. Enlargement

Enlargement occurs when an object is represented by a larger drawing while preserving its proportions.

Examples:

- Enlarging a small machine component for detailed inspection.
- Enlarging a small diagram for classroom teaching.
- Enlarging a map feature for easier observation.

Important distinction

A scale of $1:100$ represents a reduction because 1 unit on the drawing represents 100 units in reality.

A scale of $2:1$ represents an enlargement because 2 units on the drawing represent 1 unit on the original object.

13.0 PRACTICAL APPLICATIONS OF SCALE DRAWING IN KENYA

Scale drawing is used in many sectors of the Kenyan economy.

13.1 Architecture and building construction

Architects prepare scaled plans before constructing:

- Houses.
- Schools.
- Hospitals.
- Offices.
- Shopping centres.
- Churches and other public buildings.

A scale drawing helps builders understand the arrangement and dimensions of the proposed structure.

13.2 Land surveying

Surveyors use scale drawings to:

- Represent land boundaries.
- Determine the dimensions of plots.
- Identify the positions of roads and buildings.
- Prepare land subdivision plans.
- Support land-use planning.

13.3 Road construction

Engineers use scale drawings to:

- Plan roads and highways.
- Determine distances between junctions.
- Design road curves.
- Locate bridges and drainage structures.
- Estimate construction requirements.

13.4 Geography and map reading

Geographers use scale drawings to:

- Represent counties and countries.
- Identify towns and villages.
- Estimate distances between places.
- Study geographical features.
- Interpret transport networks.

13.5 Agriculture

Scale drawings are used to plan:

- Farm boundaries.
- Irrigation systems.
- Greenhouses.
- Livestock units.
- Crop fields.
- Water storage facilities.

13.6 School planning

A school can use scale drawings to plan the location of:

- Classrooms.
- Administration blocks.
- Laboratories.
- Libraries.
- Toilets.
- Playgrounds.
- Water points.
- Gardens.
- Access roads.

13.7 Transport and navigation

Pilots, drivers, sailors and other transport professionals use maps and bearings to determine directions and locations.

13.8 Importance of bearings in everyday life

Bearings are useful in:

1. Navigation and travelling.
2. Locating places on maps.
3. Land surveying.

4. Planning roads and railway lines.
5. Aviation and marine transport.
6. Search and rescue operations.
7. Military and emergency planning.
8. Determining directions between landmarks.

14.0 DIGITAL TECHNOLOGY IN SCALE DRAWING

Modern technology makes it easier to read maps, measure distances and prepare scale drawings.

Examples of digital resources include:

- Google Maps.
- Google Earth.
- GeoGebra.
- Digital atlas applications.
- Computer-aided design (CAD) software.
- Digital surveying applications.

Using digital maps to investigate scale

Learners can:

1. Open a digital map.
2. Locate their school or nearby market.
3. Identify a hospital, police post or other landmark.
4. Use the distance-measuring tool.
5. Compare the actual distance with a printed map distance.
6. Identify the scale where displayed.
7. Investigate how zooming changes the displayed map size.

Important observation

When a digital map is zoomed in or out, the map image changes size on the screen. A fixed printed measurement cannot be interpreted using an old scale unless the scale display is updated or the image is calibrated again.

PROJECT: PREPARING A SCALE DRAWING OF THE SCHOOL COMPOUND

Practical group project

Project title

Preparing a Scale Drawing of Our School Compound

Project objective

To enable learners to apply scale drawing, measurement, bearings and map interpretation in representing the layout of their school.

Project instructions

Work in groups to prepare a scale drawing of the school compound.

Stage 1: Identification of features

Identify the important features within the school compound, such as:

- Classrooms.

- Administration block.
- Library.
- Laboratory.
- Toilets.
- Playground.
- Assembly ground.
- Water tank.
- Gate.
- Flag post.
- Garden.
- Access road.

Stage 2: Collecting the required materials

The group should collect:

1. Measuring tape.
2. Ruler.
3. Graph paper.
4. Pencil.
5. Eraser.
6. Protractor.
7. Notebook.
8. Calculator.
9. Camera or mobile device, where permitted.
10. Computer or tablet, where available.

Stage 3: Measuring distances

Measure the distances between important structures.

For example:

Features	Actual distance
Main gate to administration block	30 m
Administration block to classroom block	20 m
Classroom block to playground	40 m
Playground to water tank	15 m
Library to laboratory	12 m

These are illustrative measurements. Learners should collect actual measurements from their school.

Stage 4: Selecting an appropriate scale

Suppose the longest distance in the school compound is 100 metres.

Choose a scale of:

$$1 \text{ cm represents } 10 \text{ m}$$

This gives:

$$100 \text{ m} \div 10 = 10 \text{ cm}$$

The longest distance will therefore occupy 10 cm on the drawing.

Stage 5: Converting measurements

Using the selected scale:

Actual distance	Drawing distance
30 m	3 cm
20 m	2 cm
40 m	4 cm
15 m	1.5 cm
12 m	1.2 cm

Stage 6: Preparing the drawing

Learners should:

1. Draw the boundary of the school compound.
2. Mark the entrance.
3. Draw the buildings in their approximate positions.
4. Represent paths and roads.
5. Show the playground.
6. Include the water points and other facilities.
7. Use suitable symbols.
8. Add a north arrow.
9. Write the scale.
10. Label all important features.

Stage 7: Documentation

The group should prepare a short report showing:

- Project title.
- Objectives.
- Materials used.
- Features identified.
- Measurements collected.
- Scale selected.
- Calculations carried out.
- Symbols used.
- Drawing procedure.
- Challenges experienced.
- Solutions to the challenges.
- Photographs or video clips, where permission has been granted.
- Conclusion.

Stage 8: Presentation

Each group should present its school map to the class and explain:

1. The scale used.
2. How the measurements were obtained.
3. How the drawing distances were calculated.
4. How the north direction was identified.
5. The symbols used.

6. The challenges encountered.

Suggested project assessment rubric

Assessment area	Marks
Identification of school features	5
Accuracy of measurements	5
Correct selection of scale	5
Accuracy of calculations	5
Quality and accuracy of drawing	10
Correct use of symbols and labels	5
Project report and presentation	5
Total	**40**

Competencies developed

- Critical thinking and problem-solving.
- Creativity and imagination.
- Communication and collaboration.
- Digital literacy.
- Learning to learn.

Values promoted

- Responsibility.
- Cooperation.
- Respect.
- Honesty in measurement.
- Accuracy.
- Accountability.

Pertinent and contemporary issues

- Environmental conservation through proper land-use planning.
- Safety when measuring school structures and roads.
- Responsible use of digital technology.
- Community participation in school development.
- Efficient use of land and other resources.

16.0 SUMMARY OF KEY FORMULAE AND CONCEPTS

16.1 Important formulae

Concept	Formula
Scale	Drawing distance : Actual distance
Actual distance	Map distance × Scale factor
Map distance	Actual distance ÷ Scale factor
Scale factor	Actual distance ÷ Map distance
Ratio scale	$1:n$
Smallest angle of a regular polygon's rotational symmetry	Not applicable to this sub-strand

Concept	Formula
Back bearing when forward bearing is below $180°$	Forward bearing $+180°$
Back bearing when forward bearing is above $180°$	Forward bearing $-180°$

16.2 Important facts to remember

1. A scale represents the relationship between drawing distance and actual distance.
2. Ratio scales must use the same units.
3. 1 km $= 100\,000$ cm.
4. 1 m $= 100$ cm.
5. Actual distance is obtained by multiplying map distance by the scale factor.
6. Map distance is obtained by dividing actual distance by the scale factor.
7. Bearings are measured clockwise from north.
8. Bearings are written using three digits.
9. The bearing of north is $000°$ or $360°$.
10. The bearing of east is $090°$.
11. The bearing of south is $180°$.
12. The bearing of west is $270°$.
13. Back bearings differ from forward bearings by $180°$.
14. A north arrow should be included in a properly prepared map.
15. A scale drawing should preserve the proportions of the original object.

REVISION EXERCISES

SECTION A: MULTIPLE-CHOICE QUESTIONS (30 MARKS)

Instructions: Answer all questions. Select the correct answer from alternatives A, B, C and D. Each question carries 1 mark.

1. What is a scale drawing?

A. A drawing made without measurements.

B. A drawing in which dimensions are represented proportionally according to a specified scale.

C. A drawing that changes the shape of an object.

D. A drawing made using freehand lines only.

2. Which of the following best describes a linear scale?

A. A scale that shows the relationship between lengths on a drawing and actual lengths.

B. A scale that shows the weight of an object.

C. A scale that measures temperature.

D. A scale used only for measuring angles.

3. A map uses the scale 1 cm represents 5 km. What actual distance is represented by 4 cm?

A. 10 km.

B. 15 km.

C. 20 km.

D. 25 km.

4. Convert 1 cm represents 2 km into ratio form.

A. $1:20\,000$.

B. $1:200\,000$.

C. $1:2\,000$.

D. $1:2\,000\,000$.

5. Convert the scale $1:50\,000$ into statement form.

A. 1 cm represents 50 m.

B. 1 cm represents 5 km.

C. 1 cm represents 500 m.

D. 1 cm represents 50 km.

6. A map has a scale of $1:100\,000$. What actual distance is represented by 7 cm?

A. 700 m.

B. 7 km.

C. 70 km.

D. 700 km.

7. The actual distance between two towns is 20 km. A map uses a scale of $1:200\,000$. Calculate the map distance.

A. 5 cm.

B. 10 cm.

C. 20 cm.

D. 40 cm.

8. A scale drawing uses 4 cm to represent 20 metres. Find the ratio scale.

A. $1:50$.

B. $1:100$.

C. $1:500$.

D. $1:5\,000$.

9. A school map uses a scale of 1 cm represents 4 metres. What actual distance is represented by 8.5 cm?

A. 12 m.

B. 24 m.

C. 34 m.

D. 40 m.

10. Which unit conversion is correct?

A. 1 km = 1,000 cm.

B. 1 km = 10,000 cm.

C. 1 km = 100,000 cm.

D. 1 km = 1,000,000 cm.

11. A ratio scale is written as $1:250\,000$. What distance does 1 cm represent in kilometres?

A. 0.25 km.

B. 2.5 km.

C. 25 km.

D. 250 km.

12. Which of the following is a graphical scale?

A. A scale expressed only in words.

B. A scale expressed as a ratio.

C. A graduated bar divided into sections representing actual distances.

D. A scale expressed as a percentage.

13. Which instrument is used to measure bearings?

A. Thermometer.

B. Protractor.

C. Weighing balance.

D. Measuring cylinder.

14. A bearing is measured:

A. Anticlockwise from east.

B. Clockwise from north.

C. Anticlockwise from south.

D. Clockwise from west.

15. What is the bearing of east?

A. $000°$.

B. $045°$.

C. $090°$.

D. $180°$.

16. What is the bearing of south?

A. $090°$.

B. $180°$.

C. $270°$.

D. $360°$.

17. A point is directly west of another point. What is its bearing?

A. $090°$.

B. $180°$.

C. $225°$.

D. $270°$.

18. The bearing of a market from a school is $045°$. In which general direction is the market?

A. South-East.

B. North-East.

C. South-West.

D. North-West.

19. A hospital is on a bearing of $135°$ from a school. In which direction is the hospital?

A. North-East.

B. South-East.

C. South-West.

D. North-West.

20. The bearing of B from A is $070°$. Find the bearing of A from B.

A. $110°$.

B. $180°$.

C. $250°$.

D. $290°$.

21. The bearing of a market from a school is $230°$. Find the back bearing.

A. $040°$.

B. $050°$.

C. $130°$.

D. $410°$.

22. A point is on a bearing of $315°$. Which general direction does it lie in?

A. North-East.

B. South-East.

C. South-West.

D. North-West.

23. A map uses a scale of $1:400\,000$. The distance between two towns on the map is 3 cm. Find the actual distance.

A. 4 km.

B. 8 km.

C. 12 km.

D. 16 km.

24. A building is 24 metres long. It is represented on a drawing using a scale of 1 cm represents 3 metres. Find its drawing length.

A. 6 cm.

B. 7 cm.

C. 8 cm.

D. 9 cm.

25. A rectangular classroom is 15 m long and 9 m wide. Using a scale of 1 cm represents 3 m, what are its drawing dimensions?

A. 3 cm by 5 cm.

B. 5 cm by 3 cm.

C. 6 cm by 4 cm.

D. 9 cm by 5 cm.

26. A scale drawing shows a distance of 6 cm representing 30 metres. What is the ratio scale?

A. $1:50$.

B. $1:100$.

C. $1:500$.

D. $1:5\,000$.

27. Which of the following is an important feature of a properly prepared map?

A. A north arrow.

B. Decorative pictures only.

C. Unlabelled buildings.

D. Unmeasured boundaries.

28. Which professional commonly uses scale drawings when planning building designs?

A. Musician.

B. Architect.

C. Farmer only.

D. Journalist.

29. The bearing of a health centre from a school is $180°$. What is the bearing of the school from the health centre?

A. $090°$.

B. $180°$.

C. $270°$.

D. $360°$.

30. A school compound is 120 metres long. It is to be represented using a scale of 1 cm represents 20 metres. What length should be used on the drawing?

A. 4 cm.

B. 5 cm.

C. 6 cm.

D. 8 cm.

SECTION B: STRUCTURED QUESTIONS (50 MARKS)

Instructions: Answer all questions. Show all working clearly. Use a ruler and protractor where necessary. Use graph paper for scale drawings where appropriate.

QUESTION 1: LINEAR SCALES (8 MARKS)

a) Define the term linear scale. (2 marks)

b) Convert the following statement scale into ratio form:

$$1 \text{ cm represents } 4 \text{ km}$$

(2 marks)

c) Convert the ratio scale $1:150\,000$ into statement form using kilometres. (2 marks)

d) State two applications of scale drawing in Kenya. (2 marks)

QUESTION 2: CALCULATIONS USING MAP SCALES (8 MARKS)

A map of a region has a scale of $1:200\,000$.

a) Calculate the actual distance represented by 6 cm on the map. Give your answer in kilometres. (3 marks)

b) The actual distance between two trading centres is 35 km. Calculate the distance between them on the map in centimetres. (3 marks)

c) State one reason why maps use scale. (2 marks)

QUESTION 3: DRAWING A SCHOOL COMPOUND (8 MARKS)

A rectangular classroom measures 18 metres long and 12 metres wide.

A scale of 1 cm represents 3 metres is to be used.

a) Calculate the length of the classroom on the drawing. (2 marks)

b) Calculate the width of the classroom on the drawing. (2 marks)

c) State the dimensions of the rectangle to be drawn. (2 marks)

d) Give two features that should be included when preparing a school compound plan. (2 marks)

QUESTION 4: BEARINGS (8 MARKS)

a) Define the term bearing. (2 marks)

b) State the bearings of the following compass directions:

c) East. (1 mark)

ii) South. (1 mark)

iii) The bearing of a hospital from a school is $065°$. Calculate the bearing of the school from the hospital. (2 marks)

iv) State two rules followed when measuring bearings. (2 marks)

QUESTION 5: BEARINGS AND SCALE DRAWING (9 MARKS)

A health centre is 8 km from a school on a bearing of $120°$.

A scale of 1 cm represents 2 km is to be used.

a) Calculate the distance to be represented on the drawing. (2 marks)

b) State the bearing and direction of the health centre from the school. (2 marks)

c) Describe how to construct the scale drawing using a ruler and protractor. (3 marks)

d) Calculate the back bearing of the health centre from the school. (2 marks)

QUESTION 6: SCALE DRAWING PROJECT AND MAP INTERPRETATION (9 MARKS)

A learner is preparing a map of a school compound. The following measurements are obtained:

Feature	Actual measurement
Length of administration block	30 m
Width of administration block	12 m
Distance from administration block to library	40 m
Distance from library to playground	60 m

The learner chooses a scale of 1 cm represents 10 metres.

a) Calculate the drawing length and width of the administration block. (2 marks)

b) Calculate the drawing distance between:

c) Administration block and library. (1 mark)

ii) Library and playground. (1 mark)

iii) State three items of equipment required when preparing the school compound map. (3 marks)

iv) State two reasons why a north arrow should be included in the school map. (2 marks)

ANSWERS AND MARKING SCHEME

Section A: Multiple-choice answer key

Question	Answer	Question	Answer
1	B	16	B
2	A	17	D
3	C	18	B
4	B	19	B
5	C	20	C
6	B	21	B
7	B	22	D
8	C	23	C
9	C	24	C
10	C	25	B
11	B	26	C
12	C	27	A
13	B	28	B
14	B	29	D
15	C	30	C

Section B: Suggested marking scheme

QUESTION 1 (8 MARKS)

a) A linear scale is a scale that expresses the relationship between lengths on a drawing or map and the corresponding actual lengths. (2 marks)

$$1 \text{ cm represents } 4 \text{ km}$$

Convert 4 km into centimetres:

$$4 \times 100\,000 = 400\,000 \text{ cm}$$

Therefore:

$$\boxed{1:400\,000}$$

(2 marks)

$$1:150\,000$$

This means:

$$1 \text{ cm}: 150\,000 \text{ cm}$$

$$150\,000 \div 100\,000 = 1.5 \text{ km}$$

$$\boxed{1 \text{ cm represents } 1.5 \text{ km}}$$

(2 marks)

d) Any two:

- Architecture and building construction.
- Land surveying.
- Road construction.
- Map reading.
- Agricultural planning.
- School compound planning.

(2 marks)

QUESTION 2 (8 MARKS)

a) Given:

$$\text{Scale} = 1:200\,000$$

$$\text{Map distance} = 6 \text{ cm}$$

Actual distance:

$$= 6 \times 200\,000$$

$$= 1\,200\,000 \text{ cm}$$

Convert to kilometres:

$$1\,200\,000 \div 100\,000 = 12 \text{ km}$$

Answer: 12 km. (3 marks)

b) Actual distance = 35 km.

Convert to centimetres:

$$35 \times 100\,000 = 3\,500\,000 \text{ cm}$$

Map distance:

$$= \frac{3\,500\,000}{200\,000}$$

$$= 17.5 \text{ cm}$$

Answer: 17.5 cm. (3 marks)

c) Any one explained reason:

- To represent large geographical areas on small sheets of paper.
- To estimate actual distances between places.
- To represent buildings and land accurately.

(2 marks)

QUESTION 3 (8 MARKS)

a) Length:

$$\frac{18}{3} = 6 \text{ cm}$$

(2 marks)

b) Width:

$$\frac{12}{3} = 4 \text{ cm}$$

(2 marks)

c) Drawing dimensions:

$$\boxed{6 \text{ cm} \times 4 \text{ cm}}$$

(2 marks)

d) Any two:

- Classrooms.
- Administration block.
- Library.
- Playground.
- Laboratory.
- Main gate.
- Toilets.
- Water points.

(2 marks)

QUESTION 4 (8 MARKS)

a) A bearing is the angle measured clockwise from the north direction at a given point to the direction of another point. (2 marks)

b) East = $090°$. (1 mark)

ii) South = $180°$. (1 mark)

iii) Back bearing:

$$065° + 180° = 245°$$

Answer: $245°$**.** (2 marks)

d) Any two:

- Measure clockwise from north.
- Write the bearing using three digits.
- Draw a north line at the starting point.
- Use a protractor to measure the angle.

(2 marks)

QUESTION 5 (9 MARKS)

$$\text{Drawing distance} = \frac{8}{2}$$

$$= 4 \text{ cm}$$

(2 marks)

- Bearing: $120°$. (1 mark)
- Direction: South-East. (1 mark)

c) Construction:

- Mark the school as the starting point. (1)

- Draw a north line and measure $120°$ clockwise. (1)
- Draw a ray and mark the health centre 4 cm from the school. (1)

d) Back bearing:

$$120° + 180° = 300°$$

Answer: $300°$**.** (2 marks)

QUESTION 6 (9 MARKS)

a) Administration block:

Length:

$$30 \div 10 = 3 \text{ cm}$$

Width:

$$12 \div 10 = 1.2 \text{ cm}$$

Answer: 3 cm by 1.2 cm. (2 marks)

b) Administration block to library:

$$40 \div 10 = 4 \text{ cm}$$

(1 mark)

ii) Library to playground:

$$60 \div 10 = 6 \text{ cm}$$

(1 mark)

c) Any three:

- Measuring tape.
- Ruler.
- Graph paper.
- Pencil.
- Eraser.
- Protractor.
- Calculator.
- Notebook.

(3 marks)

d) Any two explained reasons:

- It shows the direction of north on the map.
- It helps users identify other compass directions.
- It assists in locating features relative to one another.
- It supports the interpretation of bearings.

(2 marks)

STRAND 3.0: STATISTICS AND PROBABILITY

SUB-STRAND 3.1: STATISTICS II

1.0 Introduction to Statistics II

Statistics is a branch of Mathematics that deals with the collection, organisation, presentation, analysis and interpretation of data to make informed decisions.

In everyday life, people collect and use data to understand situations, identify patterns, compare information and plan for the future.

For example:

- A school headteacher analyses examination results to identify learners who need additional support.
- A farmer records the amount of rainfall received each month to plan planting activities.
- A health officer studies the number of patients visiting a health centre to determine staffing needs.
- A shopkeeper records daily sales to determine the most demanded products.
- A transport operator studies passenger numbers to plan the number of vehicles required.

Statistics helps people make decisions based on evidence rather than guesses.

1.1 Key terms used in statistics

1. Data

Data refers to facts, figures, measurements or observations collected for a particular purpose.

Examples:

- Heights of learners in a Grade 11 class.
- Shoe sizes of learners in a school.
- Number of books read by learners in one month.
- Daily temperatures recorded in a town.
- Amount of maize harvested by farmers.

2. Raw data

Raw data is information collected in its original form before it has been organised or arranged.

Example:

The following are the marks scored by 12 learners in a Mathematics test:

$45,\ 60,\ 55,\ 70,\ 45,\ 80,\ 65,\ 55,\ 50,\ 60,\ 75,\ 45$

This information is raw data because it has not yet been arranged into a frequency table.

3. Observation

An observation is a single value or item of information recorded during data collection.

For example, if the heights of five learners are 150 cm, 155 cm, 160 cm, 165 cm and 170 cm, each height is an observation.

4. Frequency

Frequency is the number of times a particular value or group of values occurs in a data set.

For example, in the data:

2, 3, 2, 4, 3, 2, 5

The frequency of 2 is 3 because it occurs three times.

5. Class interval

A class interval is a group or range into which numerical data is organised.

Examples:

- 10–19
- 20–29
- 30–39
- 40–49

6. Class width

Class width is the difference between the upper and lower class boundaries of a class interval.

For example, for the class interval 20–29:

$$\text{Class width} = 29.5 - 19.5 = 10$$

7. Grouped data

Grouped data is data that has been organised into classes or intervals together with their corresponding frequencies.

8. Ungrouped data

Ungrouped data is data presented as individual observations without being arranged into class intervals.

9. Measures of central tendency

Measures of central tendency are statistical measures used to identify a central or representative value in a set of data.

The three main measures are:

- Mean.
- Median.
- Mode.

1.2 Learning outcomes

By the end of this sub-strand, the learner should be able to:

a) Collect data from the immediate environment.

b) Group and represent data in a frequency distribution table.

c) Determine the mean, mode and median of grouped data.

d) Present grouped data using histograms with equal class widths.

e) Apply measures of central tendency in real-life situations.

f) Embrace the use of measures of central tendency in everyday life.

1.3 Essential inquiry questions

1. Why is data collection important in day-to-day life?
2. How are measures of central tendency applicable in real life?

2.0 COLLECTION OF DATA

2.1 Meaning of data collection

Data collection is the systematic process of gathering information from people, objects, events or the environment for a particular purpose.

Data collection is an important first step in statistical investigations because the accuracy of conclusions depends on the quality of the information collected.

2.2 Sources of data

Data can be obtained from different sources.

A. Primary sources

Primary data is information collected directly by the person conducting the investigation for a specific purpose.

Examples of primary data include:

- Measuring learners' heights in a classroom.
- Counting the number of vehicles passing a road.
- Interviewing farmers about their crop production.
- Recording the weights of pupils in a school.
- Asking learners about their preferred school subjects.

B. Secondary sources

Secondary data is information that has already been collected, processed or published by another person or organisation.

Examples include:

- Kenya National Bureau of Statistics (KNBS) reports.
- Ministry of Education records.
- School attendance registers.
- Published books and journals.
- Government reports.
- Reliable online databases and publications.

2.3 Methods of collecting data

1. Observation

Observation involves watching people, objects or events and recording the information obtained.

Examples:

- Counting the number of learners entering a school compound.
- Observing the number of vehicles using a road.
- Recording the number of animals in a livestock enclosure.

Advantages:

- It provides first-hand information.
- It can be used where respondents may not be able to provide answers.
- It allows the investigator to record actual events.

Limitations:

- It may take a long time.
- Some observations may be inaccurate.
- The observer may influence the results through personal judgement.

2. Interview

An interview involves asking people questions and recording their responses.

Examples:

- Interviewing farmers about their maize harvest.
- Asking shopkeepers about their daily sales.
- Interviewing learners about their preferred learning activities.

Advantages:

- Questions can be explained where necessary.
- Detailed information can be obtained.
- The interviewer can seek clarification.

Limitations:

- It may be time-consuming.
- Some respondents may give inaccurate answers.
- It may be expensive when many people are involved.

3. Questionnaire

A questionnaire is a set of written questions given to respondents to complete.

Example:

A teacher wants to establish the most popular sport among Grade 11 learners.

Sport	Tick (✓)
Football	
Volleyball	
Netball	
Athletics	
Basketball	

Advantages:

- Many people can be reached.
- Responses can be collected in a standard format.
- It is relatively inexpensive.

Limitations:

- Some respondents may not return the questionnaires.
- Respondents may misunderstand questions.
- Some answers may be incomplete.

4. Measurement

Measurement involves using appropriate instruments to obtain numerical information.

Examples:

Information required	Measuring instrument
Height	Metre rule or height metre
Mass	Weighing scale
Length	Tape measure
Temperature	Thermometer
Time	Stopwatch or clock
Volume	Measuring cylinder

5. Counting

Counting involves determining the number of items, people or events.

Examples:

- Counting the number of learners in each class.
- Counting the number of eggs collected from a poultry farm.
- Counting the number of customers visiting a shop.

2.4 Steps followed when collecting data

A statistical investigation should follow a systematic procedure.

1. **Identify the problem or purpose.** Determine what information is required.
2. **Identify the population.** Establish the entire group being investigated.
3. **Select the sample, where necessary.** Choose a suitable number of members from the population.
4. **Choose a method of data collection.** Decide whether to use observation, interviews, questionnaires, measurements or counting.
5. **Prepare the tools.** Organise questionnaires, recording sheets or measuring instruments.
6. **Collect the information.** Gather data carefully and consistently.
7. **Record the observations.** Write down the information immediately to avoid forgetting it.
8. **Check the data.** Identify missing, duplicated or obviously incorrect entries.
9. **Organise the data.** Arrange the information into a suitable table.
10. **Analyse and interpret the data.** Use appropriate statistical methods to draw conclusions.

2.5 Important considerations during data collection

- Data should be relevant to the investigation.
- Information should be accurate and reliable.
- Appropriate measuring instruments should be used.
- Respondents should be treated respectfully.
- Personal information should be kept confidential.
- Data should not be changed to produce a desired result.
- The investigator should avoid bias.
- Measurements should use consistent units.
- Permission should be obtained where necessary, especially when collecting personal information.

2.6 Importance of collecting data

Data collection helps to:

1. Make informed decisions.

2. Identify problems in a community.
3. Compare performance over time.
4. Plan the use of available resources.
5. Monitor changes in a situation.
6. Identify patterns and trends.
7. Evaluate the success of projects.
8. Support research and innovation.

Example from a Kenyan school

A school wants to determine the number of learners who use different means of transport to school. The teacher collects information from learners and organises it into a table.

The findings may help the school:

- Plan arrival and departure arrangements.
- Improve learner safety.
- Identify transport-related challenges.
- Communicate with parents and guardians.

3.0 ORGANISATION OF DATA

3.1 Meaning of data organisation

Data organisation is the process of arranging collected information into a meaningful order so that it can be easily understood, analysed and interpreted.

Raw data is often difficult to analyse, especially when many observations have been collected. Organising it into a frequency distribution table makes the information easier to study.

3.2 Why data should be organised

Data organisation helps to:

- Reduce confusion when handling large amounts of information.
- Identify the most common values.
- Identify the smallest and largest observations.
- Compare different groups of observations.
- Calculate statistical measures easily.
- Prepare graphs and charts.
- Communicate findings clearly.

3.3 Arranging data in ascending and descending order

Data may be arranged in two main ways.

A. Ascending order

Ascending order means arranging data from the smallest value to the largest value.

Example:

Raw data:

8, 5, 7, 4, 9, 6, 5, 8

Ascending order:

4, 5, 5, 6, 7, 8, 8, 9

B. Descending order

Descending order means arranging data from the largest value to the smallest value.

Using the same data:

9, 8, 8, 7, 6, 5, 5, 4

3.4 Meaning of a frequency distribution table

A frequency distribution table is a table that shows the values or class intervals in a data set and the number of observations occurring in each value or class.

It provides a systematic way of presenting data.

3.5 Frequency distribution table for discrete data

Discrete data consists of countable values, usually whole numbers.

Worked Example 1

The following are the numbers of books read by 15 learners in one month:

2, 3, 1, 2, 4, 3, 2, 5, 1, 3, 2, 4, 3, 2, 1

Construct a frequency distribution table.

Step 1: Identify the different values.

The values are 1, 2, 3, 4 and 5.

Step 2: Count the number of times each value occurs.

Step 3: Construct the table.

Number of books	Tally	Frequency
1	III	3
2	IIII/	5
3	IIII	4
4	II	2
5	I	1
Total		**15**

The total frequency is 15, which is equal to the number of learners investigated.

3.6 Frequency distribution table for grouped continuous data

Continuous data can take any value within a given range. It is commonly obtained through measurement.

Examples include:

- Height.
- Mass.
- Time.
- Distance.
- Temperature.

When many observations are collected, they can be grouped into class intervals.

Worked Example 2

The heights, in centimetres, of 30 learners in a Kenyan secondary school are recorded below:

148, 152, 155, 161, 157, 149, 163, 158, 154, 166,

151, 159, 162, 153, 167, 156, 160, 165, 150, 164,

158, 155, 168, 152, 161, 157, 163, 154, 159, 166.

Construct a frequency distribution table using class intervals of width 5 cm.

Step 1: Identify the smallest and largest values.

Smallest height = 148 cm.

Largest height = 168 cm.

Step 2: Select suitable class intervals.

Using a class width of 5 cm:

- 145–149
- 150–154
- 155–159
- 160–164
- 165–169

Step 3: Count the observations in each class.

Step 4: Construct the frequency distribution table.

Height (cm)	Class boundaries (cm)	Frequency
145–149	144.5–149.5	2
150–154	149.5–154.5	6
155–159	154.5–159.5	8
160–164	159.5–164.5	8
165–169	164.5–169.5	6
Total		**30**

The frequencies add up to 30, confirming that all observations have been included.

3.7 Class limits and class boundaries

Class limits are the lowest and highest values written in a class interval.

For the class interval 20–24:

- Lower class limit = 20.
- Upper class limit = 24.

Class boundaries are the actual dividing points between adjacent classes when continuous measurements are recorded to the nearest whole number.

For the class interval 20–24:

- Lower class boundary = 19.5.
- Upper class boundary = 24.5.

The boundaries remove gaps between consecutive classes.

3.8 Important parts of a frequency distribution table

A complete frequency distribution table may contain:

1. **Class interval:** The range of observations in a group.
2. **Class boundaries:** The actual limits separating adjacent continuous classes.
3. **Tally:** Marks used to count observations.
4. **Frequency (f):** Number of observations in a class.
5. **Class midpoint (x):** The central value of a class.
6. **Cumulative frequency (cf):** The running total of frequencies.
7. **Product of frequency and midpoint (fx):** Used when calculating the mean.

3.9 Calculating class midpoints

The class midpoint is the average of the lower and upper class limits.

$$x = \frac{\text{Lower class limit} + \text{Upper class limit}}{2}$$

Worked Example 3

Find the midpoint of the class interval 30–39.

$$x = \frac{30 + 39}{2}$$

$$x = \frac{69}{2}$$

$$\boxed{x = 34.5}$$

3.10 Cumulative frequency

Cumulative frequency is the running total of frequencies up to and including a particular class.

It is obtained by adding each frequency to the preceding cumulative frequency.

Worked Example 4

Complete the cumulative frequency column.

Class interval	Frequency (f)	Cumulative frequency (cf)
10–19	3	3
20–29	5	8
30–39	7	15
40–49	4	19
50–59	1	20
Total	**20**	

Explanation:

- First cumulative frequency = 3.
- Second = 3 + 5 = 8.
- Third = 8 + 7 = 15.
- Fourth = 15 + 4 = 19.
- Fifth = 19 + 1 = 20.

The final cumulative frequency must equal the total frequency.

4.0 PRESENTATION OF DATA USING HISTOGRAMS

4.1 Meaning of a histogram

A histogram is a graphical representation of a frequency distribution for continuous numerical data. It consists of rectangular bars drawn over class intervals.

The height of each bar represents the frequency when all class intervals have equal widths.

Unlike a bar graph, the bars in a histogram touch each other because the class intervals represent continuous data.

4.2 Characteristics of a histogram

A histogram has the following features:

1. It represents numerical data grouped into continuous class intervals.
2. The horizontal axis represents class boundaries or class intervals.
3. The vertical axis represents frequency for equal class widths.
4. The bars touch each other because there are no gaps between continuous intervals.
5. The width of each bar represents the class width.
6. The height of each bar represents the frequency when class widths are equal.
7. The bars are rectangular.
8. A suitable title should be provided.
9. Both axes should be clearly labelled.
10. An appropriate scale should be selected.

4.3 Difference between a bar graph and a histogram

Bar graph	Histogram
Usually represents categorical or discrete data.	Represents grouped continuous numerical data.
Bars are separated by gaps.	Bars touch each other.
Bars may have different widths in some presentations.	Bar widths represent class widths.
Order of categories may be changed.	Class intervals follow numerical order.
Height represents the value or frequency.	For equal class widths, height represents frequency.

4.4 Histograms with uniform class widths

Uniform class widths are class intervals that have equal widths.

For example:

- 0–9
- 10–19
- 20–29
- 30–39
- 40–49

Each class has a width of 10 units.

For uniform class widths, the frequency can be represented directly by the height of each rectangle.

This makes it easy to compare the frequencies of different classes.

4.5 Steps for drawing a histogram

1. Prepare a frequency distribution table.
2. Identify the class intervals and their frequencies.
3. Confirm that all class intervals have equal widths.
4. Determine the class boundaries.
5. Draw two perpendicular axes.
6. Label the horizontal axis with the class boundaries.
7. Label the vertical axis with frequency.
8. Select a suitable scale for the vertical axis.
9. Draw rectangular bars whose widths correspond to the class intervals.
10. Make the bars touch each other.
11. Give the histogram a suitable title.
12. Check that the bars accurately represent the frequencies.

4.6 Worked Example 5: Drawing a histogram

The following table shows the marks scored by 40 learners in a Mathematics examination.

Marks	Frequency
0-9	2
10-19	5
20-29	9
30-39	12
40-49	8
50-59	4
Total	**40**

Draw a histogram to represent the information.

Step 1: Identify the class widths.

Each class has a width of 10 marks.

Step 2: Determine the class boundaries.

Class interval	Class boundaries	Frequency
0-9	-0.5-9.5	2
10-19	9.5-19.5	5
20-29	19.5-29.5	9
30-39	29.5-39.5	12
40-49	39.5-49.5	8
50-59	49.5-59.5	4

Step 3: Draw and label the axes.

- Horizontal axis: Marks.
- Vertical axis: Frequency.

Step 4: Draw the bars.

The bars should have heights of 2, 5, 9, 12, 8 and 4 respectively.

Histogram: Marks scored by 40 learners

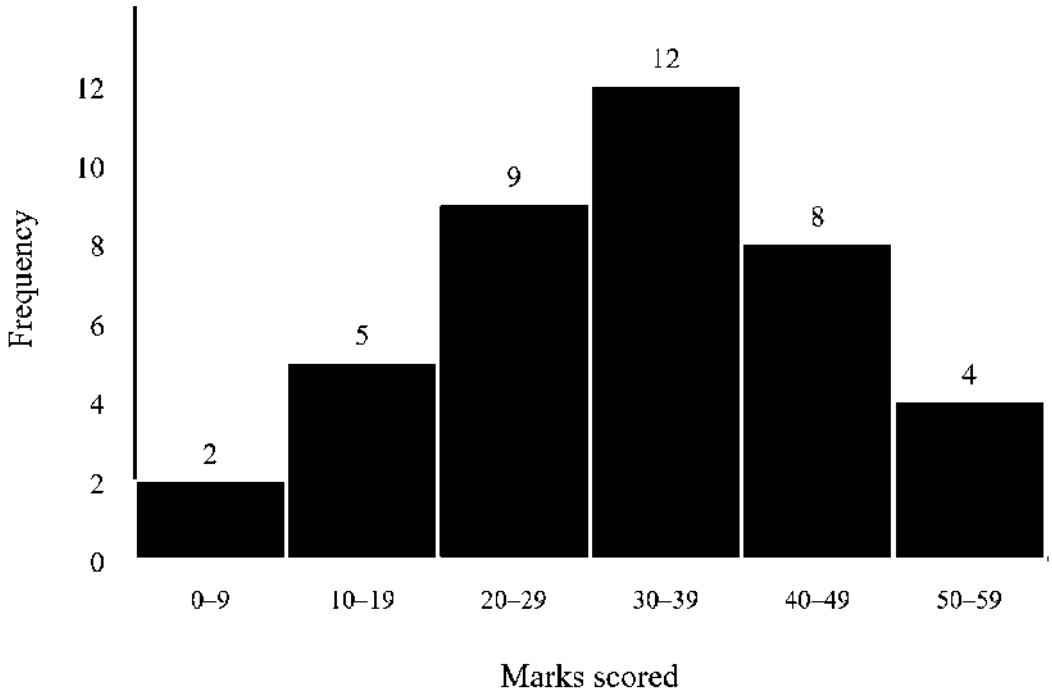


Illustration of a histogram for equal class widths. Adjacent rectangles touch.

Interpretation of the histogram

From the histogram:

- The highest frequency is 12.
- The modal class is 30–39.
- The lowest frequency is 2.
- Most learners scored between 30 and 39 marks.
- Only two learners scored between 0 and 9 marks.
- The total number of learners is 40.

4.7 Interpreting a histogram

A histogram can be used to obtain information about a data set.

It helps us to identify:

1. The class with the highest frequency.
2. The class with the lowest frequency.
3. The modal class.
4. The total number of observations.
5. The general distribution of observations.
6. Whether most observations are concentrated in lower or higher classes.
7. Whether the data is spread over a wide or narrow range.

4.8 Important note about unequal class widths

This sub-strand focuses on histograms with uniform class widths.

When class widths are unequal, the height of a histogram bar should not simply be the frequency. Instead, frequency density is used:

$$\text{Frequency density} = \frac{\text{Frequency}}{\text{Class width}}$$

The area of each bar then represents the frequency.

For equal class widths, the frequency density is proportional to frequency, so direct frequency heights can be used.

5.0 MEASURES OF CENTRAL TENDENCY

5.1 Meaning of measures of central tendency

Measures of central tendency are statistical values that describe the centre or typical value of a data set.

They help to summarise a large amount of information using a single representative value.

The three main measures of central tendency are:

1. Mean.
2. Median.
3. Mode.

5.2 Importance of measures of central tendency

Measures of central tendency are useful because they:

- Summarise large sets of data.
- Make comparison between groups easier.
- Help in planning and decision-making.
- Show typical performance.
- Help identify common patterns.
- Support the interpretation of examination results.
- Assist businesses in understanding sales patterns.
- Help farmers estimate typical production levels.
- Support government planning and resource allocation.

5.3 THE MEAN

Meaning of the mean

The mean is the arithmetic average of a set of observations. It is obtained by adding all the observations and dividing the sum by the total number of observations.

The mean is commonly called the average.

A. Mean of ungrouped data

The formula is:

$$\boxed{\bar{x} = \frac{\sum x}{n}}$$

Where:

- $\bar{x}$ = mean.
- $\sum x$ = sum of all observations.
- n = number of observations.

Worked Example 6

The marks scored by six learners in a Mathematics test were:

$40,\ 50,\ 60,\ 70,\ 80,\ 90$

Calculate the mean mark.

Step 1: Add all the observations.

$$\sum x = 40 + 50 + 60 + 70 + 80 + 90$$

$$\sum x = 390$$

Step 2: Determine the number of observations.

$$n = 6$$

Step 3: Apply the formula.

$$\bar{x} = \frac{390}{6}$$

$$\boxed{\bar{x} = 65}$$

Therefore, the mean mark is 65.

B. Mean of grouped data

When data is grouped into class intervals, the exact individual observations are not available. We therefore use class midpoints to estimate the mean.

The formula is:

$$\boxed{\bar{x} = \frac{\sum fx}{\sum f}}$$

Where:

- f = frequency.
- x = class midpoint.
- fx = product of frequency and midpoint.
- $\sum fx$ = sum of all frequency-midpoint products.
- $\sum f$ = total frequency.

Steps for calculating the mean of grouped data

1. Write down the class intervals.
2. Identify the frequency of each class.
3. Calculate the midpoint of each class.
4. Multiply each midpoint by its frequency.
5. Find the sum of all frequencies.
6. Find the sum of all fx values.
7. Divide $\sum fx$ by $\sum f$.

Worked Example 7

The table below shows the marks scored by 30 learners in a Mathematics examination.

Calculate the mean mark.

Marks	Frequency (f)
10-19	3
20-29	5
30-39	8
40-49	9
50-59	5
Total	**30**

Step 1: Calculate the class midpoints.

For the first class:

$$x = \frac{10 + 19}{2} = 14.5$$

Repeat for all classes.

Step 2: Calculate fx.

Marks	Frequency (f)	Midpoint (x)	fx
10-19	3	14.5	43.5
20-29	5	24.5	122.5
30-39	8	34.5	276
40-49	9	44.5	400.5
50-59	5	54.5	272.5
Total	**30**		**1115**

Step 3: Apply the formula.

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$\bar{x} = \frac{1115}{30}$$

$$\boxed{\bar{x} = 37.17}$$

Therefore, the estimated mean mark is approximately 37.17.

The mean is an estimate because the individual marks within each class are represented by their class midpoints.

C. Properties of the mean

- It uses all observations in the data set.
- It is easy to calculate.
- It is useful when comparing groups.
- It can be affected by extremely high or low values.
- It may not be an actual observation in the data set.

- It is suitable for many numerical data sets.

D. Limitations of the mean

- It can be distorted by extreme values.
- It may give a value that does not actually occur in the data.
- It is not always suitable for highly skewed data.
- It cannot be determined for purely categorical data such as favourite colours.

5.4 THE MEDIAN

Meaning of the median

The median is the middle value of a data set when the observations are arranged in ascending or descending order.

It divides an ordered data set into two equal parts:

- Half of the observations lie below or at the median.
- Half of the observations lie above or at the median.

A. Median of ungrouped data

The method used depends on whether the number of observations is odd or even.

Case 1: Odd number of observations

The formula for the position of the median is:

$$\boxed{\frac{n+1}{2}}$$

Worked Example 8

Find the median of:

$7,\ 3,\ 9,\ 5,\ 11,\ 8,\ 6$

Step 1: Arrange the data in ascending order.

$3,\ 5,\ 6,\ 7,\ 8,\ 9,\ 11$

Step 2: Count the observations.

$$n = 7$$

Step 3: Find the middle position.

$$\frac{n+1}{2} = \frac{7+1}{2} = 4$$

The fourth observation is 7.

$$\boxed{\text{Median} = 7}$$

Case 2: Even number of observations

When the number of observations is even, the median is the average of the two middle observations.

Worked Example 9

Find the median of:

$4,\ 8,\ 6,\ 10,\ 12,\ 14$

Step 1: Arrange the data in ascending order.

$4,\ 6,\ 8,\ 10,\ 12,\ 14$

Step 2: Identify the two middle observations.

The two middle values are 8 and 10.

Step 3: Calculate their average.

$$\text{Median} = \frac{8 + 10}{2}$$

$$\boxed{\text{Median} = 9}$$

B. Median of grouped data

For grouped data, the median is estimated using the cumulative frequency distribution.

The formula is:

$$\boxed{\text{Median} = L + \left(\frac{\frac{N}{2} - cf}{f}\right)h}$$

Where:

- L = lower class boundary of the median class.
- N = total frequency.
- cf = cumulative frequency before the median class.
- f = frequency of the median class.
- h = class width.

How to identify the median class

1. Calculate the total frequency, N.
2. Find half the total frequency, .
3. Calculate the cumulative frequencies.
4. Identify the first class whose cumulative frequency is equal to or greater than .
5. This class is the median class.

Worked Example 10

The table below shows the ages of 40 workers at a tea-processing factory in Kericho County.

Calculate the median age.

Age (years)	Frequency (f)	Cumulative frequency (cf)
20–24	4	4
25–29	7	11
30–34	12	23
35–39	10	33
40–44	7	40
Total	**40**	

Step 1: Calculate half the total frequency.

$$N = 40$$

$$\frac{N}{2} = \frac{40}{2} = 20$$

Step 2: Identify the median class.

The cumulative frequency first exceeds 20 in the class 30–34.

Therefore, the median class is 30–34.

Step 3: Identify the values required.

- $L = 29.5$
- $N = 40$
- $cf = 11$
- $f = 12$
- $h = 5$

Step 4: Substitute into the formula.

$$\text{Median} = L + \left(\frac{\frac{N}{2} - cf}{f}\right)h$$

$$= 29.5 + \left(\frac{20 - 11}{12}\right) \times 5$$

$$= 29.5 + \frac{9}{12} \times 5$$

$$= 29.5 + 3.75$$

$$\boxed{\text{Median} = 33.25 \text{ years}}$$

Therefore, the estimated median age is 33.25 years.

C. Properties of the median

- It divides an ordered data set into two equal parts.
- It is not greatly affected by extremely high or low values.
- It is useful for data with extreme values.
- It is suitable for numerical data.
- It may not be one of the original observations.
- It is useful when the data is skewed.

D. Limitations of the median

- It does not use the actual size of every observation in the same way as the mean.
- The data must first be arranged or grouped.
- For grouped data, the median is an estimate.
- It is less suitable for some mathematical operations than the mean.

5.5 THE MODE

Meaning of the mode

The mode is the value that occurs most frequently in a data set.

It represents the most common or popular observation.

A data set may have:

- One mode: unimodal.
- Two modes: bimodal.
- More than two modes: multimodal.
- No mode: when no value occurs more frequently than the others.

A. Mode of ungrouped data

Worked Example 11

Find the mode of:

$4,\ 6,\ 5,\ 4,\ 7,\ 8,\ 4,\ 6,\ 5,\ 4$

Step 1: Count the frequency of each value.

Value	Frequency
4	4
5	2
6	2
7	1
8	1

The value 4 occurs four times, which is more than any other value.

$$\boxed{\text{Mode} = 4}$$

B. Mode of grouped data

For grouped data, the mode is estimated using the modal class.

The modal class is the class interval with the highest frequency.

The formula is:

$$\boxed{\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h}$$

Where:

- L = lower class boundary of the modal class.
- f_1 = frequency of the modal class.
- f_0 = frequency of the class before the modal class.
- f_2 = frequency of the class after the modal class.
- h = class width.

Steps for calculating the mode of grouped data

1. Identify the class with the highest frequency.
2. This class is the modal class.
3. Identify the lower class boundary of the modal class.
4. Identify the frequency of the modal class.
5. Identify the frequencies of the preceding and succeeding classes.
6. Determine the class width.
7. Substitute the values into the formula.
8. Simplify to obtain the estimated mode.

Worked Example 12

The table below shows the daily milk production, in litres, of 40 dairy cows on a farm in Nyandarua County.

Calculate the estimated mode.

Milk production (litres)	Frequency
5-9	4
10-14	8
15-19	14
20-24	9
25-29	5
Total	**40**

Step 1: Identify the modal class.

The highest frequency is 14.

Therefore, the modal class is 15-19.

Step 2: Identify the required values.

- $L = 14.5$
- $f_1 = 14$
- $f_0 = 8$
- $f_2 = 9$
- $h = 5$

Step 3: Apply the formula.

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 14.5 + \left(\frac{14 - 8}{2(14) - 8 - 9}\right) \times 5$$

$$= 14.5 + \left(\frac{6}{28 - 17}\right) \times 5$$

$$= 14.5 + \frac{6}{11} \times 5$$

$$= 14.5 + 2.727$$

$$\boxed{\text{Mode} \approx 17.23 \text{ litres}}$$

Therefore, the estimated modal daily milk production is approximately 17.23 litres.

C. Properties of the mode

- It identifies the most common value.
- It is easy to understand.
- It can be used with categorical data.
- It is not affected by extreme values.
- It may not be unique.
- It may not represent the centre of the data.

D. Limitations of the mode

- A data set may have more than one mode.
- Some data sets may have no mode.
- It does not use all observations.
- It may not be suitable for further mathematical calculations.

6.0 COMPARISON OF MEAN, MEDIAN AND MODE

The three measures of central tendency describe a data set in different ways.

Feature	Mean	Median	Mode
Meaning	Arithmetic average	Middle value	Most frequent value
Uses all observations	Yes	Uses position in ordered data	No
Affected by extreme values	Yes	Usually not greatly	No
Requires ordering data	No for ungrouped data	Yes	No
Suitable for categories	No	Generally no	Yes
Can have more than one value	No	No	Yes
Grouped-data method		Median formula	Mode formula

6.1 Relationship between mean, median and mode

For a symmetrical, unimodal distribution, the mean, median and mode are approximately equal.

$$\boxed{\text{Mean} = \text{Median} = \text{Mode}}$$

For moderately skewed distributions, an empirical relationship sometimes used is:

$$\boxed{\text{Mode} \approx 3(\text{Median}) - 2(\text{Mean})}$$

This is an approximate relationship, not an exact rule for every data set.

6.2 Choosing an appropriate measure

- Use the **mean** when all numerical observations are important and there are no extreme values that distort the average.
- Use the **median** when the data contains extreme values or is strongly skewed.
- Use the **mode** when identifying the most popular or common value is important, including for categorical information.

7.0 APPLICATION OF MEASURES OF CENTRAL TENDENCY IN REAL-LIFE SITUATIONS

Measures of central tendency are useful in many activities carried out by individuals, schools, businesses, farmers, government departments and other organisations.

They help people summarise information and make informed decisions.

7.1 Application of the mean in everyday life

1. Education

Teachers use the mean to calculate the average marks scored by learners in examinations.

Example:

A learner scores 60, 70, 80 and 90 in four subjects.

$$\text{Mean} = \frac{60 + 70 + 80 + 90}{4}$$

$$= \frac{300}{4} = \boxed{75}$$

The learner's average score is 75 marks.

The mean can help teachers:

- Compare the performance of different classes.
- Monitor learners' progress.
- Identify areas that require improvement.
- Evaluate the effectiveness of teaching methods.

2. Agriculture

Farmers can calculate the average crop yield per acre to estimate production.

Example:

A farmer harvests 18, 20, 22, 16 and 24 bags of maize from five acres.

$$\text{Mean} = \frac{18 + 20 + 22 + 16 + 24}{5}$$

$$= \frac{100}{5} = \boxed{20 \text{ bags}}$$

The average yield is 20 bags per acre.

This information can help the farmer:

- Estimate future harvests.
- Compare productivity between farms.
- Plan for storage and transportation.
- Determine whether farming methods are effective.

3. Business and trade

Shopkeepers use average sales to estimate the amount of stock required.

Example:

A shop records daily sales of KSh 4,000, KSh 5,000, KSh 6,000, KSh 3,000 and KSh 7,000.

$$\text{Mean sales} = \frac{4000 + 5000 + 6000 + 3000 + 7000}{5}$$

$$= \boxed{\text{KSh } 5{,}000}$$

The shopkeeper can use this average to plan stock purchases.

4. Transport

Transport operators can calculate the average number of passengers carried daily to plan vehicle schedules.

5. Weather forecasting and environmental studies

Meteorologists can calculate average daily or monthly temperatures and rainfall amounts to study weather patterns.

7.2 Application of the median in everyday life

1. House prices

Real estate agents may use the median house price to describe typical property prices in an area.

This is useful because a few very expensive houses may greatly increase the mean price.

2. Household income

Researchers may use the median income to describe the income of a typical household.

The median is often useful when some households earn extremely high incomes compared with the majority.

3. Examination performance

A school may use the median mark to establish the middle performance level of learners.

4. Land sizes

Surveyors and agricultural researchers may use the median to describe typical farm sizes when a few farms are exceptionally large.

5. Waiting time

A hospital may use the median waiting time to describe how long patients typically wait before receiving services.

7.3 Application of the mode in everyday life

1. Retail businesses

Shopkeepers use the mode to identify the most frequently purchased product or product size.

Example:

A shoe shop records the following shoe sizes sold:

$38,\ 40,\ 39,\ 40,\ 41,\ 40,\ 39,\ 40,\ 42$

The mode is 40.

The shopkeeper may stock more shoes of size 40 because it is the most frequently purchased size in the recorded data.

2. Agriculture

Farmers may use the mode to identify the most common crop variety grown in a particular area.

3. Education

Teachers may identify the most frequently selected answer in a multiple-choice question to establish common learner responses.

4. Health services

Health officers may identify the most frequently reported symptoms or common age groups attending a health facility.

5. Clothing industries

Manufacturers may use the mode to identify the most commonly required clothing sizes.

7.4 Real-life comparison of the three measures

Consider the daily sales, in thousands of Kenya shillings, recorded by a small shop for seven days:

$2,\ 3,\ 3,\ 4,\ 4,\ 5,\ 14$

Mean

$$\bar{x} = \frac{2+3+3+4+4+5+14}{7}$$

$$= \frac{35}{7} = \boxed{5}$$

Median

The data is already arranged in ascending order.

The middle value is 4.

$$\boxed{\text{Median} = 4}$$

Mode

The values 3 and 4 each occur twice.

$$\boxed{\text{Modes} = 3 \text{ and } 4}$$

Interpretation

- Mean sales = KSh 5,000.
- Median sales = KSh 4,000.
- The most frequently occurring sales values are KSh 3,000 and KSh 4,000.
- The unusually high sales of KSh 14,000 increase the mean.

In this situation, the median may provide a better description of a typical day's sales than the mean.

7.5 Factors to consider when choosing a measure of central tendency

When choosing an appropriate measure, consider:

1. The type of data collected.
2. The purpose of the investigation.
3. Whether the data contains extreme values.
4. Whether the data is symmetrical or skewed.
5. Whether the most common value is required.
6. Whether a representative average is needed.
7. Whether the data is grouped or ungrouped.
8. The accuracy required for the decision being made.

8.0 INTERPRETATION OF STATISTICAL INFORMATION

8.1 Meaning of data interpretation

Data interpretation is the process of examining organised information and explaining what it means in relation to the purpose of an investigation.

It involves identifying patterns, comparing values, explaining differences and drawing conclusions.

8.2 Steps in interpreting statistical information

1. Identify the purpose of the investigation.
2. Examine the frequency distribution table or histogram.
3. Identify the highest and lowest frequencies.
4. Calculate appropriate measures of central tendency.
5. Compare the mean, median and mode where relevant.
6. Identify unusual values or patterns.
7. Draw conclusions supported by the data.
8. Suggest suitable actions based on the findings.

8.3 Worked Example 13: Interpreting school attendance

The following table shows the number of learners absent from a school over 30 school days.

Number absent	Frequency
0–4	5
5–9	10
10–14	9
15–19	4
20–24	2
Total	**30**

Questions

a) Which class has the highest frequency?

b) How many days had fewer than 10 learners absent?

c) How many days were investigated?

d) What is the modal class?

Solutions

a) The class 5–9 has the highest frequency of 10.

b) Days with fewer than 10 learners absent:

$$5 + 10 = \boxed{15 \text{ days}}$$

c) The total number of days is 30.

d) The modal class is 5–9.

Possible conclusion

The school recorded between 5 and 9 absent learners on most of the days investigated. The school management may investigate factors contributing to absenteeism and develop strategies to improve attendance.

8.4 Responsible use of statistical information

Statistical information should be used responsibly.

- Conclusions should be based on actual evidence.

- Data should not be manipulated to mislead people.
- The sample should be appropriate for the investigation.
- Personal information should remain confidential.
- The limitations of the data should be acknowledged.
- Graphs should use suitable scales and accurate labels.
- Comparisons should be fair and meaningful.

PRACTICAL CBE PROJECT: COLLECTING, ORGANISING AND ANALYSING DATA

Group project

Project title: A statistical investigation of learners' heights in a school

Project purpose

Learners work in groups to collect, organise, represent and analyse data from their immediate environment using a frequency distribution table, histogram and measures of central tendency.

Project learning outcomes

By completing the project, learners should be able to:

1. Identify a suitable statistical investigation.
2. Plan how to collect relevant data.
3. Select suitable methods and resources for data collection.
4. Collect and record information accurately.
5. Group data into equal class intervals.
6. Prepare a frequency distribution table.
7. Draw a histogram using equal class widths.
8. Calculate the mean, median and mode of grouped data.
9. Interpret the results.
10. Present findings and make recommendations.

9.1 Project preparation

Step 1: Identify an area or issue for investigation

Each group should select one issue from the immediate environment.

Possible project topics include:

Project area	Data to collect
Learners' heights	Height in centimetres
Learners' weights	Mass in kilograms
School attendance	Number of learners present daily
Shoe sizes	Shoe sizes of learners
Household water use	Litres used daily
Farm production	Bags of maize harvested
School transport	Time taken to reach school
Library use	Number of books borrowed

Step 2: Identify the target population

The group should identify the people or items from which data will be collected.

For example, the population may be all Grade 11 learners in a school.

Step 3: Decide on the sample

If the population is large, the group may select a manageable sample.

For example, a group may select 30 learners from the school.

The sample should be selected fairly to avoid bias.

Step 4: Identify the resources required

Possible resources include:

- Measuring tape or height metre.
- Weighing scale.
- Recording sheets.
- Pens and pencils.
- Ruler.
- Graph paper.
- Calculator.
- Computer or tablet.
- Spreadsheet software.
- Project notebook.

Step 5: Plan how data will be collected

The group should agree on:

- What information is required.
- Who will collect the data.
- Where data will be collected.
- When data will be collected.
- Which instruments will be used.
- How the information will be recorded.
- How privacy and safety will be maintained.

9.2 Project implementation

Example: Collecting the heights of 30 learners

Suppose a group collects the heights of 30 learners and obtains the following grouped data:

Height (cm)	Frequency
140–144	2
145–149	4
150–154	7
155–159	9
160–164	5
165–169	3
Total	**30**

Task 1: Calculate the mean

Height (cm)	f	Midpoint (x)	fx
140–144	2	142	284
145–149	4	147	588
150–154	7	152	1064
155–159	9	157	1413
160–164	5	162	810
165–169	3	167	501
Total	**30**		**4660**

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$= \frac{4660}{30}$$

$$\boxed{\bar{x} = 155.33 \text{ cm}}$$

Task 2: Calculate the median

Cumulative frequency:

Height (cm)	Frequency	Cumulative frequency
140–144	2	2
145–149	4	6
150–154	7	13
155–159	9	22
160–164	5	27
165–169	3	30

$$N = 30, \quad \frac{N}{2} = 15$$

The median class is 155–159 because its cumulative frequency is the first to reach or exceed 15.

Therefore:

- $L = 154.5$
- $cf = 13$
- $f = 9$
- $h = 5$

$$\text{Median} = 154.5 + \left(\frac{15 - 13}{9}\right) \times 5$$

$$= 154.5 + \frac{10}{9}$$

$$\boxed{\text{Median} \approx 155.61 \text{ cm}}$$

Task 3: Calculate the mode

The modal class is 155–159 because it has the highest frequency of 9.

- $L = 154.5$
- $f_1 = 9$
- $f_0 = 7$
- $f_2 = 5$
- $h = 5$

$$\text{Mode} = 154.5 + \left(\frac{9-7}{2(9)-7-5}\right) \times 5$$

$$= 154.5 + \left(\frac{2}{6}\right) \times 5$$

$$= 154.5 + 1.67$$

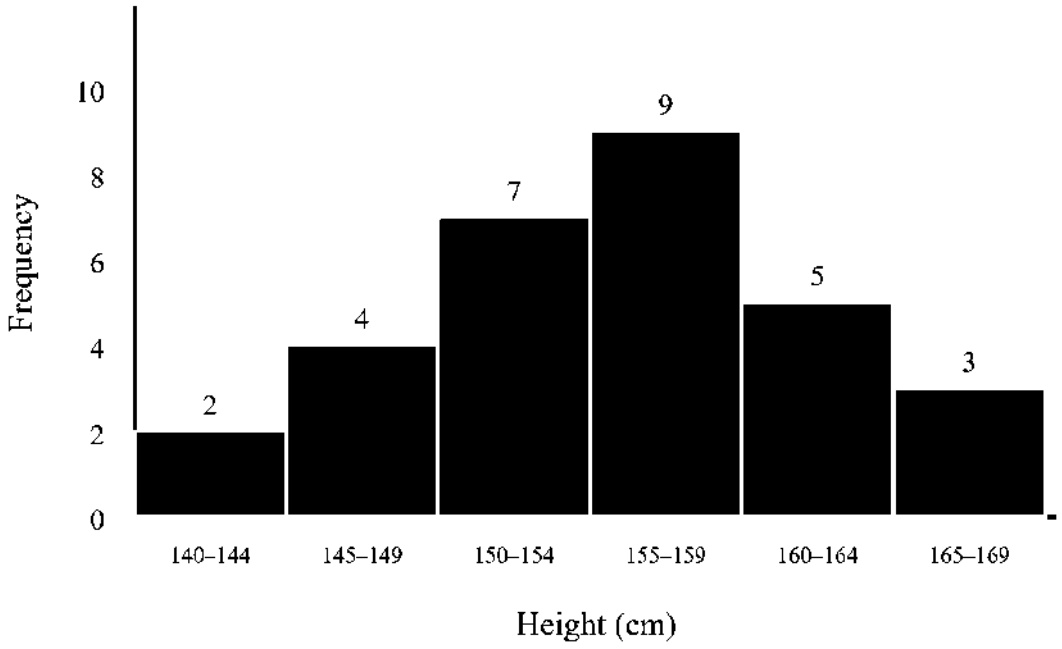
$$\boxed{\text{Mode} \approx 156.17 \text{ cm}}$$

Task 4: Draw a histogram

Learners should draw a histogram with:

- Height intervals on the horizontal axis.
- Frequency on the vertical axis.
- Equal class widths of 5 cm.
- Bars touching each other.
- An appropriate title and scale.

Histogram: Heights of 30 learners



Illustrative histogram using equal class widths.

9.3 Project analysis and discussion

After calculating the three measures, learners should discuss:

1. What is the estimated average height?
2. What is the estimated middle height?
3. Which height interval contains the largest number of learners?
4. What does the histogram reveal about the distribution of heights?
5. Why might the mean and median differ?
6. How can the results be used in planning school activities?
7. What challenges were experienced during data collection?
8. How could the investigation be improved?

Possible applications of the findings

The findings may help the school to:

- Understand the general distribution of learners' heights.
- Plan certain physical activities and sports equipment.
- Organise age-appropriate or size-appropriate resources where relevant.
- Practise accurate measurement and statistical reporting.

The results should not be used to label, embarrass or discriminate against individual learners.

9.4 Project documentation

Each group should prepare a report containing:

1. Project title.
2. Names of group members.
3. Purpose of the investigation.
4. Description of the target population and sample.
5. Data collection method.
6. Resources used.
7. Raw data or a suitable record of observations.
8. Frequency distribution table.
9. Histogram.
10. Calculations of mean, median and mode.
11. Interpretation of results.
12. Challenges encountered.
13. Recommendations.
14. Conclusion.
15. Evidence of group participation.

The report should be stored in a print or digital portfolio.

9.5 Suggested project assessment rubric

Assessment area	Marks
Identification of the problem and planning	5
Selection and use of appropriate resources	5
Accuracy of data collection and recording	5
Preparation of frequency distribution table	5

Assessment area	Marks
Drawing and presentation of histogram	5
Calculation of mean, median and mode	9
Interpretation and recommendations	4
Documentation and presentation	2
Total	**40**

SUMMARY OF IMPORTANT FORMULAE

Concept	Formula
Class midpoint	$x = \frac{\text{Lower limit}+\text{Upper limit}}{2}$
Class width	$h = \text{Upper boundary} - \text{Lower boundary}$
Mean of ungrouped data	$\bar{x} = \frac{\sum x}{n}$
Mean of grouped data	$\bar{x} = \frac{\sum fx}{\sum f}$
Median position for odd n	$\frac{n+1}{2}$
Median of grouped data	$L + \left(\frac{\frac{N}{2} - cf}{f}\right)h$
Mode of grouped data	$L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$
Frequency density	$\frac{\text{Frequency}}{\text{Class width}}$

Symbols used

- $\bar{x}$ = mean.
- $\sum$ = sum of.
- x = observation or class midpoint, depending on context.
- f = frequency.
- fx = frequency multiplied by midpoint.
- n = number of observations.
- N = total frequency.
- L = lower class boundary.
- cf = cumulative frequency before the median class.
- h = class width.
- f_1 = frequency of modal class.
- f_0 = frequency before modal class.
- f_2 = frequency after modal class.

REVISION EXERCISES

Section A: 30 marks

SECTION A: MULTIPLE-CHOICE QUESTIONS

Instructions: Answer all questions. Choose the correct answer from the alternatives A, B, C and D. Each question carries 1 mark.

1. Which of the following best describes statistics?

A. The study of shapes and sizes only.

B. The collection, organisation, presentation, analysis and interpretation of data.

C. The study of numbers without calculations.

D. The process of measuring length only.

2. Which one of the following is an example of primary data?

A. Information obtained from a published government report.

B. Information obtained from a textbook.

C. Heights measured directly from learners.

D. Information obtained from an old newspaper.

3. Which method of data collection involves asking respondents written questions?

A. Observation.

B. Measurement.

C. Counting.

D. Questionnaire.

4. The number of times an observation occurs in a data set is called:

A. Class width.

B. Frequency.

C. Midpoint.

D. Range.

5. Which of the following is continuous data?

A. Number of learners in a class.

B. Number of books in a library.

C. Height of a learner.

D. Number of vehicles in a parking area.

6. The class midpoint of the interval 20–30 is:

A. 20.

B. 25.

C. 30.

D. 50.

7. Find the class width of the interval 35.5–40.5.

A. 4.

B. 4.5.

C. 5.

D. 6.

8. Which of the following is the correct lower class boundary for the class interval 40–49, when observations are recorded to the nearest whole number?

A. 39.

B. 39.5.

C. 40.5.

D. 49.5.

9. The cumulative frequency is obtained by:

A. Multiplying all frequencies.

B. Dividing the total frequency by the number of classes.

C. Adding frequencies successively.

D. Subtracting the lowest frequency from the highest.

10. Which graphical representation is most suitable for grouped continuous data?

A. Pictogram.

B. Pie chart.

C. Histogram.

D. Flow chart.

11. One important characteristic of a histogram is that:

A. Its bars must be separated by gaps.

B. Its bars touch each other.

C. It contains only circular shapes.

D. It does not require labelled axes.

12. In a histogram with equal class widths, the height of each bar represents:

A. The class midpoint.

B. The class boundary.

C. The frequency.

D. The cumulative frequency.

13. Which of the following is the formula for the mean of ungrouped data?

A. $\frac{\sum x}{n}$

B. $\frac{\sum f}{\sum x}$

C. $\frac{\sum fx}{n}$

D. $\frac{n}{\sum x}$

14. Calculate the mean of 4, 6, 8, 10 and 12.

A. 6.

B. 7.

C. 8.

D. 10.

15. Find the median of 3, 5, 7, 9 and 11.

A. 5.

B. 6.

C. 7.

D. 9.

16. Find the median of 2, 4, 6, 8, 10 and 12.

A. 6.

B. 7.

C. 8.

D. 9.

17. The mode of 3, 4, 5, 4, 6, 4, 7 is:

A. 3.

B. 4.

C. 5.

D. 6.

18. Which measure of central tendency is most affected by extreme values?

A. Mode.

B. Median.

C. Mean.

D. All are equally affected.

19. Which measure is most appropriate for identifying the most popular shoe size sold in a shop?

A. Mean.

B. Median.

C. Range.

D. Mode.

20. The total frequency of a grouped frequency distribution is 50. The value of is:

A. 20.

B. 25.

C. 30.

D. 50.

21. Which of the following is used to calculate the mean of grouped data?

A. $\frac{\sum fx}{\sum f}$

B. $\frac{\sum f}{\sum fx}$

C. $\frac{\sum x}{\sum f}$

D. $\frac{\sum f}{n}$

22. The class interval with the highest frequency is called the:

A. Median class.

B. Mean class.

C. Modal class.

D. Midpoint class.

23. The median class is identified using:

A. The highest frequency only.

B. The class midpoint.

C. Half the total frequency and cumulative frequencies.

D. The lowest class boundary.

24. Which of the following is a limitation of the mean?

A. It cannot be calculated from numerical data.

B. It is affected by extreme values.

C. It does not use any observations.

D. It always gives the most common value.

25. A data set has a mean of 15. The sum of 8 observations is:

A. 100.

B. 110.

C. 120.

D. 130.

26. A shopkeeper records shoe sizes sold as 38, 39, 40, 40, 41, 40 and 42. The most appropriate measure for identifying the most common size is:

A. Mean.

B. Median.

C. Mode.

D. Range.

27. Which measure of central tendency is generally more appropriate when a data set contains extremely high or low values?

A. Mean.

B. Median.

C. Midpoint.

D. Frequency.

28. A histogram has equal class widths of 5 units. If the frequencies of two classes are 6 and 10, the heights of their bars should be in the ratio:

A. 3:5.

B. 5:3.

C. 6:10, but with the first bar taller.

D. 10:6, but with the second bar shorter.

29. Which of the following is an appropriate use of the mean in agriculture?

A. Identifying the most popular crop colour.

B. Determining the most common type of farm tool.

C. Estimating the average maize yield per acre.

D. Identifying the most frequently used fertiliser brand.

30. Why should data be organised into a frequency distribution table?

A. To make the data difficult to understand.

B. To remove all observations.

C. To change the actual information collected.

D. To make data easier to analyse and interpret.

SECTION B: STRUCTURED QUESTIONS

Instructions: Answer all questions. Show all necessary working.

QUESTION 1: DATA COLLECTION AND ORGANISATION (8 MARKS)

A teacher collected information on the number of books read by 15 learners during a school holiday.

The observations were:

$$2,\ 3,\ 1,\ 2,\ 4,\ 3,\ 2,\ 5,\ 1,\ 3,\ 2,\ 4,\ 3,\ 2,\ 1$$

a) Define the term raw data. (1 mark)

b) State two methods that can be used to collect data in a school. (2 marks)

c) Construct a frequency distribution table for the information. (4 marks)

d) State one importance of organising data into a frequency distribution table. (1 mark)

QUESTION 2: CLASS INTERVALS AND HISTOGRAMS (8 MARKS)

The following table shows the marks scored by 30 learners in an examination.

Marks	Frequency
0–9	2
10–19	4
20–29	8
30–39	10
40–49	6
Total	**30**

a) Determine the class width. (1 mark)

b) Write down the class boundaries for the interval 20–29. (2 marks)

c) Identify the modal class. (1 mark)

d) Draw a histogram to represent the information. (4 marks)

QUESTION 3: MEAN OF GROUPED DATA (8 MARKS)

The following table shows the masses, in kilograms, of 25 bags of maize harvested by a farmer.

Mass (kg)	Frequency
40–49	3
50–59	5
60–69	8

Mass (kg)	Frequency
70–79	6
80–89	3
Total	**25**

a) Explain the meaning of class midpoint. (1 mark)

b) Calculate the midpoint of the class interval 60–69. (1 mark)

c) Complete a table showing the class midpoints and the products fx. (4 marks)

d) Calculate the estimated mean mass of the bags of maize. (2 marks)

QUESTION 4: MEDIAN OF GROUPED DATA (8 MARKS)

The table below shows the ages of 40 workers in a flower farm in Naivasha.

Age (years)	Frequency
20–24	4
25–29	8
30–34	12
35–39	10
40–44	6
Total	**40**

a) Complete the cumulative frequency column. (2 marks)

b) Identify the median class. (1 mark)

c) State the values of L, cf, f, N and h. (2 marks)

d) Calculate the estimated median age. (3 marks)

QUESTION 5: MODE OF GROUPED DATA (9 MARKS)

The following table shows the daily milk production of dairy cows on a farm in Nyandarua County.

Milk production (litres)	Frequency
5–9	4
10–14	7
15–19	12
20–24	9
25–29	3
Total	**35**

a) Define the term modal class. (1 mark)

b) Identify the modal class. (1 mark)

c) State the values of L, f_1, f_0, f_2 and h. (3 marks)

d) Calculate the estimated mode of the milk production. (4 marks)

QUESTION 6: APPLICATION OF MEASURES OF CENTRAL TENDENCY (9 MARKS)

A shopkeeper in Kisumu recorded the daily sales of a particular product, in thousands of Kenya shillings, over seven days as follows:

$$2,\ 3,\ 3,\ 4,\ 4,\ 5,\ 14$$

a) Calculate the mean daily sales. (2 marks)

b) Determine the median daily sales. (1 mark)

c) Determine the mode or modes. (1 mark)

d) Explain why the mean is higher than the median. (2 marks)

e) State two ways in which measures of central tendency can help the shopkeeper make business decisions. (2 marks)

f) Which measure would you recommend the shopkeeper to use when describing typical daily sales? Give a reason. (1 mark)

ANSWERS AND MARKING SCHEME

SECTION A: MULTIPLE-CHOICE ANSWERS

Question	Answer	Question	Answer	Question	Answer
1	B	11	B	21	A
2	C	12	C	22	C
3	D	13	A	23	C
4	B	14	C	24	B
5	C	15	C	25	C
6	B	16	B	26	C
7	C	17	B	27	B
8	B	18	C	28	A
9	C	19	D	29	C
10	C	20	B	30	D

SECTION B: SUGGESTED ANSWERS AND MARKING SCHEME

QUESTION 1: DATA COLLECTION AND ORGANISATION (8 MARKS)

a) Raw data is information collected in its original form before it is organised or processed. (1)

b) Any two:

- Observation.
- Interview.
- Questionnaire.
- Measurement.
- Counting.

(1 mark each = 2)

c) Frequency distribution table:

Number of books	Tally	Frequency

Number of books	Tally	Frequency
1	III	3
2	IIII/	5
3	IIII	4
4	II	2
5	I	1
Total		**15**

- Correct values or class categories (1)
- Correct tallies (1)
- Correct frequencies (1)
- Correct total (1)

d) Any one:

- Makes data easier to analyse.
- Makes data easier to interpret.
- Helps identify common values.
- Simplifies the presentation of information.

QUESTION 2: CLASS INTERVALS AND HISTOGRAMS (8 MARKS)

$$\text{Class width} = 10$$

b) For the interval 20–29:

Lower boundary = 19.5.

Upper boundary = 29.5.

(1 mark each = 2)

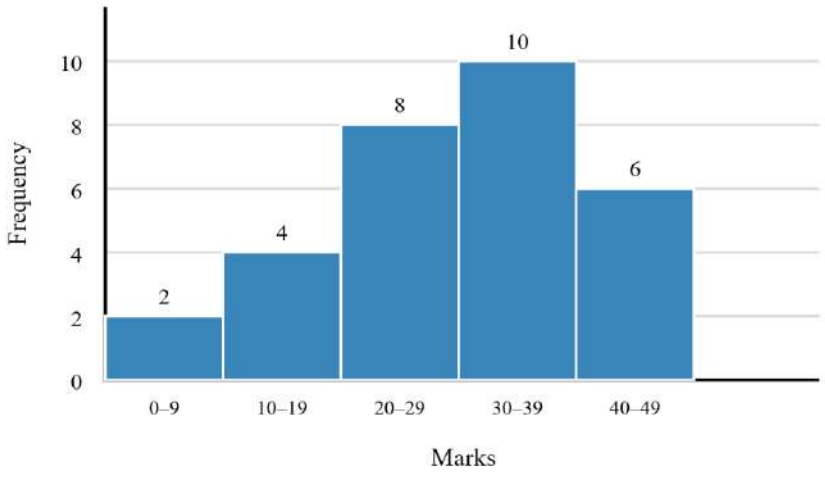
c) Modal class = 30–39. (1)

d) Histogram (4 marks):

- Correctly labelled horizontal axis: Marks (1).
- Correctly labelled vertical axis: Frequency (1).
- Correct bar heights and class intervals (1).
- Bars touching and suitable title (1).

Suggested histogram:

Marks scored by 30 learners



QUESTION 3: MEAN OF GROUPED DATA (8 MARKS)

a) Class midpoint is the central value of a class interval, obtained by finding the average of its lower and upper class limits. (1)

$$x = \frac{60 + 69}{2}$$

$$\boxed{x = 64.5}$$

c) Completed table:

Mass (kg)	f	x	fx
40–49	3	44.5	133.5
50–59	5	54.5	272.5
60–69	8	64.5	516
70–79	6	74.5	447
80–89	3	84.5	253.5
Total	**25**		**1622.5**

- Correct midpoints (2)
- Correct products fx (2)

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$= \frac{1622.5}{25}$$

$$\boxed{\bar{x} = 64.9 \text{ kg}}$$

- Correct substitution (1)
- Correct answer (1)

QUESTION 4: MEDIAN OF GROUPED DATA (8 MARKS)

a) Cumulative frequency:

Age	Frequency	Cumulative frequency
20-24	4	4
25-29	8	12
30-34	12	24
35-39	10	34
40-44	6	40

b) Median class = 30-34. (1)

c) Required values:

- $L = 29.5$
- $cf = 12$
- $f = 12$
- $N = 40$
- $h = 5$

$$\text{Median} = L + \left(\frac{\frac{N}{2} - cf}{f}\right)h$$

$$= 29.5 + \left(\frac{20 - 12}{12}\right) \times 5$$

$$= 29.5 + \frac{8}{12} \times 5$$

$$= 29.5 + 3.333$$

$$\boxed{\text{Median} \approx 32.83 \text{ years}}$$

- Correct formula and substitution (1)
- Correct calculation (1)
- Correct answer with units (1)

QUESTION 5: MODE OF GROUPED DATA (9 MARKS)

a) Modal class is the class interval with the highest frequency. (1)

b) Modal class = 15-19. (1)

c) Required values:

- $L = 14.5$
- $f_1 = 12$
- $f_0 = 7$
- $f_2 = 9$
- $h = 5$

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 14.5 + \left(\frac{12 - 7}{2(12) - 7 - 9}\right) \times 5$$

$$= 14.5 + \left(\frac{5}{24 - 16}\right) \times 5$$

$$= 14.5 + \frac{5}{8} \times 5$$

$$= 14.5 + 3.125$$

$$\boxed{\text{Mode} = 17.625 \text{ litres}}$$

- Correct formula (1)
- Correct substitution (1)
- Correct simplification (1)
- Correct answer with units (1)

QUESTION 6: APPLICATION OF MEASURES OF CENTRAL TENDENCY (9 MARKS)

a) Mean:

$$\bar{x} = \frac{2 + 3 + 3 + 4 + 4 + 5 + 14}{7}$$

$$= \frac{35}{7}$$

$$\boxed{5 \text{ thousand KSh}}$$

Equivalent to KSh 5,000. (2)

b) Median:

The data is already arranged in ascending order.

The middle observation is 4.

$$\boxed{4 \text{ thousand KSh}}$$

c) The modes are 3 and 4 thousand KSh. (1)

d) The mean is higher than the median because the unusually high daily sales value of 14 increases the arithmetic average. (2)

e) Any two:

- To estimate the typical daily sales.
- To plan the amount of stock to purchase.
- To compare sales performance across weeks.
- To identify the most common sales level.
- To plan business expenditure and cash flow.

(1 mark each = 2)

f) The median is recommended because it is less affected by the unusually high sales value of 14 and therefore gives a better indication of typical daily sales. (1)

Total Section B = 50 marks.

13.0 TEACHER'S IMPLEMENTATION GUIDANCE

13.1 Suggested teaching and learning resources

The teacher may use:

- Real objects and measurements from the school environment.

- Measuring tapes and height metres.
- Weighing scales.
- Recording sheets and questionnaires.
- Graph books and mathematical instruments.
- Calculators.
- Computers, tablets and spreadsheet applications.
- Locally available data from farms, markets, schools and health centres.

STRAND 3.0: STATISTICS AND PROBABILITY

SUB-STRAND 3.2: PROBABILITY

1.0 INTRODUCTION TO PROBABILITY

Probability is an important branch of Mathematics that deals with the study of chance and uncertainty. It helps us determine how likely an event is to occur.

In everyday life, people make decisions based on the likelihood of certain events happening. For example:

- A farmer estimates the likelihood of receiving rainfall before planting maize.
- A weather forecaster predicts the possibility of rain.
- A football supporter estimates the chances of a team winning a match.
- A teacher predicts the likelihood of a learner passing an examination.
- A businessperson estimates the possibility of making a profit.
- A family may consider the likelihood of having a boy or a girl when expecting a child.

Probability provides mathematical methods for analysing such situations.

Definition of probability

Probability is a numerical measure of the likelihood or chance that a particular event will occur.

Probability is usually represented by the letter P.

For an event A, its probability is written as:

$$P(A)$$

This is read as the probability of event A.

1.1 Importance of studying probability

Probability is useful in the following ways:

1. **Weather forecasting:** Helps meteorologists estimate the likelihood of rainfall, drought and other weather conditions.

2. **Agriculture:** Helps farmers make decisions about planting, harvesting and managing agricultural risks.
3. **Business:** Helps businesspeople estimate risks and make informed financial decisions.
4. **Sports:** Helps players, coaches and supporters analyse the chances of winning or losing.
5. **Education:** Helps in analysing examination performance and interpreting educational data.
6. **Medicine:** Helps researchers study the likelihood of diseases and the effectiveness of treatments.
7. **Insurance:** Helps insurance companies estimate risks and determine premiums.
8. **Transport:** Helps in estimating the likelihood of accidents and delays.
9. **Government planning:** Helps in analysing population trends and making informed decisions.
10. **Daily decision-making:** Helps people compare possible outcomes before making choices.

1.2 Key terms used in probability

1. Experiment

An experiment is an activity or process that produces one or more possible outcomes.

Examples:

- Tossing a coin.
- Throwing a die.
- Selecting a card from a pack.
- Picking a ball from a bag without looking.
- Recording the gender of a newborn child.

2. Outcome

An outcome is a possible result of an experiment.

For example, when a coin is tossed, the possible outcomes are:

- Head (H)
- Tail (T)

3. Event

An event is a particular outcome or a collection of outcomes of an experiment.

For example, when a die is thrown, getting an even number is an event consisting of the outcomes 2, 4 and 6.

4. Sample space

A sample space is the complete set of all possible outcomes of an experiment.

It is commonly represented by S.

For example, the sample space when throwing a standard six-sided die is:

$$S = \{1,2,3,4,5,6\}$$

5. Favourable outcomes

These are outcomes that satisfy the condition specified in an event.

For example, when throwing a die, the favourable outcomes for getting an even number are:

$$\{2,4,6\}$$

There are three favourable outcomes.

6. Equally likely outcomes

These are outcomes that have the same chance of occurring.

For example, a fair coin has two equally likely outcomes: head and tail.

7. Trial

A trial is one performance of a probability experiment.

For example, tossing a coin once is one trial.

8. Random experiment

A random experiment is an experiment whose exact outcome cannot be predicted with certainty before it is performed, although its possible outcomes are known.

2.0 EXPERIMENTS INVOLVING PROBABILITIES

2.1 Meaning of probability experiments

A probability experiment is an activity performed to investigate chance and observe the possible outcomes of an uncertain event.

In probability experiments, the outcome of an individual trial is uncertain, but the possible outcomes can be identified.

Common probability experiments include:

- Tossing coins.
- Throwing dice.
- Spinning a spinner.
- Drawing coloured balls from a bag.
- Selecting cards from a pack.
- Recording the gender of children in a family.

2.2 Experiment 1: Tossing a coin

A standard fair coin has two sides:

- Head (H)
- Tail (T)

Activity

Materials:

- One fair coin.
- A recording sheet.

- A pen or pencil.

Procedure:

1. Hold a coin in your hand.
2. Toss the coin into the air.
3. Allow it to land on a flat surface.
4. Record whether the outcome is a head or a tail.
5. Repeat the experiment 20 times.
6. Record the results in a frequency table.
7. Calculate the experimental probability of obtaining a head and a tail.

Sample results

Outcome	Frequency
Head (H)	11
Tail (T)	9
Total	20

The experimental probability of obtaining a head is:

$$P(H) = \frac{11}{20} = 0.55$$

The experimental probability of obtaining a tail is:

$$P(T) = \frac{9}{20} = 0.45$$

Notice that the two probabilities add up to 1.

$$0.55 + 0.45 = 1$$

Theoretical probability

For a fair coin, each side is equally likely to occur.

Therefore:

$$P(H) = \frac{1}{2}$$

$$P(T) = \frac{1}{2}$$

Important observation: Experimental probabilities may differ from theoretical probabilities because of random variation. As the number of trials increases, experimental results often become closer to theoretical probabilities.

2.3 Experiment 2: Throwing a die

A standard die has six faces numbered 1 to 6.

The sample space is:

$$S = \{1,2,3,4,5,6\}$$

Consider the following events:

Event A: Obtaining an even number

$$A = \{2,4,6\}$$

There are three favourable outcomes.

Event B: Obtaining an odd number

$$B = \{1,3,5\}$$

There are three favourable outcomes.

Event C: Obtaining a number greater than 4

$$C = \{5,6\}$$

There are two favourable outcomes.

Event D: Obtaining a number less than 7

$$D = \{1,2,3,4,5,6\}$$

All six outcomes are favourable.

Activity

In groups, learners should throw a die 30 times and record the frequency of each number.

Number obtained	Tally	Frequency
1		
2		
3		
4		
5		
6		
Total		30

Discuss whether all numbers occurred equally often.

2.4 Experiment 3: Selecting a coloured ball

Consider a bag containing:

- 3 red balls.
- 2 blue balls.
- 5 green balls.

Total number of balls:

$$3 + 2 + 5 = 10$$

If one ball is selected randomly, the probability of obtaining a red ball is:

$$P(R) = \frac{3}{10}$$

Probability of obtaining a blue ball:

$$P(B) = \frac{2}{10} = \frac{1}{5}$$

Probability of obtaining a green ball:

$$P(G) = \frac{5}{10} = \frac{1}{2}$$

Check:

$$P(R) + P(B) + P(G) = 1$$

$$\frac{3}{10} + \frac{2}{10} + \frac{5}{10} = 1$$

2.5 Experimental and theoretical probability

There are two important ways of determining probability.

A. Theoretical probability

Theoretical probability is the probability calculated using the known possible outcomes of an experiment, assuming the outcomes are equally likely.

The formula is:

$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of equally likely outcomes}}$$

Example 1

A fair die is thrown once. Find the probability of obtaining a number greater than 3.

Solution:

Sample space:

$$S = \{1,2,3,4,5,6\}$$

Favourable outcomes:

$$A = \{4,5,6\}$$

Number of favourable outcomes = 3.

Total number of outcomes = 6.

Therefore:

$$P(A) = \frac{3}{6}$$

$$\boxed{P(A) = \frac{1}{2}}$$

B. Experimental probability

Experimental probability is the probability obtained from the actual results of carrying out an experiment.

Formula:

$$P(A) = \frac{\text{Frequency of event A}}{\text{Total number of trials}}$$

Example 2

A learner tosses a coin 40 times and obtains 23 heads. Determine the experimental probability of obtaining a head.

Solution:

Frequency of heads = 23.

Total trials = 40.

$$P(H) = \frac{23}{40}$$

$$\boxed{P(H) = 0.575}$$

Differences between theoretical and experimental probability

Theoretical probability	Experimental probability
Based on mathematical reasoning.	Based on actual experiments.
Uses possible outcomes.	Uses observed frequencies.
Can be determined without performing the experiment.	Requires performing the experiment.
Assumes the probability model is correct.	May vary from one experiment to another.

3.0 RANGE OF PROBABILITY

3.1 Meaning of the range of probability

The range of probability refers to the possible numerical values that the probability of an event can take.

The probability of any event lies between 0 and 1, inclusive.

$$\boxed{0 \leq P(A) \leq 1}$$

This means that probability cannot be negative or greater than 1.

Probability can be expressed as:

- A fraction.
- A decimal.
- A percentage.

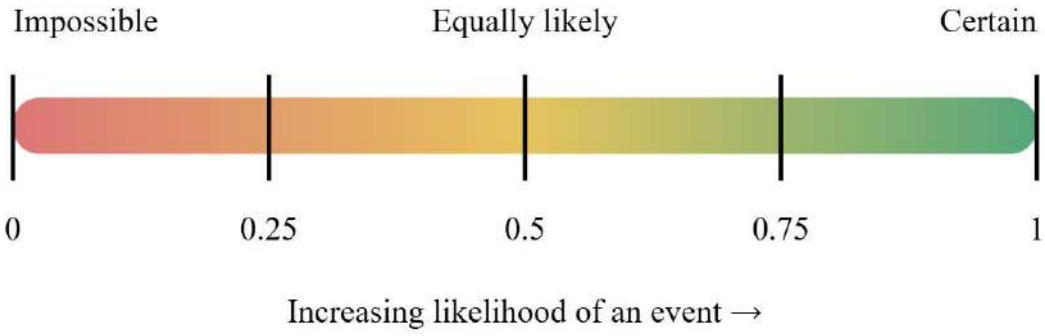
For example:

$$\frac{1}{2} = 0.5 = 50\%$$

3.2 Understanding the probability scale

The probability scale helps us interpret the likelihood of an event.

The probability scale



Increasing likelihood of an event →

Probability	Interpretation
0	Impossible event
0.1	Very unlikely
0.25	Unlikely
0.5	Equally likely to occur or not occur
0.75	Likely
0.9	Very likely
1	Certain event

3.3 Types of events according to probability

1. Impossible event

An impossible event is an event that cannot occur.

Its probability is zero.

$$P(A) = 0$$

Example:

Obtaining 8 when throwing an ordinary six-sided die.

2. Certain event

A certain event is an event that must occur.

Its probability is one.

$$P(A) = 1$$

Example:

Obtaining a number less than 7 when throwing an ordinary six-sided die.

3. Equally likely events

Two events are equally likely if they have the same probability of occurring.

Example:

Obtaining a head or a tail when tossing a fair coin.

$$P(H) = P(T) = \frac{1}{2}$$

4. Likely event

A likely event has a probability greater than one-half but less than one.

Example:

A bag contains 8 red balls and 2 blue balls. Selecting a red ball is likely.

$$P(R) = \frac{8}{10} = 0.8$$

5. Unlikely event

An unlikely event has a probability greater than zero but less than one-half.

Example:

Using the same bag, selecting a blue ball is unlikely.

$$P(B) = \frac{2}{10} = 0.2$$

3.4 Converting probability into fractions, decimals and percentages

Probability can be represented in different forms.

To convert a fraction into a decimal, divide the numerator by the denominator.

To convert a decimal into a percentage, multiply by 100.

To convert a percentage into a decimal, divide by 100.

Example 3

Express the probability $\frac{3}{5}$ as a decimal and a percentage.

Solution:

$$\frac{3}{5} = 0.6$$

Percentage:

$$0.6 \times 100 = 60\%$$

Therefore:

$$\boxed{\frac{3}{5} = 0.6 = 60\%}$$

3.5 Complementary events

The complement of an event is the event that occurs when the original event does not occur.

If A is an event, its complement is written as A' or A^c.

The probability of the complement is:

$$\boxed{P(A') = 1 - P(A)}$$

This is one of the most important probability rules.

Example 4

The probability that a student arrives at school late is $\frac{2}{9}$. Find the probability that the student does not arrive late.

Solution:

$$P(A) = \frac{2}{9}$$

$$P(A') = 1 - \frac{2}{9}$$

$$= \frac{9}{9} - \frac{2}{9}$$

$$\boxed{P(A') = \frac{7}{9}}$$

Example 5

The probability that a farmer's maize crop is destroyed by pests is 0.15. Find the probability that the crop is not destroyed by pests.

Solution:

$$P(\text{not destroyed}) = 1 - 0.15$$

$$\boxed{0.85}$$

3.6 Main properties of probability

1. The probability of an impossible event is zero.
2. The probability of a certain event is one.
3. The probability of any event lies between zero and one.
4. The sum of the probabilities of all possible outcomes in a sample space is one.
5. The probability of an event and its complement adds up to one.
6. A probability can be written as a fraction, decimal or percentage.

4.0 PROBABILITY SPACE

4.1 Meaning of probability space

A probability space is a mathematical description of a random experiment. It consists of all possible outcomes and the probabilities assigned to those outcomes.

At this level, learners mainly work with listing sample spaces and assigning probabilities to outcomes.

A probability space can be represented using:

- Lists.
- Tables.
- Diagrams.
- Sample-space grids.

4.2 Generating a sample space

Generating a sample space means identifying and listing all possible outcomes of a random experiment.

Example 1: Tossing one coin

Possible outcomes:

$$S = \{H, T\}$$

There are two possible outcomes.

Example 2: Throwing one die

$$S = \{1,2,3,4,5,6\}$$

There are six possible outcomes.

Example 3: Tossing two coins

Consider an experiment in which two fair coins are tossed simultaneously.

The possible outcomes are:

$$S = \{HH, HT, TH, TT\}$$

Where:

- HH means both coins show heads.
- HT means the first coin shows head and the second shows tail.
- TH means the first coin shows tail and the second shows head.
- TT means both coins show tails.

There are four possible outcomes.

Each outcome has probability:

$$\frac{1}{4}$$

4.3 Using a table to generate a probability space

Example 4: Tossing two coins

The outcomes can be arranged in a table.

First coin	Second coin	Outcome
H	H	HH
H	T	HT
T	H	TH
T	T	TT

Determine the probability of:

(a) Obtaining two heads

Only HH satisfies the condition.

$$P(\text{two heads}) = \frac{1}{4}$$

(b) Obtaining exactly one head

Favourable outcomes are HT and TH.

$$P(\text{exactly one head}) = \frac{2}{4} = \frac{1}{2}$$

(c) Obtaining at least one head

Favourable outcomes are HH, HT and TH.

$$P(\text{at least one head}) = \frac{3}{4}$$

(d) Obtaining no heads

The only favourable outcome is TT.

$$P(\text{no heads}) = \frac{1}{4}$$

4.4 Sample-space grid for two dice

When two standard dice are thrown, the total number of possible ordered outcomes is:

$$6 \times 6 = 36$$

The table below shows the sum of the numbers obtained.

Die 1 Die 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Example 5

Two dice are thrown. Find the probability that their sum is 7.

The favourable ordered outcomes are:

$$(1,6), (2,5), (3,4),$$

$$(4,3), (5,2), (6,1)$$

Number of favourable outcomes = 6.

Total outcomes = 36.

$$P(\text{sum of 7}) = \frac{6}{36}$$

$$\boxed{\frac{1}{6}}$$

Example 6

Two dice are thrown. Find the probability that their sum is greater than 10.

The favourable outcomes are:

$$(5,6), (6,5), (6,6)$$

There are 3 favourable outcomes.

$$P(\text{sum greater than 10}) = \frac{3}{36}$$

$$\boxed{\frac{1}{12}}$$

4.5 Probability space involving gender

Consider a family with two children. Assume that the probability of having a boy or a girl is equal and that the gender outcomes for the two births are independent.

Let:

- B represent a boy.
- G represent a girl.

The sample space is:

$$S = \{BB, BG, GB, GG\}$$

Outcome	Description	Probability
BB	Two boys	1/4
BG	First boy, second girl	1/4
GB	First girl, second boy	1/4
GG	Two girls	1/4

Probability of having two boys:

$$P(BB) = \frac{1}{4}$$

Probability of having one boy and one girl:

$$P(BG \text{ or } GB) = \frac{2}{4} = \frac{1}{2}$$

Probability of having at least one girl:

$$P(\text{at least one girl}) = \frac{3}{4}$$

Mathematical note: This is an idealised probability model. Real-world birth outcomes are not necessarily exactly 50:50.

4.6 Addition rule of probability

The addition rule is used to determine the probability that event A or event B occurs.

For mutually exclusive events:

$$\boxed{P(A \text{ or } B) = P(A) + P(B)}$$

For events that are not mutually exclusive:

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Where:

- $A \cup B$ means A or B or both.
- $A \cap B$ means both A and B occur.

Example 7

A die is thrown once. Find the probability of obtaining a 2 or a 5.

$$P(2) = \frac{1}{6}$$

$$P(5) = \frac{1}{6}$$

Since the two outcomes cannot occur on the same throw:

$$P(2 \text{ or } 5) = \frac{1}{6} + \frac{1}{6}$$

$$\boxed{\frac{1}{3}}$$

Example 8

A die is thrown once. Find the probability of obtaining an even number or a number greater than 3.

Let:

$$A = \{2,4,6\}$$

$$B = \{4,5,6\}$$

The intersection is:

$$A \cap B = \{4,6\}$$

Therefore:

$$P(A \cup B) = \frac{3}{6} + \frac{3}{6} - \frac{2}{6}$$

$$= \frac{4}{6}$$

$$\boxed{\frac{2}{3}}$$

5.0 MUTUALLY EXCLUSIVE AND INDEPENDENT EVENTS

5.1 Mutually exclusive events

Two or more events are mutually exclusive if they cannot occur at the same time in a single trial of an experiment.

In other words, the occurrence of one event prevents the occurrence of the other event in that trial.

Mathematically:

$$\boxed{P(A \cap B) = 0}$$

Examples of mutually exclusive events

1. Obtaining a head and a tail in a single toss of one coin.
2. Obtaining an even number and an odd number in one throw of a die.
3. Selecting a red ball and a blue ball in a single selection of one ball.
4. A football team winning and losing the same match.
5. A learner obtaining a grade A and a grade E in the same examination under a single grading system.

5.2 Addition rule for mutually exclusive events

For two mutually exclusive events A and B:

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

Worked Example 1

A fair die is thrown once. Find the probability of obtaining an even number or a 5.

Solution:

Even numbers:

$$A = \{2,4,6\}$$

Therefore:

$$P(A) = \frac{3}{6}$$

Getting a 5:

$$B = \{5\}$$

$$P(B) = \frac{1}{6}$$

The events are mutually exclusive.

Therefore:

$$P(A \cup B) = \frac{3}{6} + \frac{1}{6}$$

$$= \frac{4}{6}$$

$$\boxed{\frac{2}{3}}$$

Worked Example 2

A bag contains 4 red balls, 3 green balls and 5 yellow balls. One ball is selected randomly. Find the probability of selecting a red or green ball.

Total balls:

$$4 + 3 + 5 = 12$$

Probability of red:

$$P(R) = \frac{4}{12}$$

Probability of green:

$$P(G) = \frac{3}{12}$$

Since one ball is selected, the events are mutually exclusive.

$$P(R \text{ or } G) = \frac{4}{12} + \frac{3}{12}$$

$$\boxed{\frac{7}{12}}$$

5.3 Independent events

Two events are independent if the occurrence of one event does not affect the probability of the occurrence of the other event.

This means that knowing the result of one event does not change the probability of the other event.

Mathematically:

$$\boxed{P(A \cap B) = P(A) \times P(B)}$$

Examples of independent events

1. Tossing a coin and throwing a die.
2. Tossing a coin twice.
3. Throwing a die twice.
4. Selecting a ball from a bag, replacing it and then selecting another ball.
5. Spinning a fair spinner twice.

5.4 Multiplication rule for independent events

For two independent events A and B:

$$\boxed{P(A \text{ and } B) = P(A) \times P(B)}$$

Worked Example 3

A fair coin is tossed twice. Find the probability of obtaining a head on both tosses.

Solution:

Probability of a head on the first toss:

$$P(H) = \frac{1}{2}$$

Probability of a head on the second toss:

$$P(H) = \frac{1}{2}$$

The tosses are independent.

Therefore:

$$P(HH) = \frac{1}{2} \times \frac{1}{2}$$

$$\boxed{\frac{1}{4}}$$

Worked Example 4

A fair die is thrown and a fair coin is tossed. Find the probability of obtaining a 6 on the die and a head on the coin.

Solution:

$$P(6) = \frac{1}{6}$$

$$P(H) = \frac{1}{2}$$

Therefore:

$$P(6 \text{ and } H) = \frac{1}{6} \times \frac{1}{2}$$

$$\boxed{\frac{1}{12}}$$

5.5 Difference between mutually exclusive and independent events

This distinction is very important in probability.

Mutually exclusive events	Independent events
Cannot occur together in one trial.	Can occur together.
Occurrence of one prevents the other in that trial.	Occurrence of one does not affect the probability of the other.
$P(A \cap B) = 0$.	$P(A \cap B) = P(A)P(B)$.
Addition rule is used.	Multiplication rule is used for both events occurring.
Example: Getting 2 or 5 in one die throw.	Example: Getting a head and a 6 when tossing a coin and throwing a die.

Important mathematical note

Two events with non-zero probabilities cannot be both mutually exclusive and independent. For the usual school probability model, mutually exclusive events are dependent because the occurrence of one rules out the other.

5.6 Dependent events

Two events are dependent when the occurrence of one event changes the probability of the other event.

For example, selecting two balls from a bag without replacing the first ball.

Worked Example 5

A bag contains 4 red balls and 3 blue balls. Two balls are selected one after another without replacement. Find the probability that both balls are red.

Solution:

Total balls = 7.

Probability that the first ball is red:

$$P(R_1) = \frac{4}{7}$$

After selecting a red ball, 3 red balls remain out of 6 balls.

Probability that the second ball is red:

$$P(R_2|R_1) = \frac{3}{6}$$

Therefore:

$$P(R_1 \text{ and } R_2) = \frac{4}{7} \times \frac{3}{6}$$

$$= \frac{12}{42}$$

$$\boxed{\frac{2}{7}}$$

Important: When sampling without replacement, the probabilities change after the first selection.

6.0 PROBABILITY TREE DIAGRAMS

6.1 Meaning of a probability tree diagram

A probability tree diagram is a branching diagram used to represent all possible outcomes of a probability experiment involving successive events.

It shows:

- The possible outcomes at each stage.
- The probability of each outcome.
- The different combinations of outcomes.
- The probability of a particular sequence of events.

Tree diagrams are useful when dealing with two events, especially independent and mutually exclusive events.

6.2 Rules for drawing probability tree diagrams

1. Begin with a single starting point.
2. Draw a branch for every possible outcome of the first event.
3. Write the probability on each branch.
4. From each first branch, draw branches representing possible outcomes of the second event.
5. Write the probabilities of the second event on the branches.
6. Ensure that the probabilities of branches from the same point add up to 1.
7. Multiply probabilities along a particular path to determine the probability of that sequence.
8. Add the probabilities of different paths when they represent the required event.

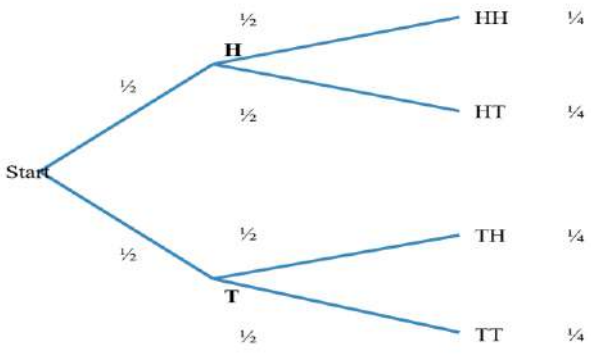
6.3 Tree diagram for tossing a coin twice

Consider tossing a fair coin twice.

Each toss has two possible outcomes:

- Head (H), probability $\frac{1}{2}$.
- Tail (T), probability $\frac{1}{2}$.

Probability tree: Two coin tosses



Each complete path represents one possible outcome of the two tosses.

Sample space:

$$S = \{HH, HT, TH, TT\}$$

Probability of each outcome:

$$P(HH) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(HT) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(TH) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(TT) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Worked Example 1: Probability of getting exactly one head

Favourable outcomes are:

$$HT,\ TH$$

Therefore:

$$P(\text{exactly one head}) = \frac{1}{4} + \frac{1}{4}$$

$$\boxed{\frac{1}{2}}$$

Worked Example 2: Probability of getting at least one head

Favourable outcomes are:

$$HH,\ HT,\ TH$$

Therefore:

$$P(\text{at least one head}) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

$$\boxed{\frac{3}{4}}$$

Worked Example 3: Probability of getting two tails

Only TT is favourable.

$$P(TT) = \frac{1}{2} \times \frac{1}{2}$$

$$\boxed{\frac{1}{4}}$$

6.4 Tree diagram for a coin and a die

A fair coin is tossed and a fair die is thrown.

The coin has two possible outcomes:

- Head (H) = $\frac{1}{2}$
- Tail (T) = $\frac{1}{2}$

The die has six possible outcomes, each with probability $\frac{1}{6}$.

Find the probability of obtaining a head and an even number.

Solution:

$$P(H) = \frac{1}{2}$$

Even numbers on a die are 2, 4 and 6.

$$P(E) = \frac{3}{6} = \frac{1}{2}$$

Since the events are independent:

$$P(H \text{ and } E) = \frac{1}{2} \times \frac{1}{2}$$

$$\boxed{\frac{1}{4}}$$

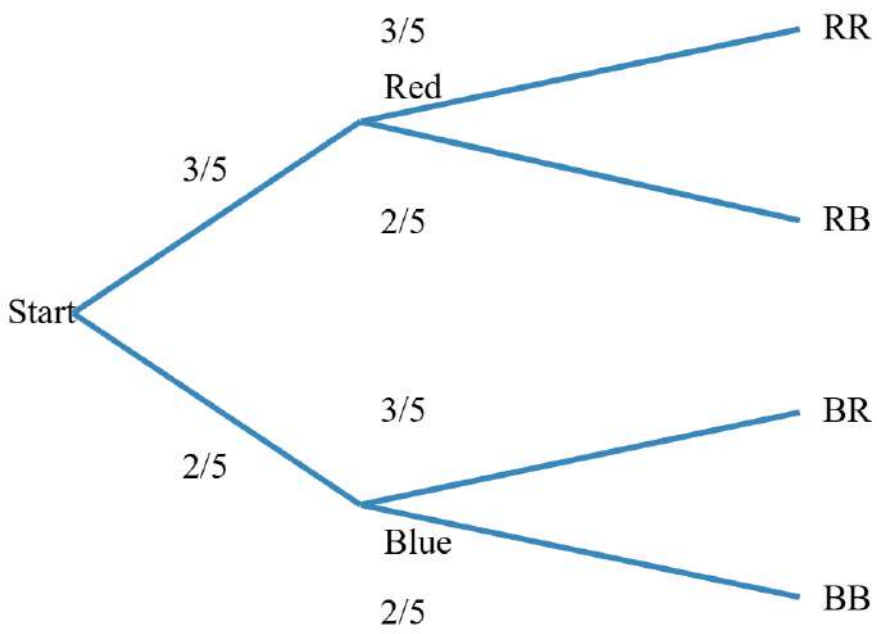
6.5 Tree diagrams involving mutually exclusive outcomes

A tree diagram can also represent successive stages in which the outcomes at each stage are mutually exclusive.

For example, a bag contains 3 red balls and 2 blue balls. One ball is selected and returned before a second selection.

The possible colours at each selection are red and blue.

Tree diagram: Two selections with replacement



Since the first ball is replaced, the probabilities remain unchanged on the second selection.

Probability of selecting two red balls:

$$P(RR) = \frac{3}{5} \times \frac{3}{5}$$

$$\boxed{\frac{9}{25}}$$

Probability of selecting one red and one blue in any order:

$$P(RB) + P(BR)$$
$$= \left(\frac{3}{5} \times \frac{2}{5}\right) + \left(\frac{2}{5} \times \frac{3}{5}\right)$$
$$= \frac{6}{25} + \frac{6}{25}$$
$$\boxed{\frac{12}{25}}$$

6.6 Tree diagrams without replacement

Consider a bag containing 3 red balls and 2 blue balls. Two balls are selected without replacement.

The probability of selecting a red ball first is:

$$P(R_1) = \frac{3}{5}$$

If the first ball is red, the remaining balls are 2 red and 2 blue.

$$P(R_2|R_1) = \frac{2}{4}$$

If the first ball is blue, the remaining balls are 3 red and 1 blue.

$$P(R_2|B_1) = \frac{3}{4}$$

Worked Example 4

Find the probability of selecting two red balls without replacement.

$$P(RR) = \frac{3}{5} \times \frac{2}{4}$$
$$= \frac{6}{20}$$
$$\boxed{\frac{3}{10}}$$

Worked Example 5

Find the probability of selecting one red and one blue ball.

There are two possible paths:

- Red then blue (RB).
- Blue then red (BR).

$$P(RB) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20}$$
$$P(BR) = \frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$$

Therefore:

$$P(RB \text{ or } BR) = \frac{6}{20} + \frac{6}{20}$$
$$\boxed{\frac{3}{5}}$$

Important points when using tree diagrams

- Multiply probabilities along branches of the same path.
- Add probabilities of separate paths representing the same required event.
- When there is replacement, the probabilities normally remain unchanged.
- When there is no replacement, the total number of items decreases after each selection.
- The probabilities of all final complete outcomes must add up to one.

7.0 APPLICATION OF PROBABILITY IN REAL-LIFE SITUATIONS

Probability is widely used in different areas of society. It enables people to understand uncertainty and make better decisions.

7.1 Application in agriculture

Farmers use probability when making decisions about crop production and livestock management.

Examples:

1. Estimating the probability of rainfall before planting.
2. Predicting the likelihood of pest and disease attacks.
3. Estimating the possibility of successful seed germination.
4. Planning irrigation based on rainfall patterns.
5. Estimating the chances of livestock producing healthy offspring.
6. Assessing the risk of crop failure.

Worked Example

A farmer observes that rainfall occurs on 18 out of 30 days during a particular period. Estimate the experimental probability of rainfall on a randomly selected day from that period.

Solution:

$$P(\text{rain}) = \frac{18}{30}$$

$$\boxed{0.6 \text{ or } 60\%}$$

This is an experimental estimate, not a guarantee of future rainfall.

7.2 Application in weather forecasting

Meteorologists use probability to communicate the likelihood of weather conditions.

Examples:

- Probability of rainfall.
- Probability of thunderstorms.
- Probability of strong winds.
- Probability of extreme temperatures.

If the probability of rainfall is 70%, it means that the forecast assigns a 70% chance to rainfall under the forecasting model and specified conditions.

It does not mean that it will rain for 70% of the day.

7.3 Application in sports

Probability helps sportspeople analyse performance and possible results.

Examples:

1. Estimating the probability of a football team winning a match.
2. Analysing the likelihood of scoring a penalty.
3. Predicting the possible results of a competition.
4. Evaluating performance trends in athletics.
5. Comparing players' performance records.

Example

A football team wins 12 matches out of 20 matches played.

Experimental probability of winning:

$$P(\text{win}) = \frac{12}{20}$$

$$\boxed{0.6 = 60\%}$$

7.4 Application in education

Probability can help schools analyse learning outcomes.

Examples:

- Estimating the probability of a learner passing an examination based on a defined data model.
- Analysing attendance patterns.
- Predicting participation in school activities.
- Evaluating assessment performance.

7.5 Application in health

Probability is used in medical research and public health.

Examples:

1. Estimating disease occurrence in a population.
2. Studying the likelihood of treatment success.
3. Analysing the risk of disease transmission.
4. Evaluating the effectiveness of vaccination programmes.
5. Planning health resources.

7.6 Application in business and finance

Businesspeople use probability to make decisions under uncertainty.

Examples:

- Estimating the likelihood of profit or loss.
- Analysing customer demand.
- Predicting stock shortages.
- Assessing financial risks.

- Planning insurance policies.

7.7 Application in transport

Probability is useful in transport management.

Examples:

- Estimating the possibility of road accidents.
- Analysing traffic congestion.
- Predicting transport delays.
- Assessing the reliability of public transport.

7.8 Application in games and entertainment

Probability is used in:

- Board games.
- Card games.
- Dice games.
- Spinning games.
- Lotteries.

It helps players understand the chances of obtaining particular results.

7.9 Ethical use of probability

Probability should be used responsibly.

1. Probabilities are estimates and should not always be treated as certainties.
2. Decisions should consider reliable information.
3. Probability should not be used to mislead people.
4. In health and education, predictions should be interpreted carefully.
5. People should understand that a likely event is not guaranteed to occur.

8.0 SUMMARY OF IMPORTANT PROBABILITY FORMULAE

Concept	Formula
Basic probability	$P(A) = \frac{n(A)}{n(S)}$
Range of probability	$0 \leq P(A) \leq 1$
Certain event	$P(S) = 1$
Impossible event	$P(\varnothing) = 0$

Concept	Formula
Complement	$P(A') = 1 - P(A)$
Experimental probability	$\frac{\text{Frequency of event}}{\text{Total trials}}$
Mutually exclusive events	$P(A \cup B) = P(A) + P(B)$
General addition rule	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Independent events	$P(A \cap B) = P(A) \times P(B)$
Dependent events	$P(A \cap B) = P(A) \times P(B \mid A)$

Where:

- $n(A)$ represents the number of favourable outcomes.
- $n(S)$ represents the total number of equally likely outcomes.
- $P(A')$ represents the probability that A does not occur.
- $P(A \cap B)$ represents the probability that both A and B occur.
- $P(A \cup B)$ represents the probability that A or B or both occur.
- $P(B \mid A)$ represents the probability of B occurring given that A has occurred.

REVISION EXERCISES

Strand: Statistics and Probability
Sub-strand: Probability
Total Marks: 80

Instructions to learners

1. Answer all questions in Section A.
2. Answer all questions in Section B.
3. Show all necessary working in Section B.

4. Give probability answers in their simplest fractional form unless otherwise instructed.
5. Use appropriate mathematical diagrams where necessary.

SECTION A: MULTIPLE-CHOICE QUESTIONS (30 MARKS)

Choose the correct answer from the alternatives A, B, C and D.

1. What is probability?

A. The study of shapes and sizes.
B. The numerical measure of the likelihood of an event occurring.
C. The study of numbers and operations.
D. The process of measuring distances.

2. Which of the following is a random experiment?

A. Adding 5 and 4.
B. Measuring the length of a ruler.
C. Tossing a coin and observing the outcome.
D. Multiplying 6 by 3.

3. A fair coin is tossed once. What is the sample space?

A. {1, 2}
B. {H, T}
C. {H, H}
D. {T, T}

4. Which of the following represents the probability of an impossible event?

A. 1
B. 0.5
C. 0
D. 2

5. The probability of a certain event is:

A. 0
B. 0.25
C. 0.5
D. 1

6. Which of the following is the correct range of probability?

A. $P(A) < 0$
B. $0 \leq P(A) \leq 1$
C. $P(A) > 1$
D. $-1 \leq P(A) \leq 0$

7. A fair die is thrown once. What is the probability of obtaining a 4?

A. $\frac{1}{2}$
B. $\frac{1}{3}$
C. $\frac{1}{6}$
D. $\frac{2}{3}$

8. A fair die is thrown once. What is the probability of obtaining an even number?

A. $\frac{1}{6}$
B. $\frac{1}{3}$

C. $\frac{1}{2}$
D. $\frac{2}{3}$

9. A bag contains 3 red balls and 7 blue balls. What is the probability of selecting a red ball?

A. $\frac{7}{10}$
B. $\frac{3}{10}$
C. $\frac{1}{3}$
D. $\frac{1}{2}$

10. The probability of an event is 0.35. What is the probability of the event not occurring?

A. 0.35
B. 0.45
C. 0.55
D. 0.65

11. A coin is tossed 50 times and a head occurs 28 times. What is the experimental probability of obtaining a head?

A. $\frac{28}{50}$
B. $\frac{22}{50}$
C. $\frac{50}{28}$
D. $\frac{1}{2}$

12. Which of the following pairs represents mutually exclusive events when one die is thrown?

A. Getting an even number and getting a 4.
B. Getting an odd number and getting a 3.
C. Getting a 2 and getting a 5.
D. Getting a number greater than 2 and getting a 6.

13. Two events are mutually exclusive if:

A. They always occur together.
B. They cannot occur at the same time in one trial.
C. They have equal probabilities.
D. They are independent.

14. Two events A and B are independent if:

A. One event prevents the other from occurring.
B. Both events have probability zero.
C. The occurrence of one does not affect the probability of the other.
D. Both events must occur simultaneously.

15. Which formula represents the probability of two independent events occurring together?

A. $P(A) + P(B)$
B. $P(A) - P(B)$
C. $P(A) \times P(B)$
D. $P(A) \div P(B)$

16. A fair coin is tossed twice. What is the probability of obtaining two tails?

A. $\frac{1}{2}$
B. $\frac{1}{4}$
C. $\frac{3}{4}$
D. 1

17. A coin is tossed twice. What is the probability of obtaining exactly one head?

A. $\frac{1}{4}$
B. $\frac{3}{4}$
C. $\frac{1}{2}$
D. 1

18. A fair die is thrown once. What is the probability of obtaining a number greater than 4?

A. $\frac{1}{6}$
B. $\frac{1}{3}$
C. $\frac{1}{2}$
D. $\frac{2}{3}$

19. A bag contains 5 green balls and 5 yellow balls. What is the probability of selecting a green ball?

A. 0.2
B. 0.4
C. 0.5
D. 0.8

20. Which of the following is the complement of obtaining an even number when throwing a die?

A. Obtaining a prime number.
B. Obtaining an odd number.
C. Obtaining a number greater than 3.
D. Obtaining a number less than 5.

21. A probability of 75% is equivalent to:

A. 0.075
B. 0.75
C. 7.5
D. 75

22. Two dice are thrown simultaneously. How many possible ordered outcomes are there?

A. 6
B. 12
C. 24
D. 36

23. A tree diagram is mainly used to:

A. Measure angles.
B. Show successive possible outcomes of an experiment.
C. Calculate areas of shapes.
D. Draw straight lines.

24. In a probability tree diagram, probabilities along one complete path are:

A. Added.
B. Subtracted.
C. Multiplied.
D. Divided.

25. When a ball is selected from a bag, replaced and another ball selected, the events are generally:

A. Mutually exclusive.
B. Independent.
C. Impossible.
D. Certain.

26. A bag contains 4 red balls and 6 blue balls. One ball is selected and not replaced. What is the probability of selecting a second red ball if the first ball was red?

A. $\frac{4}{10}$
B. $\frac{3}{9}$
C. $\frac{4}{9}$
D. $\frac{6}{9}$

27. If $P(A) = 0.4$, find $P(A')$.

A. 0.2
B. 0.4
C. 0.6
D. 1.4

28. Which of the following is an application of probability in agriculture?

A. Calculating the perimeter of a farm.
B. Predicting the likelihood of rainfall.
C. Drawing a farm boundary.
D. Measuring the height of a tree.

29. A football team wins 15 matches out of 25 matches played. What is its experimental probability of winning?

A. 0.25
B. 0.40
C. 0.60
D. 0.75

30. Two mutually exclusive events A and B have probabilities 0.3 and 0.4 respectively. Find the probability that A or B occurs.

A. 0.1
B. 0.7
C. 0.12
D. 1.2

SECTION B: STRUCTURED QUESTIONS (50 MARKS)

Answer ALL questions. Show your working clearly.

QUESTION 1: BASIC PROBABILITY AND SAMPLE SPACE (8 MARKS)

(a) Define the following terms:

(b) Random experiment. (1 mark)

(ii) Sample space. (1 mark)

(b) A fair six-sided die is thrown once.

(c) Write down the sample space. (1 mark)

(ii) Determine the probability of obtaining an odd number. (2 marks)

(iii) Determine the probability of obtaining a number greater than 4. (1 mark)

(iv) A fair coin is tossed twice.

(v) List all the possible outcomes. (1 mark)

(vi) Determine the probability of obtaining two heads. (1 mark)

QUESTION 2: RANGE AND COMPLEMENTARY PROBABILITY (8 MARKS)

(a) State the range of probability. (1 mark)

(b) A farmer estimates that the probability of rainfall on a particular day is 0.65.

Determine:

(i) The probability that it will not rain. (2 marks)

(ii) Express the probability of rainfall as a percentage. (1 mark)

(iii) A bag contains 5 red balls, 4 green balls and 3 yellow balls. One ball is selected randomly.

Calculate the probability of selecting:

(i) A green ball. (1 mark)

(ii) A red or yellow ball. (2 marks)

(iii) A ball that is not red. (1 mark)

QUESTION 3: EXPERIMENTAL PROBABILITY (8 MARKS)

A learner tosses a fair coin 60 times and records the following results:

Outcome	Frequency
Head	34
Tail	26
Total	60

(a) Calculate the experimental probability of obtaining:

(b) A head. (2 marks)

(ii) A tail. (2 marks)

(b) Explain why the experimental probability of obtaining a head may differ from the theoretical probability. (2 marks)

(c) State the theoretical probability of obtaining a tail when a fair coin is tossed. (1 mark)

(d) What happens to experimental probability as the number of trials increases significantly? (1 mark)

QUESTION 4: MUTUALLY EXCLUSIVE AND INDEPENDENT EVENTS (8 MARKS)

(a) Distinguish between mutually exclusive events and independent events. (2 marks)

(b) A fair die is thrown once.

Determine the probability of obtaining:

(i) An even number or a 5. (2 marks)

(ii) A number less than 3 or a 6. (1 mark)

(iii) A fair coin is tossed and a fair die is thrown.

Calculate the probability of obtaining:

(i) A head and a 4. (2 marks)

(ii) A tail and an odd number. (1 mark)

QUESTION 5: PROBABILITY TREE DIAGRAMS (9 MARKS)

A bag contains 3 red balls and 2 blue balls. Two balls are selected one after another without replacement.

(a) Draw a probability tree diagram showing all possible outcomes and the probabilities of each branch. (4 marks)

(b) Calculate the probability of selecting:

(c) Two red balls. (2 marks)

(ii) Two blue balls. (1 mark)

(iii) One red ball and one blue ball in any order. (2 marks)

QUESTION 6: APPLICATION OF PROBABILITY (9 MARKS)

A school conducted a survey involving 100 learners to establish their preferred co-curricular activities. The results were as follows:

Activity	Number of learners
Football	40
Athletics	25
Volleyball	20
Music	15
Total	100

A learner is selected randomly from the group.

(a) Determine the probability that the learner prefers:

(b) Football. (1 mark)

(ii) Athletics. (1 mark)

(iii) Music. (1 mark)

(b) Calculate the probability that the learner prefers football or volleyball. (2 marks)

(c) Calculate the probability that the learner does not prefer football. (2 marks)

(d) Explain two ways in which probability can be applied in real-life situations in Kenya. (2 marks)

11.0 SUGGESTED PRACTICAL CLASSROOM ACTIVITIES

These activities help learners develop mathematical reasoning, collaboration, communication and problem-solving skills.

Activity 1: Coin-toss experiment

Learners work in groups to toss coins 50 times, record the outcomes and calculate experimental probabilities.

Activity 2: Dice experiment

Learners throw dice, record the results and compare the probabilities of obtaining odd, even and prime numbers.

Activity 3: Coloured-ball experiment

Learners select coloured balls from a bag and investigate probability with and without replacement.

Activity 4: Tree diagram construction

Learners construct probability tree diagrams for two successive coin tosses and two ball selections.